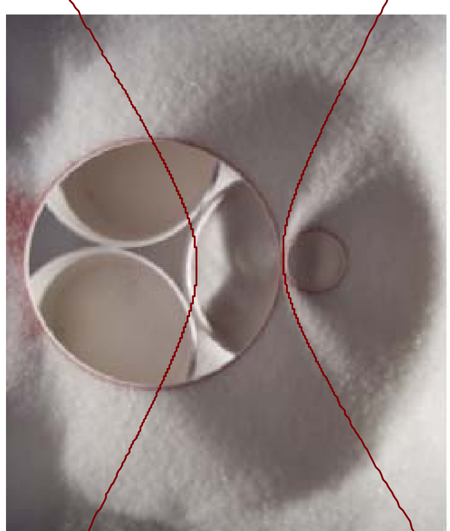
Utah Mathematics Teacher

Fall 2008

The Geometry of Piles of Salt



http://www.uctmonline.org/

Utah Council of Teachers of Mathematics

2008-2009 Mission and Membership

MISSION: To promote quality teaching and learning of mathematics in Utah, UCTM will:

- Provide essential opportunities for professional collaboration among mathematics educators and advocates.
- Communicate and publicly advocate for mathematics education in Utah.
- Recognize and honor exemplary performance in mathematics by Utah students and teachers.

MEMBERSHIP: Create an account (uctmonline.org) to receive a FREE e-membership to UCTM with the following benefits:

- Monthly emails with important information about Utah mathematics education including
- Conferences
- Latest web resources
- Professional Development opportunities
- Updates from the State Office of Education
- Access to monthly web discussions and conversations about ongoing topics of interest (see Forums)
- More… (please feel free to suggest your ideas)

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Letter from UCTM President

Utah Mathematics Educators,

Welcome to the first issue of <u>The Utah Mathematics Teacher</u>, a professional journal for Utah mathematics educators produced by the Utah Council of Teachers of Mathematics (UCTM). This journal features research and vision from Utah mathematics leaders to advance our work as mathematics educators in a rapidly changing world.

We invite you to join with us to read the journal, dialogue, and collaborate professionally toward providing meaningful, challenging, and worthwhile mathematics for ALL students in Utah. Please consider the following ways you might participate in the UCTM network.

- ***Read and discuss the UCTM journal** in your school and professional development teams. Recognize and value the leadership and professionalism within our Utah mathematics education network.*
- ***Promote UCTM e-membership** (no cost) among your local colleagues. Imagine the advantage of having 1850 secondary mathematics teachers and all elementary teachers networked, collaborating and accessing up-to-date resources and information. Create a new account at http://uctmonline.org/.*
- ***Attend the UCTM fall conference**. Access the best ideas from mathematics educators across the state. Present your exemplary instructional and curricular knowledge and strategies at the conference.*
- ***Nominate exemplary mathematics teachers for UCTM awards**. Help us promote excellent mathematics educators. UCTM will publicize these awards in local as well as prominent newspapers.*
- ***Join with UCTM to focus on research.** Research is key to informing our practice. We can learn about pairing or teaming with university colleagues in order to collect and analyze data about student learning.*
- ***Join in the ongoing UCTM forum discussions via uctmonline.org.** Ask and respond to current topics such as writing and receiving grants for the mathematics classroom, formative assessment, new teacher issues, technology integration, and engaging students in reasoning and sense-making.*

We welcome your suggestions for how UCTM might better serve the mathematics community in Utah. Please contact a board member with questions, suggestions, or comments.

Sincerely,

Camille Baker
UCTM President

UCTM Fall Keynote – Johnny Lott

Reasoning and Proof or Lack of Reasoning and Proof:

Where Do Your Students Fit?

Johnny W. Lott
University of Mississippi

From *Standards and Curriculum: A View from the Nation: A Joint Report by the National Council of Teachers of Mathematics and the Association of State Supervisors of Mathematics* (Lott &Nishimura, 2004), we know that there has been little consistency in state standards concerning the teaching of reasoning and proof in schools. In the high school geometry course under the standard of reasoning in 2004, the following generally agreed upon standards were found:

- Use inductive reasoning to make conjectures and deductive reasoning to justify conclusions
- Use of/justify theorems related to congruence
- Use of/justify theorems related to properties of parallel and perpendicular lines
- Use of/justify theorems related to relationships among, chords and secants,
- Use of/justify theorems

These standards were only common to about 10 states and give no clear picture of an overall national picture on reasoning and proof. Proof as a separate standard was rarely found in the 2004 survey. In 2008, the National Council of Teachers of Mathematics (NCTM) drafted and released for public comment a document entitled *Focus in High School Mathematics: Reasoning and Sense Making, Public Draft*. This document (expected to be in final print in April 2009) is to be a fundamental part of a series about high school mathematics and what is important for students. With reasoning and sense making and their implications for proof to be a part of the fundamental cornerstone of secondary school mathematics, it is important to have an understanding of students' knowledge of reasoning and proof today. The purpose of this paper is to consider where students are and where NCTM might want them to be in the future.

Where students are

Research (Clements & Battista, 1992; Senk, 1989) has shown us that most students leave high school at van Hiele levels 2 or 3 as described below:

- Level 2--Descriptive/analytic: They recognize shape characteristics and identify properties of shape families.
- Level 3--Abstract/relational: They form abstract definitions and provide informal arguments to justify classifications

And Hoffer (1983) reported that students' levels of development do not match with the expected level of most textbooks.

Further as reported by McCrone in a presentation to the Knoxville MathFest (2006), students often interpret drawings or diagrams as providing general proofs (Chazan, 1993; Martin et al., 2005). Only about 30% of all students in proof-based geometry courses achieve 75% mastery in proof writing (Senk, 1985). Students who enter geometry at or near van Hiele level 3 are most successful at proof writing after taking a geometry course (Senk, 1989). And classroom opportunities to conjecture, write and critique proofs make a difference in learning proof writing (Martin et al., 2005). McCrone summarized by saying that proof-based courses may be neither appropriate nor beneficial to most students.

Where students might be

With this background, NCTM is recommending in its draft document: "A high school curriculum that focuses on reasoning and sense making will help to satisfy the increasing demand for scientists, engineers, and mathematicians while preparing students for whatever professional, vocational, or technical needs may arise" (p. 2). To this end, NCTM has described reasoning as taking "many forms, ranging from informal explanation and justification to formal deduction, as well as inductive observations" (p. 3). Similarly sense making is described as "developing understanding of a situation, context, or concept by connecting it with existing knowledge" (p. 3)" And it appears that NCTM sees reasoning and sense making as being intertwined from informal setting to more formal proof-type settings.

With these understandings, there are implications for the classroom and instruction. The view of mathematics may be more global than in much of the current state of assigning reasoning and proof to a geometry course. Reasoning and sense making will be viewed consistently across the entire mathematics spectrum, students will be actively involved in reasoning and sense making from early grades through their entire mathematical careers, all students will have the opportunity to experience this type of study (and not just the college-bound set), curriculum may need to be more aligned from preK-16, and high stakes testing will reflect the importance of both reasoning and sense making.

Whether NCTM's proposed recommendations come to fruition remains to be seen, but it is clear that students are not achieving at high levels of reasoning, sense making or writing proofs. If changes are to be made along the lines suggested by NCTM, it will be necessary for all involved with mathematics and mathematics education to consider what changes are needed at all levels.

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Voices from the Classroom

The Geometry of Piles of Salt

Troy Jones

Willowcreek Middle School

Introduction

A few summers ago I attended a math workshop where participants were encouraged to share some of their teaching ideas. A professor from Brazil shared some ideas that forever changed the way I look at the world. I'll never forget the exhilaration I experienced as I predicted in my mind what would happen next as he performed his experiments. French novelist and philosopher Marcel Proust (1871-1922) penned these sentiments which captured the way I felt: "The real voyage of discovery lies not in finding new lands, but in seeing with new eyes."

As I have investigated his ideas, and come up with proofs of each case, I have had to research many topics in order to tie loose ends together. I have experienced much satisfaction in this process. Although I will not supply the proofs in this article, I would encourage those interested to prove the propositions put forth in each experiment.

A Few Preliminaries

When granular material is poured on a surface from a fixed source, it begins to form a conical pile. The angle that the surface of the pile of granular material makes with horizontal is referred to as the angle of repose. The *angle of repose* is the minimum angle at which this granular material can no longer support itself, and will flow under the influence of gravity. (The term "granular" covers a wide range, since even large boulders that accumulate at the foot of a mountain have an angle of repose, and a rockslide or avalanche occurs if this angle is exceeded.)

The steepness of the angle of repose is affected by such properties as the size and angularity of the grains, density of the grains, cohesion between the grains (due to electrostatic energy, magnetism, water film, etc.), substrate roughness, shear-stress, and gravity. Ordinary table salt has an angle of repose of about 32°.

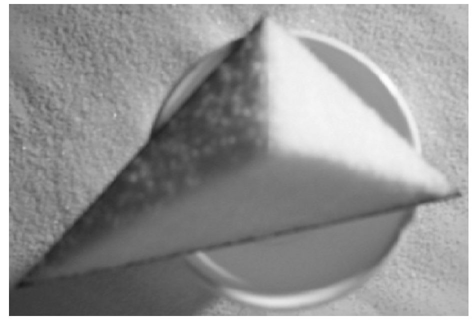
The Method of Investigation

In order to perform these experiments with students, you will need several boxes of table salt which you can sometimes find on sale 3 for a dollar, and several paper cups. I like to have2 sheets of poster paper or butcher paper on the table to catch salt and then lift it back up and funnel it into the paper cups to reuse on the subsequent trials.

You will need to cut out of cardboard or other stiff material various polygons and other circular shapes as described later on in this article. I like to keep the shapes fairly small so as not to need too much salt to cover them completely. There are several layers of mathematical sophistication that can be pointed out in each experiment, depending on the background of the students.

First, after placing the two sheets of poster paper on the table, one on top of the other, I place one or more paper cups on the poster paper and then suspend one of the polygons on the paper cups. I begin pouring the salt on the polygon until the shape is completely filled with salt and it begins to slide off the edges of the polygon into the cups and onto the paper. As you are beginning to pour, and throughout the experiment, you should be asking students to make conjectures and predict what will happen. As their predictions are either verified or contradicted, you should try to have students explain what is happening and guide them into discovering as much of the mathematics as their background allows.

Triangle Shape



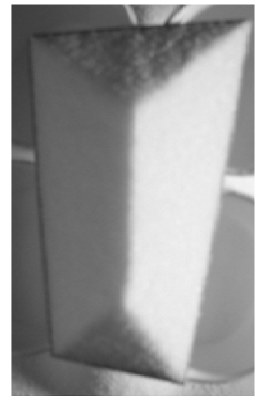
As the salt begins to fill the fill the interior of the triangle and then slide off the edges, ridgelines begin to form. These ridgelines appear to all meet at a point, forming a triangular pyramid (see figure 1). It is interesting to try to guess what this point is where the ridgelines meet. As is the case with all of these experiments, an observer looking straight down on the salt pile could imagine projecting the ridgelines and points directly down to the base figure. Then the ridgelines and points can be talked about in a two-dimensional environment.

The most common points of concurrency discussed in a geometry class are the centroid, incenter, circumcenter, and orthocenter. The centroid is the point of concurrency of the medians, the segments connecting the midpoint of a side with the opposite vertex. The incenter is the point of concurrency of the angle bisectors. The circumcenter is where the perpendicular bisectors of the sides concur. And the altitudes of a triangle concur at the orthocenter.

Look at the physical model of what is happening to verify the geometry. As the salt slides off the edges of the triangle, the ridgelines stabilize at a location that is equidistant from the edges of the triangle. In a triangle the line that is equidistant from two sides of a triangle is the bisector of the angle formed by those two sides. Therefore, each ridge line of the pyramid, when projected down to the base triangle, is an angle bisector. The angle bisectors meet at the incenter, the point that is equidistant from the sides of the triangle. So the peak of the pyramid is directly above the incenter of the triangular base.

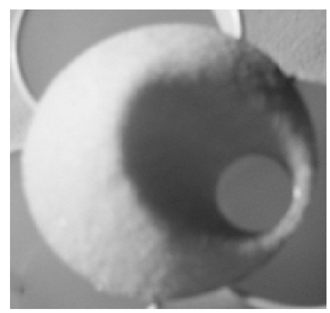
Quadrilateral Shape

When salt is poured on an arbitrary quadrilateral shape, ridgelines are also formed, but they do not necessarily all meet at a single point (see figure 2). As with the triangle, we can see that the ridgelines of the quadrilateral originating from each vertex are angle bisector of the angles at those vertices. The ridgelines originating from the two closest vertices meet at a point. The ridgelines originating from the next pair of closest vertices also meet at a point. These two points are then connected by another ridgeline. After some thought, it can be determined that this new ridgeline is equidistant from the two closest sides of the quadrilateral, and is therefore the angle bisector of the angle formed by extending these two sides until they meet. It is an interesting investigation to determine which types of quadrilaterals have ridgelines that all concur at a single point.

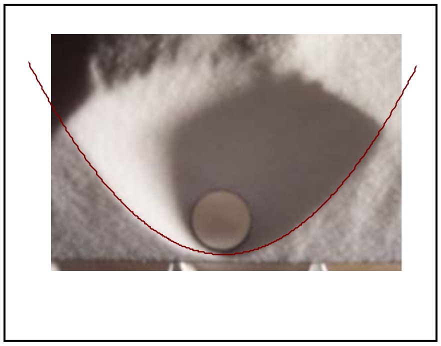


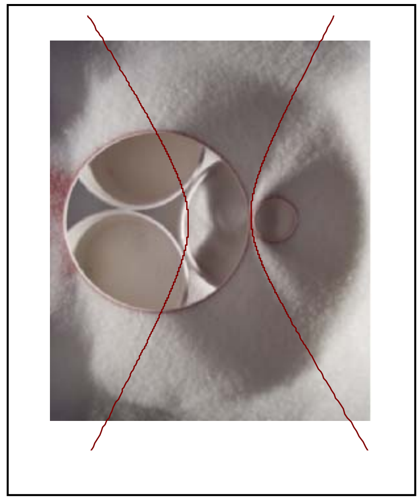
Circular Shapes

If a circle is cut out of cardboard, and then another circle is cut out of the interior of the circle, interesting investigations can be performed. The inner circle can be concentric with the outer circle, or offset towards the edge. As salt is poured on the circular shape and slides off the edges of the outer and inner circle, a ridgeline is formed that is equidistant from both edges (see figure 3). It is interesting to prove from the geometry of this model that the ridgeline is actually an ellipse.



Both of the other conic sections, the parabola and the hyperbola, can also be modeled with salt piles. I have attempted to take photographs where the lighting and shading emphasize the ridgelines. The photographs are imported into Geometer's Sketchpad where a construction can be manipulated to fit on top of the photographs.





The difficulty with modeling the parabola and the hyperbola is that both of these curves are infinite. Figure 4 shows a portion of the parabola, while figure 5 shows a portion of the hyperbola. Because we can't use an infinite amount of salt, secondary ridgelines begin to form where we taper off the salt pile.

Conclusion

These salt piles provide a rich source of investigation into both simple and deep geometric relationships. While this article is not long enough to contain the proofs of the proposed phenomena, nor a more detailed explanation of my procedures, I would encourage interested readers to investigate these relationships and provide the proofs for themselves. I would be happy to be a resource to anyone wanting to know more on these topics.

New Teacher Corner

The Secrets to a Great Start
Nicole Robertson

Timberline Middle School

Two years ago I received my first offer to be a full-time teacher. As I hung up the phone, I pictured my first classroom. There I was in the front of a classroom of perfect students, all with their hands raised, anxious to answer my every query; each one perfectly prepared and eager for me to dispense the wisdom and knowledge that would guide them through the rest of their lives. Little did I know that while there are many students eager to learn, there are also many parents to call, papers to grade, assessments to evaluate, amazing lessons to plan, meetings to attend, and a life balance to attain, administrative work to complete, remediation to conduct, and all by the end of that first day. After *surviving* the last few years, here is what I have learned:

- **Simplify** – Simplify your grading, your time, your class structure, and your routines. Some of the things I have done to simplify my life and classroom are:
  - Have the students grade each other's papers. This not only cuts down on the amount of things that you have to do, but also allows the students to see what the correct answers are.
  - Set up your class structure at the beginning of the year and make expectations very clear. If you want them to put their papers in a specific place, make sure they know the location from the beginning.
  - Label things around the room. This cuts down on constant repetition and direction.
  - Establish a simple routine from the beginning starting with: when you input grades into the computer, grade quizzes, call parents, and plan lessons.
  - If calling parents seems too time consuming, use e-mail and make a weekly goal for how many parents you will contact.
- **Get Help** – If you have a student aide, let them input grades on a paper copy of your grade book. If you do not have that luxury, ask for parent volunteers at the beginning of the year to come in once or twice a week during school.

- **Start Small –** Begin with simple straightforward lessons. Use the time before you begin teaching, (summer or winter breaks) to plan at least your first full unit or chapter. You will appreciate the time that this pre-planning frees so you can establish routines in your classroom. As the year progresses, improve on one unit at time. Do not think that every lesson must be flashy and impressive. Stick with interesting and understandable, then improve as you go.
- **Steal, Borrow, and Learn From Others –** Use your Professional Learning Communities to learn about and use new and amazing ideas that have been tried and tested in the classroom. Ask questions to find where your students may have misconceptions. Ask for copies of lessons and activities from those who are willing to help. (They were once first year teachers too!)
- **Strive for More Than Survival –** Pre-plan, preview, and prepare. Know the end from the beginning. Look for multiple ways to explain something so that if a question arises you can be a little more prepared. Be flexible and look for those teaching moments that can be clarifying but have a general plan of what you need to cover by certain dates in the year. Even being planned a few days in advance is an accomplishment. Do not get discouraged when you have a horrible day. Look at each of those as a day of learning and move on.
- **Have Fun –** Kids are amazing and very fun to be around. Take those moments when the student says something that is on topic but completely random to laugh along with the class. As teachers we have one of the best jobs in the world and we get paid to do it. Enjoy your time in the classroom.

Many things must be learned once you are in your own classroom. Remember this profession has many rewards. It helps to record weekly (if not daily) all the neat, funny, and educational experiences that you have in these first few years of teaching. As long as you keep teaching simple, start small, steal, borrow, learn from others, strive for more than survival, and have fun you can be successful in your first years of teaching. Good Luck.

Oh, and by the way, one day it happened, I was teaching and had that great moment where every student was participating and enjoying math. They asked applicable questions, and when that one student you know struggles with everything, finally feels complete success, the feeling you experience as a teacher is indescribable. I live for days like that one.

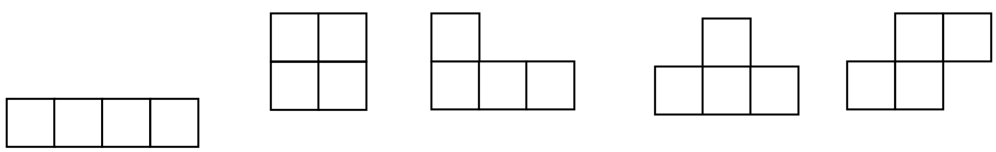
Elementary Corner

Tumbling Tetrominoes

April Leder

Alpine School District Grades 3-4 Mathematics Specialist

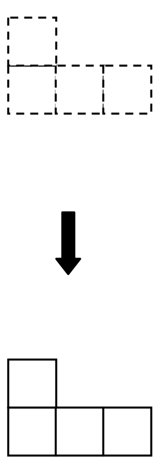
Geometry is more than naming shapes. This game provides students with the opportunity to experience the movement of two-dimensional shapes called tetrominoes. Tetrominoes are arrangements of four squares with full sides touching. There are only five possible tetrominoes.

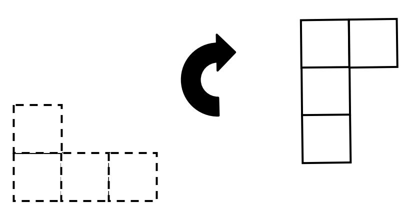


Learning goals, rationale, and pedagogical context

In this task students will be investigating various transformations. These include slides (translations), flips (reflections), and turns (rotations) Students work with these motions as they fit shapes together to cover a particular area. In addition, they will investigate ideas about patterns, congruence, and area.

by sliding: by flipping: and by turning:





This activity addresses the Utah Elementary Mathematics Curriculum in third, fourth, and sixth grades. It meets Standard III, Objective 2 a. in third grade, Standard III, Objective 3 a. in fourth grade if they identify what type of transformation they are completing, and is a precursor for Standard III, Objective 2 a. & b. in sixth grade.

Objectives

The students will-

- Visualize how different shapes can be moved to fit in a determined space
- Measure area by finding the number of square units that cover a flat surface
- Describe physical motions as a series of slides, flips, and turns

Materials

10-by-12 grid paper

Set of tetrominoes created from paper or interlocking cubes for each student (*Each tetromino should be a solid color that is different from the other shapes. These can be created ahead of time by the teacher or by students in a previous lesson.*)

Number cubes

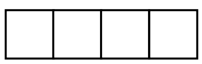
Crayons

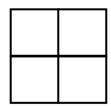
Previous Knowledge

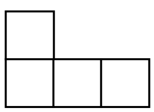
Students will explore all possible arrangements of four squares with full sides touching to create all possible tetrominoes. In order to compare two shapes to make sure they are not the same, they can slide, flip, and turn them. If one tetromino can fit on top of another, it is not a new shape. The following day, they can choose a single tetromino and use it repeatedly to see if they can use it to cover a 10-by-12 grid.

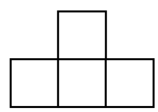
Lesson

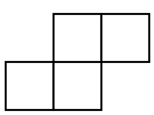
This game is a paper version of the computer game Tetris. The object of the game is to place tetrominoes in a way that will cover as many of the squares in the 10-by-12 grid as possible. One, two, three, or four students can play together. Each player is given a 10-by-12 grid as a game board. The 10-unit edge is at the bottom. One player rolls the number cube to determine which tetromino to use. Make a key for numbers one through six where each number 1-5 stands for a particular tetromino; six is player’s choice.

1: 

2: 

3: 

4: 

5: 

6: Free choice

Then each player determines where to “place” the tetromino on his or her game board. It must touch the bottom or another tetromino already in place. Encourage students to slide, flip, or turn the paper or interlocking cube version of the tetromino first so they can get the best fit. Then each player records their move by coloring in the squares to match the placement. Play continues until no more tetrominoes can be placed on a game board. Students calculate the number of squares covered at the end of the game to determine their score. The player with the highest score wins.

While students are playing, you will have the opportunity to observe their mathematical thinking. What you learn should guide your decision about how to proceed. How do they determine where to place their tetromino? What motions do they use to get it into place? Are they planning ahead? What strategies do they use to fit more shapes on a grid? Are they systematic in their approaches or more random? At first, it is fine for students to use trial and error until they become familiar with the shapes. Eventually, however, they should be able to visualize where a piece will fit and where they will place it.

Summarize the game with a class discussion. Ask students how they determined where to place a tetromino. Place a partially filled game board on the overhead. Roll the dice to determine which tetromino will be used next. Have students visualize where they would place it. Ask them to turn to partner and explain their choice. What would they hope to roll next and why?

After students have played the game several times, they invent a variety of strategies for determining the total number of squares in the rectangle and the number of squares they have covered. Ask the following questions in a whole class discussion: How did you figure out the number of squares there are in a rectangle? How did you figure out how many of those square you covered?

Extensions

Students can continue to explore covering area with tetrominoes by using a 5-by-24 grid created by cutting the 10-by-12 rectangle in half lengthwise and taping the short ends together. Other rectangles that can be created are 15-by-8 and 6-by-20. They then play the game on different grids to explore whether the same tetrominoes can cover these new rectangles.

Reflections

This task provides an opportunity to introduce the term congruent as the idea of "having exactly the same size and shape." Any of the tetrominoes can be slid, flipped, or turned to fit exactly on top of the same tetromino. The emphasis should be on becoming familiar with ways of moving the tetrominoes, however, and not the terminology.

There are many practical aspects to being able to visualize how different shapes fit into a space. These include being able to read a map, assembling furniture and other items, arranging furniture in a living space, and cutting cookies using a cookie cutter. These processes are difficult and complex for students to develop; therefore, they will need experience over many years of schooling. Even though your students will begin and end this task with different skill levels, all of them will have gained a greater knowledge of transformations and area.

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10-by-12 grid paper

Elementary Corner

The LAST Division Method
You'll Ever Need.

Kristy Aycock

This lesson is intended for teachers of 3rd to 5th grade students. This lesson was developed because most of my students struggled with Long Division. I felt like I needed a way for the students to feel successful while they were developing the deeper understanding of division. My students love this method and are constantly asking me to present harder problems for them to solve. If our lesson for that day is not on division, they beg to do at least one division problem.

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The LAST Division Method You'll Ever Need

Developed by Kristy Aycock

This division method was developed for 4th grade students who have trouble understanding division and all the steps it involves. This method is a way for students to be successful while they gain the deeper understanding of division.

STEP 1

Begin by demonstrating how multiplication is related to division. In multiplication two factors are multiplied to create the product. With division, you have the product and one factor. To solve division, you must find what the other factor is. Use of the multiplication table is an excellent way to show this relationship between multiplication and division.

Students should then practice using this principle while solving simple division problems, such as: 48 ÷ 4, 69 ÷ 3, & 46 ÷ 2.

STEP 2

Explain that not all division problems can be divided evenly. LAST is a way to help solve these more difficult problems. The "L" in LAST stands for List. The "A" in LAST stands for

Answer. The "S" in LAST stands for Subtraction. The "T" in LAST stands for Take Down. Start with the "A". The Answers always start with 0 and goes to 9.

EX:

L	A	S	T
	0		
	1		
	2		
	3		
	4		
	5		
	6		
	7		
	8		
	9		

Next, explain and demonstrate that the list is constructed from the divisor.

EX:

3

L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		

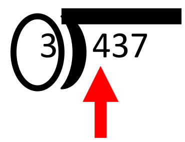
3) 437

This number is then multiplied to all of the answers. Practice making the List and Answers several times with different divisors before beginning the division problem.

To begin the division problem, you start with the first number of the dividend. In the example, it is the 4. You look at the List and say, "What is closest to 4 without being more." In this example you would choose the 3.

EX:

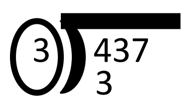
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



This number from the list is put under the dividend.

EX:

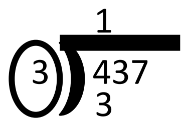
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



The answer that goes with the List number is put on the line.

EX:

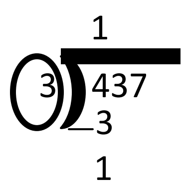
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



Now you go to the "S" and you subtract.

EX:

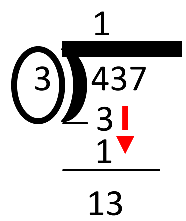
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



Now we are at the "T", Take Down. In the example we would take down the 3.

EX:

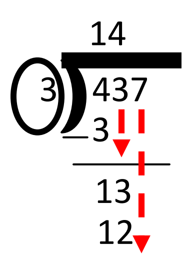
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



We have completed the first time through the LAST process. We now start again with the "L", the List. Say, "What is closest to 13 without being more?" From the List we would choose 12, because 15 would be more. The "A" (answer) that goes with 12 is 4.

EX:

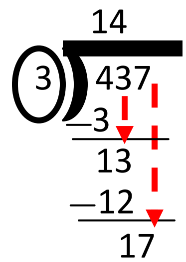
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



The next step is "S" (subtract). 13 – 12 is 1. Then we go to the "T" (take down) and we take down the 7.

EX:

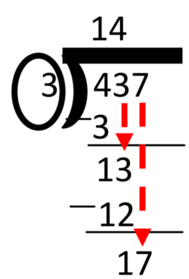
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



We now go through the process for the third time. Start with the "L" (List) and say, "What is closest to 17 without being more?" We chose 15 because 18 would be more. The "A" (answer) that goes with 15 is 5. Next "S" (subtract) 15 from 17 and we get 2.

EX:

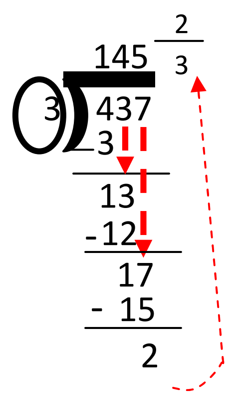
L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



Next, is “T” (take down). There is nothing left to take down, so we must take up. We take the 2 up and we are left with 2 out of 3

EX:

L	A	S	T
0	0		
3	1		
6	2		
9	3		
12	4		
15	5		
18	6		
21	7		
24	8		
27	9		



The quotient is $145\frac{2}{3}$.

Middle/Junior High Corner

The Amazing Race

Scott Gregory

Olympus Junior High

How can we make math fun, exciting, worthwhile, and involve other content areas? Maybe it can't be done but we can still keep trying as math teachers. With the popularity of the television show "The Amazing Race" comes an opportunity to engage students while they learn different math concepts. This can also involve other teachers and other content areas in the school if you would like. This can be a useful teaming activity.

The idea is that there is a goal to be reached that involves some work on the student's part along with cooperation with others on the team. There are stops and tasks to be accomplished for each team. The first team to finish all the tasks and all the clues is the winning team and receives the prize. I have done this as an after school or extracurricular activity so it does not take away from class time. However, teams need to talk to teachers so they will have to do some of the work during the school day. Let's look at the procedure.

1. Students find teammates - 4 students is the maximum on a team.
2. The first clue is given to the team. The clue could be a math problem or a puzzle. Something that involves whatever concepts you would like the students to work on.
3. The teams solve the clue which will lead them to a teacher, administrator, or room in the building. All the stops will be "Pit Stops" where the students have to do a task created by the person at the stop. For example, if the stop was at a History class, the teacher there may ask them to create something that has to do with whatever they are studying in History at that point in time. The tasks can take time to complete. Preferably, the task should require the students to do some work at home.

4. When the team has completed the task to the satisfaction of the teacher at the stop, they are given the next clue.
5. The first team to complete all the clues and the "Pit Stops" wins the event. This can take a couple of weeks or longer if you would like.

Students are involved and engaged in this activity. Make the reward something they would really like if you can. The PTSA might help you out on this. I give them this paper below before they start:

A few things to note when working on the Amazing Race:

1. *Do not interrupt a class to get the teacher to help you.*

2. *Be careful with the clues. Some are easy, but some are a little tricky. If you are stuck, come see Mr. Gregory for a hint.*

3. *Carry a notebook and pencil with you to get instructions at each stop.*

4. *Do not skip a stop!!! The order you go is very important!! If you are out of order in the final paper, you will be disqualified!*

5. *Have Fun!!!*

Middle/Junior High Corner

Different Parts of the Room Lesson
Converse, Inverse and Contrapositive Forms of Conditional Statements

Valerie Chambers

Willowcreek Middle School

In April 2005, Drusilla Willhite and I had been discussing classes we had attended at the National Council of Teachers of Mathematics (NCTM) 2005 Annual Meeting. Drusilla shared with me a few things that she had learned in a session entitled “Algebraic Thinking and Brain Research” given by Eleanor Ennis and Michele Harbinson, Mathematics Professional Development Coaches in Wicomico County, Maryland. Part of what Drusilla Willhite shared was that when you were teaching closely related concepts that can be easily confused it helps the students if you teach the concepts in different parts of the room. The example Mrs. Willhite’s shared pertained to perimeter and area. The next year, I was teaching a logic unit in my Geometry class. The part of the unit focused on taking a conditional statement and writing the converse, inverse and contrapositive of the statement. I had taught this concept for many years and I knew that students confuse the different forms of a conditional statement. I was racking my brain to determine what I could do this time, when suddenly I remembered the information Drusilla Willhite shared on using different parts of the room.

Materials:	4 – IF signs (pink)	1 – Statement sign (white)
	4 – THEN signs (yellow)	1 – Converse sign (white)
	4 – P signs (green)	1 – Inverse sign (white)
	4 – Q signs (goldenrod)	1 – Contrapositive sign (white)
	4 – NOT signs (blue)	

Procedure:

1. I had a student hold the **Statement** sign in the front of the room (north), a student hold the **Converse** sign on the east side of the room, a student hold the **Inverse** sign on the south side of the room, and a student hold the **Contrapositive** sign on the south side of the room.
2. Four students formed the statement **IF P THEN Q** by the **Statement** sign.
3. Four students formed the statement **IF P THEN Q** by the **Converse** sign. We discussed that the ***converse*** of a statement of a statement switched the **IF** part of the statement with the **THEN** part of the statement. So, the **P** student switched places with the **Q** statement. The resulting statement was IF Q THEN P.
4. Four students formed the statement **IF P THEN Q** by the **Inverse** sign. We discussed that the ***inverse*** of a statement inserts the word **NOT** in the **IF** part of the statement and the **THEN** part of the statement. So, two students with **NOT** signs inserted themselves between the **IF** and the **P** and between the **THEN** and the **Q**. The resulting statement was IF NOT P THEN NOT Q.
5. Four students formed the statement **IF P THEN Q** by the **Contrapositive** sign. We discussed that the ***contrapositive*** of a statement switched the **IF** part of the statement with the **THEN** part of the statement AND inserts the word **NOT** in the **IF** part of the statement and the **THEN** part of the statement. So, the **P** student switched places with the **Q** statement and two students with **NOT** signs inserted themselves between the **IF** and the **Q** and between the **THEN** and the **P**. The resulting statement was IF NOT Q THEN NOT P.
6. We quickly went around the room reminding ourselves of what we had done. The students then sat down.
7. We wrote statements and their converse, inverse and contrapositive statements on the board. Referring to the different parts of the room as we wrote the statements.

High School Corner

Seamless Transition to College

Gary Turner, Casey Dudley, Jon Warnick

Wasatch High School

Wasatch High School is a 3A school in Heber City, on the other side of the mountains from Salt Lake. We serve about 950 students from grades 10 - 12. As a math department we've been concerned about students who do well in high school mathematics then go to college and have to retake courses they've already had. This has become more of a problem as the colleges have become more restrictive in what they allow students to sign up for. Often students' ACT is not high enough for placement in a specific math course. In this case, students must take a placement test. For most students the test is taken in the fall after a summer of doing little or no math. Consequently, they score poorly on the placement test and are required to take a remedial course they don't really need, thus wasting time and money. We've had a concurrent enrollment College Algebra/Trigonometry class available through Utah Valley University (math 1050 and 1060) but most of our students don't qualify to take it because of the 23 required on the ACT math portion or placement by the COMPASS exam. So the math 1050/1060 was only meeting the needs of a small portion of the students we serve.

As a department we've adopted several new procedures to address this weakness. We would like all of our students to leave high school with a score or a class that would allow them to be placed in at least a math 1010 class. One of the strategies we've adopted is to stress concurrent enrollment math 1010 (Intermediate Algebra) for our seniors who score the required 19 or higher on their math ACT score, but not a 23. That will allow them to qualify for math 1050 when they enter college and gives them the chance to work on some of their fundamentals before moving on. Another strategy has been to develop an ACT preparation class for those who are scoring below 19 on the ACT after completing intermediate algebra.

We've placed an increased emphasis on ACT scores and their role in college placement with our students. A key component of the plan begins in Geometry and Intermediate Algebra. Practice ACT tests are given on a regular basis and progress is charted. After the Intermediate Algebra core test students are encouraged to take the ACT to try to qualify for concurrent enrollment math 1050. We began doing this last year and our numbers for 1050 nearly doubled this year. Not only did more students take the ACT, but our ACT math scores rose this last year. We believe this increase was because of the extra practice the students have had and the emphasis placed on the importance ACT scores.

After Intermediate Algebra students are placed according to their ACT score and teacher recommendation. Parents have the right to change what class their child takes, but most follow the teacher's recommendation. Those who are not scoring at least 19 on the ACT are placed in an ACT preparation class where they work on fundamentals and test taking strategies. Juniors who score 19 or higher, but not 23, are placed in math 1010 for their senior year so they will be able and ready to take math 1050 when they enter college. Those who score 23 or higher are placed in math 1050. Sophomores who score 19 or higher, but not 23, are recommended for pre-calculus as juniors. That allows them to move on to either AP calculus or statistics as a senior and generate college credit by means of the AP exam, or to retest and take 1050/1060 as seniors, depending on their progress.

In every case the point is to allow them to leave high school with a college placement. Those who take the ACT preparation class may transfer out, into math 1010, at any time by taking the ACT exam and scoring 19 or higher, allowing them to earn college credit and qualifying them to move on to math 1050 the next year. Whatever route they take they leave Wasatch High with a score or course allowing them to move forward with their college courses. They also receive the review or "filling in the cracks" that some need.

Last year one of our teachers, Jon Warnick, kept track of the scores of those who stayed in his ACT prep class for the year. Many of those who worked hard raised their scores by several points. The average increase was approximately 1 ½ points. Those who's scores were already close to 19 were able to get out at the quarter or semester to get into math 1010 with their new scores, so he was working with those who needed the most help by the second semester. The actual concurrent enrollment 1010 course begins with the second semester. Those who transferred in to 1010 did as well as those who were already in the class. This year we've had time to put together a curriculum for the course, so he doesn't have to make it up on the fly, and hope to improve on last year's results.

It's our goal to create a seamless transition for our students as they move from high school to college. We believe the ACT preparation class and math 1010 class are helping those who had holes in their knowledge to be better prepared, and that the concurrent enrollment 1010 and 1050 classes are helping our students to experience the demanding pace of a college math class before entering college - while they still have regular contact with their teachers and the support of parents.

Six Principles of High-Quality Instruction

Douglas L. Corey, Blake E. Peterson, Ben Merrill Lewis, and Jared Bukarau

Brigham Young University

Steven R. Covey, author and management expert, explains in his book *Principle-Centered Leadership* (1989) about an interesting phenomenon. In surveying 200 years of success literature he found that around 1940 the literature began to change from a *character ethic* to a *personality ethic*. In the character ethic the core push was to develop fundamental traits such as service, honesty, industry, patriotism, integrity, self-discipline, and benevolence. The personality ethic changed the focus to "human relations techniques, on influence strategies, on image building, on getting what you want, . . . on success programming and people manipulation tactics" (Covey, 1989, p xi). The continuation of this trend has yielded many quick-fix and band-aid approaches to increase success in various enterprises, both individual and institutional. However, these latter approaches, rarely succeed when separated from the principles associated with the character ethic, especially when evaluated with the test of time.

Focusing on foundational principles is a change from the current conversations around mathematics education. Debates have focused around the effectiveness of methods (lecture, group work, manipulatives, writing about mathematics, seatwork, etc) and the implementation of various educational philosophies (reform, traditional, etc.). Current research findings have found that effective instruction is largely independent of instructional method or strategy. For example, when educational researchers judge the quality of instruction by observing lessons, both instruction that is considered "reform" as well as "traditional" are rated at the highest level, as well as rated at the lower levels (Weiss, 2003). We can conclude that different "styles" or "methods" can be equally effective.

Another example can be found in international studies of middle school instructional quality. The two highest achieving countries in the latest TIMSS video studies, Japan and Hong Kong SAR, teach very differently from each other when comparing teaching styles. Japanese instruction often looks similar to what many refer to "reform" instruction with more time spent on fewer problems and lots of student-to-student interaction with presentation of student work as a central focus of discussion.

Mathematics instruction in Hong Kong looks, on the surface, a lot like US instruction with teachers giving interactive lectures, having students doing some seatwork during the lesson, and largely focusing on understanding and performing mathematical procedures (Hiebert et al., 2005; NCES, 2003).

These findings raise the question, what is it about instruction, if not the style or the method, that determines effectiveness? We argue that the answer is found, at least partially, in the ideas of Stephen R. Covey mentioned earlier: foundational principles. In this case it is foundational principles of high-quality instruction. Principles are much like a compass, showing the direction you should go but not dictating how to get there. The principles are not so explicit as to tie a conception of high-quality instruction to a particular form, but, if the principles are understood well, they can be used to evaluate instructional quality of varying forms.

In the remainder of this short paper we explain six principles of effective instruction that emerged from a study of Japanese middle school mathematics teachers. These principles come from a study of mathematics teacher education in Japan. Student teachers in Japan teach fewer lessons than their US counterparts. This enables the cooperating teachers to spend a lot of time discussing lesson plans and instructional decisions with the student teacher before the lesson is ever taught. In our study the student teachers and the cooperating teachers spent about three one-hour sessions discussing the lesson plan before the lesson was taught and about an hour after it is taught. We used the conversations, 19 pre-lesson conversations and 7 post-lesson conversations, to explore what the cooperating teachers viewed as important in designing and teaching a good lesson.

Principle 1: *High-quality mathematics instruction intellectually engages students with important mathematics.*

This principle appears to be the most central feature of a high-quality mathematics lesson. This was a topic in every single one of the 19 pre-lesson conversations between cooperating teachers and student teachers. Not only was it most frequently discussed across conversations but the other five principles are all closely tied to this single central principle of high-quality mathematics instruction. Although US teachers also emphasize engaging students as important, they tend to emphasize physical engagement rather than intellectual engagement (Wilson, Cooney, & Stinson, 2005; Wang & Cai, 2007). Japanese teachers focus explicitly on intellectual engagement. Below is an excerpt of a conversation where this is illustrated. CT U is looking at ST M’s lesson plan for the first time. The lesson is introducing the idea of a variable to the students. After looking at the lesson plan for a few minutes and asking some clarifying questions the following conversation takes place.

CT U: Are there any places that students use their head?

ST M: There is no such a place. Nothing at all.

CT U: Your plan is to do this and this, right? Students won't use their head at all. I don't know what you plan is in this part but they won't use their head either. This part only requires them to fill in blank. I don't know if you really want to do that yet. You didn't plan to stimulate student's "thinking process." So it will be quite a mediocre lesson. Do you think this [problem?] will make students think?

ST M: No, I don't think this will make them think.

It is clear that to the CT this lesson would be "mediocre" because there is nothing in the lesson to stimulate student's "thinking process." He asks a simple question "Are there any places where students use their head?" This one question summarizes this first principle well, because if the answer is no, then there is no hope of it being a good lesson.

Principle 2: *High-quality instruction is guided by an explicit and specific set of goals that consist of student motivation, student performance, and student understanding.*

Every Japanese lesson plan begins with a set of goals. The goal statements are very important to Japanese teachers. The goal statements help Japanese teachers balance between mathematical skills and conceptual understanding, something that is often dichotomized in the mathematics education literature. The goals also helped the teachers balance two other issues, to make the mathematics interesting and meaningful to the students while maintaining high mathematical standards.

The goals help to guide teachers in developing a lesson. The cooperating teachers continually referred back to the goal of the lesson to see if the activities suggested by the student teacher were aligned with the goals. They even went beyond checking for alignment but they challenged to student teachers to come up with the best activities that they could that would accomplish all of the goals of the lesson.

Principle 3: *The flow of high-quality instruction begins from a question or a problem that students see as problematic. As students intellectually engage in the problem or in answering the question they learn the lesson's* hatsumon *or big mathematical idea. The flow of mathematical ideas follows a natural path from what students currently understand and know to the new material of the day's lesson.*

In our analysis of the data we found frequent references (13 of the 19 conversations) to a concept that the Japanese call the "flow" of a lesson. Flow includes the whole logical structure of the lesson as planned (how it builds on students' ideas, how the task creates a need for the mathematics, etc) as well as how the lesson actually plays out (building on specific student comments, transitions, etc).

The lesson flow answers the natural questions raised by principles one and two. In which problem, questions, or activities will the students intellectually engage (principle 1) that will best address the goals of the lesson (principle 2), and in particular, will raise the *hatsumon* or big idea of the lesson. Ideally, the hatsumon can be developed largely based on work the students do, but lessons vary on how well the Japanese teachers reach this ideal.

Principle 4: *High-quality instruction is created with close connections to past lessons and to build a basis for future lessons. The lessons have strong connections within a unit as well as connections across grades. The lessons in a unit help students progress to ways of thinking, writing, and representing mathematics evident in the discipline of mathematics.*

One interesting result here is that lessons within a unit changed depending on the placement within a unit: beginning, middle, or end. Japanese teachers teach lessons at the beginning of the unit in a very open-ended, exploratory fashion. However, at the end of the unit they lesson are more "focused" and are taught in a more explicit fashion. Although the analogy is not perfect, the beginning lessons look more like proposed "reform" instruction while the ending lessons look more "traditional." However, all lessons still strive to have students doing intellectual work in a way that naturally builds connections to the new material.

Principle 5: *High-quality instruction adapts so that all students are engaged in mathematical work that appropriately challenges their current understanding.*

It is clear from the pre-lesson conversations that differentiating instruction is important to these Japanese mathematics teachers. More than half of the conversations, 10 out of 19, discussed adapting instruction for different kinds of students. Adaptations of the lesson are done differently than the current US differentiated instruction literature recommends. Much of the current US literature pushes for differentiation along learning style classification, focuses on differences between individuals, and is not content specific (Gregory & Chapman, 2002).

Japanese teachers emphasize commonalities among students rather than the differences. They craft lessons based on knowledge common to all students in the class, but challenge all students. Of course the instruction is more effective for some students than others and is more challenging for some than others. Part of the lesson planning process in Japan is to consider how to adapt the lesson to students who are struggling or who are not challenged. The Japanese then adapt instruction by considering two groups of students, those that understand specific content and those that are struggling to understand. For those that understand and are not challenged, they adapt the material to make it more challenging. For the students that are struggling they provide hints or carefully adapt the task so it is still challenging these students, but at their level. Below is a quote from a student teacher who explains how she failed to do this in a lesson she had just taught.

> There were some that solved the problem very quickly, and there were others who couldn't do anything at all. I wasn't able to follow up on those two groups. Now I can, but at that time I wasn't sure what should have been said. Nobody was able to come up with all four methods, but there were groups that used two or three methods. To those groups, I said, "Are there any other ways?" or "How would elementary school students solve this problem?" But, there was little reaction to those questions, and I wonder if my questions weren't appropriate.

Principle 6: *High-quality instruction requires a well thought out, detailed lesson plan that addresses the previous five principles and ties them together in a coherent lesson.*

We admit that this principle is less about instruction itself and more about what is needed for good instruction to take place. However, it was clear that this was an extremely important principle that cooperating teachers wanted student teachers to learn. It is also clear from the post-lesson conversations that both the cooperating teachers and student teachers thought that many of the disappointments in the lesson could have been solved by better preparation and more “research” on the part of the student teacher.

Conclusion

These principles seem to be a good basis for understanding what is necessary for students to learn mathematics with understanding. A couple of researchers surveyed all examples they could find of projects, programs, curriculum, or systems where students successfully learned mathematics with understanding (Hiebert & Grouws, 2007). They could only find two things that were common among all of these efforts. The first one is that learning with understanding was an explicit focus. This finding tells us that learning for understanding will not come as a by product of focusing on something other than understanding. This focus is part of principle 2. The second commonality was that students had to struggle with the mathematics, that is, they had to do some intellectual work during the lesson. This corresponds to principle 1. These two principles then are necessary for students to learn with understanding. So if we want instruction to help *all* students learn mathematics with understanding we need to ensure that these two principles are present in each lesson. The other four principles we found the Japanese teachers focused on mainly support the implementation of these first two principles.

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The Comprehensive Mathematics Instruction (CMI) Framework:
A new lens for examining teaching and learning in the mathematics classroom

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Along with previously released NCTM *Standards* (NCTM 1989, 1991, 1995), the NCTM publication *Principles and Standards for School Mathematics* identifies "key issues in contemporary mathematics education" and "sets out a carefully developed and ambitious but attainable set of expectations for school mathematics" (NCTM, 2000, p. 379), including a vision of mathematics instruction in which "students confidently engage in complex mathematical tasks chosen carefully by teachers, [as] teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures." (p. 3)

To what extent is this vision and set of expectations being realized in mathematics classes throughout the United States? Observations based on the TIMSS 1995 and 1999 Video Studies (Jacobs et al., 2006) led the TIMSS researchers to conclude "the nature of classroom mathematics teaching observed in the videotapes reflects the kind of traditional teaching that has been documented during most of the past century, . . . the nature of mathematical thinking and reasoning, and the conceptual mathematical work, remain unaligned with the intent of *Principles and Standards.*" (pp. 28, 30)

Although the most recent of the TIMSS Video Studies is over nine year old, experience suggests that not much has changed from the classroom episodes observed in 1999. Results from the video study suggested the following:

- Teachers often feel that their lessons are in accord with current ideas about the teaching and learning of mathematics if they involve externally observable practices such as the use of technology, inclusion of real-world problems, or collaborative group work.

- While U. S. eighth grade mathematics teachers devote approximately one-third of their lesson time to “private interaction” (time when students work on their own or in small groups), over 75% of this time consisted of working on “repeating procedures that had been demonstrated earlier in the lesson or learned in previous lessons”, rather than “engaging in complex mathematical tasks, working productively and reflectively [to] make, refine and explore conjectures on the basis of evidence” as envisioned by the NCTM *Standards.*
- While more eighth grade teachers in 1999 incorporated such lesson features as sharing alternative strategies and examining solution methods than in 1995, the actual number of opportunities for students to do so was extremely limited. Only 5% of the problems discussed publicly in the 1999 eighth grade mathematics lessons involved the sharing of alternative solution methods, and only 2% of shared problems were actively discussed, examined, critiqued and compared.
- None of the U. S. lessons in the 1999 Video Study showed evidence of students “developing a rationale, making generalizations, or using counterexamples” as methods of mathematical reasoning.

One might wonder, what has contributed to this lack of understanding and limited implementation of the NCTM *Standards* and other recommendations for mathematics education reform? In 1999 Chazan and Ball expressed frustration with “current math education discourse about the teacher’s role in discussion-intensive teaching.” They argue that educators are often left “with no framework for the kinds of specific, constructive pedagogical moves that teachers might make.” (p. 2) While the NCTM *Standards* have promoted a vision and set of expectations for mathematics instruction, the conscientious lack of a prescriptive pedagogy often leaves teachers without a clear sense of direction.

The *Comprehensive Mathematics Instruction* (CMI) Framework was designed to provide access to reform-based pedagogical strategies for K-12 mathematics instructors. It was also designed to bridge the gap between the good pedagogical strategies of traditional instruction and the recommendations of reform-based instruction. The CMI Framework was developed over several years of collaborative efforts between professors from four departments (Educational Leadership, Mathematics, Mathematics Education, and Teacher Education) at Brigham Young University and representatives from five surrounding school districts representing one-third of the students in Utah. The CMI Framework consists of three major components: a *Teaching Cycle*, a *Learning Cycle*, and a *Continuum of Mathematical Understanding*.

<u>The Teaching Cycle</u>

Successful inquiry-based teaching moves through phases of a *Teaching Cycle* (Figure 1) that begins by engaging students in a worthwhile mathematical task (*Launch*), allows students time to grapple with the mathematics of the task (*Explore*), and concludes with a class discussion in which student thinking is examined and exploited for its potential learning opportunities (*Discuss*).

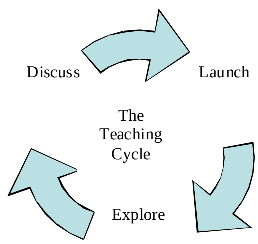

Figure 1: The Teaching Cycle

The *Teaching Cycle* is not a new idea being proposed by the CMI Framework, although the phases of the cycle might be referred to by different names in the literature, such as the Launch-Explore-Summarize instructional model of the *Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006; Schroyer & Fitzgerald, 1986). The CMI Framework contains a careful articulation of the purposes for each phase of the *Teaching Cycle* and a description of teacher and student roles during these phases of guided-inquiry. What is new, however, is the CMI Framework explicitly recognizes that the phases of the *Teaching Cycle* look and function differently depending on the purpose of the lesson, as determined by the *Learning Cycle*.

The Learning Cycle

Student learning progresses through a *Learning Cycle* (Figure 2) that first surfaces students' thinking relative to a selected mathematical purpose (*Develop Understanding*), then extends and solidifies correct and relevant thinking (*Solidify Understanding*), and finally refines thinking in order to acquire fluency consistent with the mathematical community of practice both inside and outside the classroom (*Practice Understanding*).

The *Learning Cycle* is unique to the CMI Framework and suggests both how understanding develops and how it can be guided through different types of lessons. It is particularly important to realize that the phases of the *Learning* Cycle influence and modify the *Launch, Explore* and *Discuss* phases of the *Teaching Cycle*. For example, the *Launch* during a *Develop Understanding* lesson may consist of engaging students in an open-ended task designed to elicit a variety of alternative solution strategies; whereas, the *Launch* of a *Solidify Understanding* lesson may consist of a string of related problems designed to elicit and examine a particular strategy.

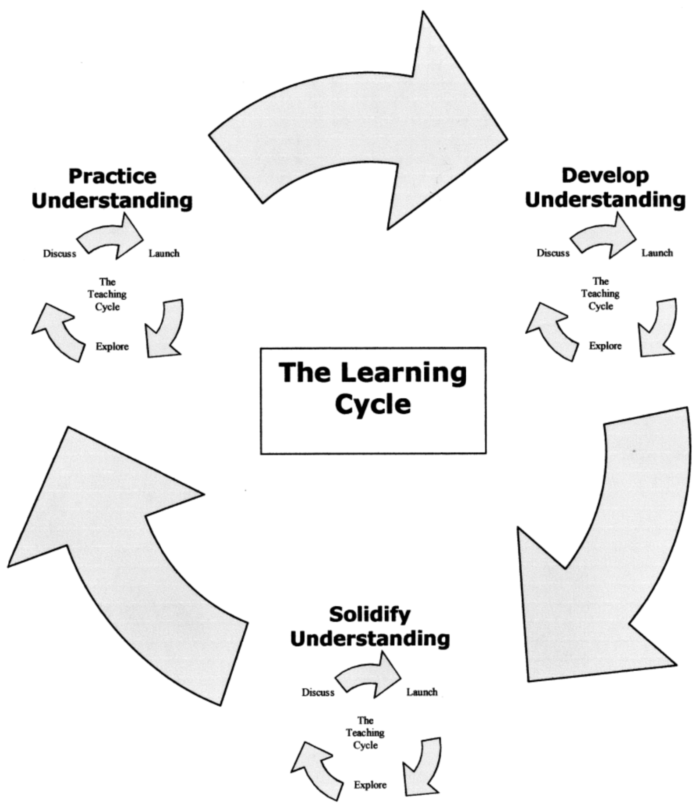

Figure 2: The Learning Cycle

Continuum of Mathematical Understanding

Mathematical understanding encompasses at least three connected but distinct domains as represented by the horizontal lines of a *Continuum of Mathematical Understanding* (Figure 3): conceptualizing mathematics, doing mathematics, and representing mathematics. Mathematical understanding progresses continually along the continuum, but it is useful to note three sets of distinct landmarks of progression along the continuum that are associated with each of the three phases of the *Learning Cycle*.

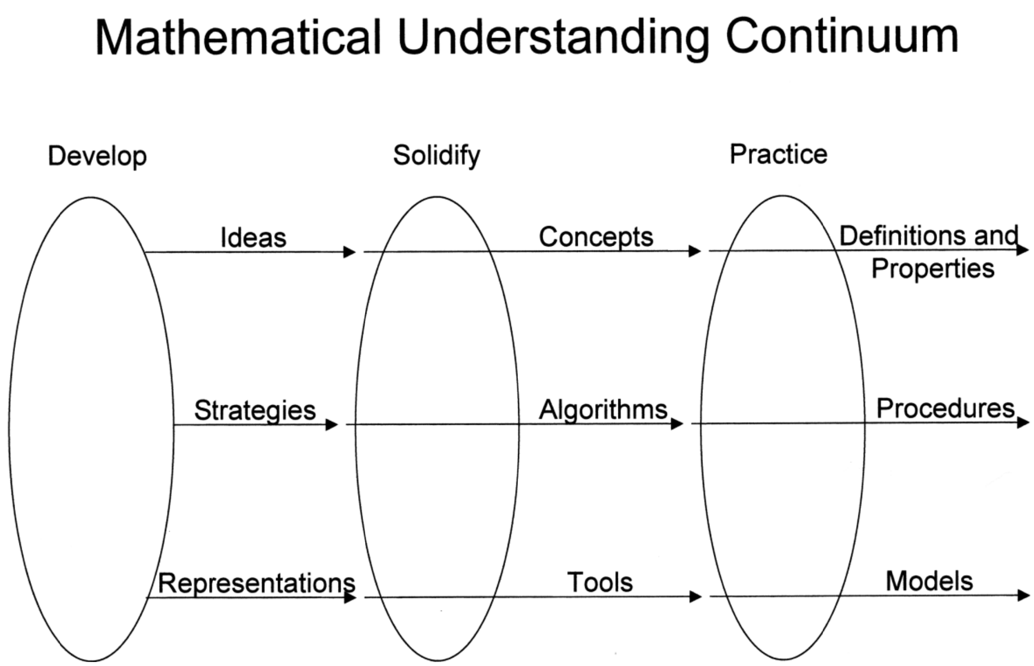

Figure 3: The Continuum of Mathematical Understanding

Emerging mental images are fragile as they are surfaced during students' initial experiences with tasks designed to elicit those images (*Develop Understanding*). We call these fragile images: ideas, strategies, and representations. These ideas, strategies and representations need to be examined for accuracy and completeness, as well as extended and connected through multiple exposures and experiences until they become more tangible, solid and useful (*Solidify Understanding*). In the CMI Framework, ideas that become more solid and firm are called concepts; solid strategies become algorithms; and useful representations become tools. Although understanding has been developed and solidified, it needs further refinement to become fluent (*Practice Understanding*). In the CMI Framework concepts that are refined become definitions or properties; algorithms that become fluent are called procedures; and refined tools become models. These definitions and properties, procedures, and models must be consistent with the broader mathematical “community of practice”.

We have tried to capture this progression of conceptualizing, doing, and representing mathematics through the *Continuum of Mathematical Understanding*. While the words used on the *Continuum* mean different things to different people, we have adopted these words to try to capture the progression of the often stated goal of mathematical instruction: deepening mathematical understanding.

Potential Uses of the CMI Framework

The CMI Framework was designed to help classroom teachers strengthen their instructional practices in order to deepen students’ mathematical understanding. To this end, the CMI Framework can be used by the classroom teacher as a pedagogical tool before, during, and after teaching. Prior to teaching, the Framework provides a planning model for designing lessons to meet intended purposes and desired learning outcomes. Teachers are expected to anticipate student thinking and to anticipate how they will orchestrate a discussion from this thinking in order to develop and solidify correct understanding.

Teaching episodes in which students are allowed to explore mathematical ideas can appear chaotic and seem unmanageable for teachers who are accustomed to presenting ideas and taking on the role of sole mathematical authority in the classroom. The CMI Framework provides a method for teachers to make sense of student work that is taking place in the classroom and to decide what to do with the ideas that are emerging.

The CMI Framework also provides language and structure for a classroom teacher (and other observers of the lesson) to reflect upon what took place during the lesson. Being able to name instructional strategies and pedagogical moves allows them to become objects of reflection and refinement; therefore, teacher intent, student understanding, and the transactions that occurred between and among students and the classroom teacher can be discussed using the language and structure of the CMI Framework. Finally, the CMI Framework focuses teachers on future work—suggesting where they can go next with student thinking and suggesting possible paths for how to get there.

Implications of the CMI Framework

The CMI Framework encompasses both the procedural goals of traditional mathematics instruction and the conceptual goals of reform-based mathematics instruction; however, it does not advocate an equal balance between these two approaches to instruction. For example, traditional mathematics instruction usually *begins and ends* at the far right of the *Continuum of Mathematical Understanding* by presenting students with definitions and properties, procedures, and models without providing students with opportunities to explore, examine or refine the conceptual underpinnings of these learning outcomes. On the other hand, implementation of reform-based instruction often *begins and ends* at the far left of the *Continuum of Mathematical Understanding* by only helping students surface ideas, strategies and representations without providing students with opportunities to solidify and practice them. The CMI Framework provides a structure and model for teachers to guide students' individually-constructed ideas towards a community of shared mathematical definitions and properties, procedures, and models. By using the *Teaching Cycle,* teachers guide students through the *Learning Cycle* in order to help them progress along the *Continuum of Mathematical Understanding*.

An instructional model that focuses on the conceptual, procedural, and representational understanding of mathematics demands more of teachers than just turning to the next page of the math textbook. It requires a "pedagogical content knowledge" (Shulman, 1986) different from the mathematical content and pedagogical knowledge most teachers receive in their preservice education or develop through teaching experience. Therefore, it is necessary for teachers who want to use the CMI Framework to have the opportunity through in-depth professional development to deepen both their mathematical content knowledge and their mathematical pedagogical knowledge.

The CMI Framework was designed to help both preservice and classroom teachers implement instructional practices that will lead to student mathematical understanding. We have discovered, however, that it can also be used to unify the discourse of mathematics education researchers who are trying to shift instructional practices to accommodate deeper student thinking and conceptual understanding.

Of necessity, this article has only introduced the major components of the CMI Framework; the Framework itself includes specific language describing purposes, particular instructional practices, and teacher and student roles within both the *Teaching* and *Learning Cycles.* We recognize, of course, that good mathematics instruction cannot be reduced to a recipe. The CMI Framework is intended to be a “framework”—a structured set of basic principles and productive practices that can lead students to deeper mathematical understanding. The authors encourage inquiries and feedback as we continue to build a community of practice worthy of the mathematical education of our students.

The authors wish to acknowledge the contributions of current and former members of the CITES/CMI Committee for their work in developing the CMI Framework.

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Lessons Learned From Research

Writing Our Way into Mathematics

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Writing is a way to think our way into mathematics and make it our own.

(Zinsser, 1988, paraphrased)

In literacy instruction, most teachers have long seen the value of writing. Their students write to discover ideas and understand topics more clearly as well as to share their ideas with others. During writing time, an elementary classroom may be a beehive of activity in which children prewrite, write, revise, edit, and publish their work. In these classrooms, the writing process is alive and well. In that same classroom, when the content switches to mathematics, writing may not have found a rightful place. Are teachers of the opinion that writing is of little utility in teaching and learning mathematics? Or might it be that teachers simply may not have given attention to incorporating writing in mathematics instruction? Or are there schools and classrooms in which teachers know the value of teaching writing in various contexts, but mandated programs prevail, with little time left for anything else? This article is designed to help teachers see the value of using writing in the mathematics classroom and to give a few practical ideas for integration.

Reform-based mathematics emphasizes the importance of this integration. As early as 1989, the National Council of Teachers of Mathematics (NCTM) identified learning to communicate mathematically as a major goal for students. In *Principles and Standards for School Mathematics* (NCTM, 2000), communication was characterized as “an essential part of mathematics and mathematics education” (p. 60) and included as one of five process standards for learners. The NCTM affirmed the role of students’ written communication in helping learners

- Organize and consolidate their mathematical thinking . . .
- Communicate their mathematical thinking coherently and clearly . . .
- Analyze and evaluate the mathematical thinking and strategies of others, and
- Use the language of mathematics to express mathematical ideas precisely. (p. 60)

The inclusion of mathematical communication in the curriculum is a widespread recommendation and has been for almost two decades. Still, teachers struggle with this idea, particularly the integration of writing in their mathematics lessons. Usually they find it more natural to make connections between writing and science (e.g., Varelas, Pappas, Kokkino, & Ortiz, 2008) or writing and social studies (e.g., Jones & Thomas, 2006). Yet integration of writing and mathematics can be just as natural.

There are two levels of integration teachers might want to try. The first level, writing without revision, can be worked into mathematics instruction quickly and readily without much attention to the writing process. The second level of integration may take more time but enables teachers to connect the writing process more fully with mathematics instruction, thus meeting the goals of both writing and mathematics. Students benefit from having multiple experiences at both levels.

Level One: Writing Without Revision

Learning Logs

As students begin class, they find a question written at the front of the room to which they respond in writing. They can write in a special notebook kept for this purpose or in their regular notes. The prompt is not a problem *per se*, but something that encourages students to focus their minds on mathematics: “Think about a time you have used mathematics outside of school. Why was it important to you at that moment?” or “Think of an early experience in mathematics at school. What made it positive or negative?” Students are given a few minutes to write their responses. They can share what they have written with partners or in small groups. If the purpose is for students to summarize their knowledge of a specific mathematical term, such as *data*, they might complete a word map or construct their own definition as part of their learning log for the day. Whatever the goal, the whole experience need not take longer than 5 minutes.

Think, Write, Share

Teachers often ask questions and then call on a student to respond. They can count on always having at least one of two students with raised hands. When lots of hands are raised, the teacher will still usually call on only one or two. Try to involve everyone by asking fewer questions and expecting students to respond in writing. Give the whole class a few minutes to think and then write their responses before calling on students to share. A time limit can provide motivation and help with the pacing.

Note Making

Students may be accustomed to taking notes, but now ask them to *make* notes as well. Along with listing the main points of a lesson, students can write their own thoughts and insights. Some teachers ask students to fold their papers in half vertically and then use one half as a place to take notes and the other as a place to record their own mathematical connections. They can also write questions or points they wish to clarify later.

Level Two: Process Writing

Shared Writing

As students explain a mathematical concept or discuss their problem solving strategies, validate their thinking by writing what they say on the board or on chart paper at the front of the room, but do not stop there. Read what has been written and say, “That’s a good start. Now can you clarify (or extend) your ideas a bit more?” Ask students to help revise what has been written, considering both content and mechanics. For example, should the ideas be listed in a different order, or does a major point need to be expressed earlier in the paragraph? Are there sentences that should be divided or combined? Are there mathematical terms that would make the meaning clearer and more precise? Are the spelling and punctuation correct?

Once revisions have been made, read the piece again. Try having every student now copy the short paragraph, making a final draft from the rough draft completed as a class. Mathematics teachers may believe it is not their place to focus on revising and editing writing, but they will find that by so doing they are also helping students clarify and refine their thinking.

Class Book

Teachers may see the value of assigning writing in mathematics, but some may cringe when they consider the extra time required to help every member of a class to write an essay about a famous mathematician or create a concept book (e.g., plane shapes; fact families; metric measurement). Also, where can they find the time to evaluate such assignments? They are overlooking a simple solution to the problem: Write the essay or book as a class. In this way, teachers minimize their own time and efforts, and students can still benefit from being engaged in writing.

Simply complete a shared writing or several shared writings over a week or month. Then number the sentences or paragraphs and assign each student a numbered part or segment to write as a final draft. Very little time is required for the teacher to move from student to student and edit the sentence or short paragraph because each student has completed only one segment of the whole text. As the teacher edits, students can be encouraged to create representations for the ideas in their segment. When the students are finished, gather the pages in order and add a cover with a title. In one class period or less, students have reviewed important mathematics concepts and have created a simple class picture book at the same time.

If the shared writing has only a few sentences, assign several students the same sentence and ask for several covers as well. In this way students produce multiple copies of the same book. Keep one in the classroom and offer the rest to students in younger grades, where they can be kept in their reading centers. Increase motivation by encouraging students to actually publish the books, using an inexpensive online publishing tool such as the one found at www.Mightyauthors.com.

ABC Books

Many mathematics units have new and complex vocabulary. As a class or in small groups, create alphabet books in which words used in the unit are highlighted and explained. Assign students letters of the alphabet and send them on a search for vocabulary words in their mathematics textbooks, notes, online, or in a mathematics dictionary (e.g., Monroe, 2006). Students can use each word in a meaningful sentence, draw an appropriate representation, or make a real-world connection. Because the format of an alphabet book requires brevity, teachers can readily respond to what students have written individually, offering both compliments and suggestions for improvement. It is not too much to ask that students both revise and edit as they prepare a final draft.

A Call to Implement Writing in Mathematics

In 1992, Joan Countryman's landmark book on writing and mathematics, *Writing to Learn Mathematics: Strategies that Work, K-12*, captured the attention of mathematics educators across the nation. Countryman's work prompted a flurry of interest, followed by numerous articles, several books, and much discussion during the ensuing years. Many ideas and resources are now available, both in print and electronically, especially resources that provide ideas for writing without revision. Unfortunately, integrating the writing process in mathematics has received far less attention. Teachers are left on their own, more or less, to craft such lessons.

When teachers ask children to write in mathematics, they not only give them structured opportunities to "think their way into mathematics and make it their own," but they also help their students value writing in learning content. Such assignments are not superfluous or simply nice to do if there is enough time. The benefits of integrating both writing without revision and the writing process in the mathematics classroom are too rich to ignore.

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State News

Constructed Response Item Workshop
Nolan Fawcett

Secondary Assessment Mathematics Specialist

June 2008

In June 2008 a talented group of teachers and district mathematics directors convened in the Nebo School District's Grant building to realign some previously written constructed response questions to the newly established secondary mathematics core. After realignment and clean up of the previously written questions the group took on the task of writing new questions.

The guidance and instruction for this workshop was taken from <u>Grades 9-12 Mathematics Assessment Sampler</u> copyright 2005 by NCTM. (Pgs xi-3)

The problems that were realigned and developed were evaluated against the following criteria:

- Good problems are those that are mathematically rich
- Can be solved in multiple ways
- Promote critical thinking
- Can be evaluated in a consistent manner

"Therefore items designed to assess specific standards and expectations should be incorporated into the classroom repertoire of assessment tasks."
" Without students' work, answers do not adequately inform teachers about what students do and do not know and students need to explain their answers or ask comparative questions among answer choices."
"Short -response and extended-response items help students explain their mathematical communication skills by explaining answers and writing solutions."

The first responsibility of the group was to realign the previously constructed problems to the Utah Mathematics Core then review the previously constructed problems and write new problems that met the described criteria. Because of the new change in the core this became a very rich and rewarding experience for those involved. Many discussions and decisions as to how the questions could and should be used were investigated. Many of the problems were identified for use in various standards.

Upon completion of the alignment task the discussion turned to how the questions could be used by teachers and students in the classroom. Again we returned to the sampler of instruction and guidance.

Four purposes for Assessment:

1. Monitoring students' progress toward learning goals
2. Making instructional decisions
3. Evaluating students' achievement
4. Evaluating programs

"Assessment should not merely be done to students; rather it should be done for students" (NCTM 200, p. 22)

The group came to the conclusion that the most probable use for the problems would be for formative assessment purposes-that is:

- Assessments that help teachers learn how their students think about mathematical concepts
- How students' understanding is communicated
- How such evidence can be used to guide instructional decisions

The group agreed that students' review of their own work and others work would be the most probable and practical use of the problems once the standards(s) had been taught. Realizing that "often, if students understand what is expected of them on individual extended-response problems, they tend to answer the questions more fully or provide greater detail than when they have no idea about the grading rubric being used." The decision was made to develop a single rubric (see page 53) that could be used for each problem where students could evaluate others work. This rubric would be posted at the top of each problem so that the student would know what their expectations were. If a more detailed rubric was needed for teacher use then the teacher could develop one that would evaluate their specific needs.

It was felt that if students or groups of students could see and evaluate others work it would enhance their own work and responses to their mathematical questions. "When students are asked to explain their reasoning or justify their responses, the assessment assumes a more informative role than when the correct answer is the only focus."

Much time and effort was given to the creation of a rubric that could be used to evaluate student work on this type of problem.

Thanks and appreciation to all who contributed their expertise and knowledge to this very successful workshop.

The items are in the process of being loaded on to our UTIPS programs and servers. We hope that teachers will get involved with using such items in their curriculum and class work. We encourage teachers to develop other assessment items that teachers can use as a basis for instructional changes and improvement in the classroom.

If individual copies of the constructed response are desired please feel free to contact:

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Constructed Response Rubric for Secondary Mathematics

The following rubric is to be used by teachers and students in peer review of constructed response questions for secondary mathematics. For problems without extensions or applications, the three point rubric should be used. For problems that specifically ask for extensions and applications, the four point rubric is appropriate.

Level of Understanding	**Points**	**Description of Student Response**
Extension and Application	4	A correct process, model and/or complete justification, including correct notation, definition of variables and use of vocabulary leads to a correct answer, extensions and applications.
Proficient	3	A correct process, model and/or complete justification, including correct notation, definition of variables and use of vocabulary leads to a correct answer.
Partially Effective	2	A partially correct process, model and/or justification may or may not lead to a correct answer.
Insufficient	1	An insufficient process, partial model and/or weak justification leads to an incorrect answer.
Nonexistent	0	No response or nonsense response. An answer without justification.

University News

The McKay School of Education at BYU is supporting two elementary professional development initiatives in its Partner Districts. These are two-year programs that focus on both elementary content and pedagogy and enable teachers to acquire part or all of the state elementary mathematics license endorsement. Seven schools have been or are involved at this point along, with three additional cohorts of teachers from a variety of schools. Data relative to both teacher and student effects is being gathered in order to evaluate the quality of the initiatives. Tentative informal analyses suggest that significant effects will be observed.

THE UNIVERSITY OF UTAH

What does the U of U have for secondary teachers?

I am glad you asked. Here are just a few of the great opportunities we have:

- Math circles for high school students - http://www.math.utah.edu/mathcircle/
- Summer math program for high school students - http://www.math.utah.edu/hsp/
- Math Circles for teachers - http://www.math.utah.edu/teacherscircle/
- Last summer we ran a geometry workshop for teachers. It was great fun. http://www.math.utah.edu/mathed/geometry08/
- Math 4090, Methods for Secondary Teachers is offered in afternoons this spring (09). http://www.math.utah.edu/schedule/winter09/
- The MSSST degree, a Master's degree from the college of science - http://www.science.utah.edu/degrees.html#second
- MSSST – Jordan District collaboration. Maggie.comings@jordan.k12.ut.us
- Endorsement classes offered in the summer on a teacher friendly schedule. http://www.math.utah.edu/schedule/summer08/ (example of last year's schedule)

Contact Marilyn Keir (keir@math.utah.edu) or Emina Alibegovic (emina@math.utah.edu).

Summer Geometry Workshop for Teachers from University of Utah

Nineteen teachers from around the state were hosted for a week by the Department of Mathematics at the University of Utah doing geometry problems and discussing their practice of teaching it. One of the goals of the workshop was to provide teachers without a large support network or regular professional development opportunities with an immersion program that would revitalize and challenge them. The participants included teachers from San Juan, Tintic, Alpine, Nebo, Iron, Jordan, Granite, Park City and Salt Lake districts. The hope was to start building a community of teachers who enjoy doing mathematics as much as they enjoy teaching it. We have taken first steps towards that goal, and we intend to continue organizing workshops in this format in subsequent summers, since the first one was well received.

What was it that we did during this week? We started each day with a session modeled on teachers' circle that the Department of Mathematics organizes during school year. Teacher's circle has its roots in a similar program for high school students, Math Circle. Math circle's goal is to expose students to higher level thinking and the kinds of problems that are often absent in standard curricula. Our department has run this program for the past eight years making it one of the best established circles in the country. The idea for teacher circle was born in San Francisco, where teachers were taking students from their schools to attend Math Circle at San Francisco State University, and while they were waiting to take them back home, they started working together on solving the same problems. They had a lot of fun, were learning from each other and the teachers' circle was born. During each session teachers are given a problem to think about and eventually are lead by one of mathematics faculty in solving the problem and learning about new topic. We took slightly different approach in our workshop. Each morning the teachers were presented with a list of problems, and it was up to each group to decide which problems they wanted to work on. The following problem opened the week of our geometry workshop:

A city bus network runs in the following fashion:

I. One can get from any bus stop to any other without transferring;

II. For any pair of routes there is one and only one bus stop where one can transfer from one route to the other;

III. There are exactly three bus stops on each route.

How many bus routes are there in town?

A heated discussion ensued in some of the groups about what the problem was really saying, how it was saying it, what should be done about it, how best to represent the problem, and ultimately whether a solution was really a solution. Sometimes a group would solve just one of the problems, but most often they would do several. The hope was that participants' personal preferences would ensure that all the problems would be at least tackled, so that we could share and show the attempts at our solutions to majority of problems. While the groups often attempted different problems, some were challenging enough that they remained unsolved, although not for the lack of trying. The input from a figure of authority during this portion of the workshop was minimal and the major reason was that while teachers say that one of the characteristics they want their students to have is perseverance when faced with challenging problems, they themselves are not often faced with the same challenge. Their having to get to the solution on their own, or not at all, puts all the responsibility in their own hands. Students know that if they do not solve a problem, their teacher will do it for them. Our teachers did not have that option, and some questions will have to remain unsolved. That is an important part of doing mathematics. We do, however, plan on revisiting the questions from the workshop on the wiki page we built for the occasion and where we hope the solutions of the problems that were posed during the workshop will slowly accumulate.

The teachers' circle did not end the mathematics component of the workshop. We had the pleasure of hearing Kevin Wortmann, Mladen Bestvina, Ellena Marchisotto (CSU Northridge), Dennis Allison, and Troy Jones give talks whose topics ranged from geometry of salt piles to what wind has to do with the shape of a surface. Our visitor, Elena Marchisotto gave several talks. Some lectures were about finite geometries, a topic that tied in nicely and explained many of the problems we were working on in the morning sessions. In fact, she provided some compelling arguments for teaching finite geometries in a high school classroom, as a way of answering an everlasting question "Why do we need to prove things?" Elena also talked about use of literature in the mathematics classroom and provided our participants with an extended list of articles that they might find useful for their personal and their students' mathematical growth.

Teachers told us that as much as they enjoyed learning from our professors they enjoyed learning from each other. We reserved a portion of each day for teachers to share their teaching ideas with one another, and were told that we need more time in this segment of each day. The teachers shared lesson plans, problems, games and puzzles. They also spent part of the day working with each other making lesson plans that they simulated and shared at the end of the workshop. This portion of the workshop would have been more successful had we secured actual students from whom we could get some feedback. It is our hope that in future we will be able to arrange for an environment that would be more conducive to the actual lesson study in its original form.

Reflecting on practice is an important part of the teaching profession and to encourage it in our participants we asked Dr. Patricio Herbst from the School of Education at the University of Michigan to participate in our workshop. His research is centered on practices of teaching geometry. His research group has developed a series of animations that describe classroom episodes and whose goal is to spark conversations and reflections about one's own teaching. The comments we received from our participants indicate that the animations are successful on several fronts. Besides thinking about their own practice, teachers are made aware of other teachers' reactions and work. It is not a frequent practice for teachers to visit each other's classroom, but through conversations about the animations they tend to get a glimpse of what their individual classroom communities are like, what they each value in their students, and in their own teaching. Further, they are presented an alternative to their own style which they may not have been aware of, and which they might want to try in their own classroom. Having a fictional character as a teacher in these animations allows teachers to be more critical, as they are not criticizing a real person, and to identify more with the teacher, as they are not distracted with the teacher's gender, personality, and other character traits that may be different from their own.

We were excited to learn that teachers enjoyed the workshop, and the community we created during that time. Our hope is that they will continue to collaborate through the wiki page we have set up for the workshop, and through other activities we will continue to organize. Many have expressed a desire to participate in our future workshops, and several will be joining us in our regular Teachers' Circle sessions. We are open to suggestions about the topics of our future workshops, and we are using this opportunity to solicit your opinions, and to invite you to apply for next year's workshop. You can find the contact information and some of the documents we shared on the following pages:

http://www.math.utah.edu/mathed/geometry08
http://uugeometryworkshop08.wikispaces.com

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UTCM would like to extend a warm welcome to the state and to our organization to Dr. Patricia Moyer-Packenham, a professor in the School of Teacher Education and Leadership in Mathematics Education at Utah State University. Patricia received her Ph.D. at the University of North Carolina at Chapel Hill and has a long history of significant contributions to the field. She specializes in school mathematics for Grades K-8. Patricia's research focuses on uses of mathematics representations and tools (including virtual, physical, pictorial, and symbolic) and teacher development in mathematics. Her publications Include a book titled What Principals Need to Know about Teaching Mathematics, numerous journal articles on manipulatives and virtual manipulative, book chapters, refereed proceedings, and contributions to mathematics methods textbooks. She is currently working on the development of a book titled Teaching K-8 Mathematics with Virtual Manipulatives. Currently, and for the past four years, Dr. Moyer- Packenham serves as Co-PI on the NSF-funded Mathematics and Science Partnership (MSP) Program Evaluation where her primary inquiries examine teacher quality, quantity and diversity in the program. She has also been the principal investigator of numerous professional development grants for mathematics teachers.

Email Address: patricia.moyer-packenham@usu.edu

DIXIE STATE
COLLEGE OF UTAH

Dixie State College does not currently offer a 4 year degree in Mathematics or Mathematics Education. However, we do offer the courses needed so that an inservice teacher can earn the level two, three or Four endorsement. Each semester we offer two higher level courses. The schedule is as following: http://new.dixie.edu/math/course_scheduling.phpAny questions, please
Contact Dr. Clare Banks at banks@dixie.edu.

UVU UTAH VALLEY UNIVERSITY

Utah Valley University is proud to announce a new Master of Education Degree (M.Ed.) It is an applied master's program aimed at building the instructional skills and professional competency of teachers. The goal is to enable participants to become more proficient in selecting optimum research-based curriculum design strategies that best apply to specific teaching situations. There are two options for participants: (a) models of instruction or (b) English as a Second Language (ESL) within the curriculum and instruction program. (The ESL option leads to an ESL endorsement from the State of Utah.)

While core coursework in research, theory, and instructional models or ESL issues is required of all participants, emphases allow students to select any of the teaching content areas accepted by the Utah State Office of Education for either an elementary education or secondary education instructional project. This breadth includes any of the following content areas, (a) fine arts, (b) foreign language, (c) health, P.E., safety, (d) mathematics, (e) language arts, (f) science, (g) social studies, or (h) technology. These content areas allow students to tailor their program to their academic interests and desired skill development.

Participants enter as cohorts and progress through the degree program in a group. The culminating applied instructional project is unique to each student, but generally course work and many activities are done with fellow students. The learning outcomes are enhanced by having opportunities for students to work together and grow as a result of shared intellectual challenges.

The M.Ed. requires 30 semester hours of graduate course work and completion of a culminating applied instructional project. Work toward and completion of the graduate instructional project is a necessary part of the program for a student to demonstrate the successful ability to determine, select, and implement instructional applications for learning at the school, district, or state level.

Call for Articles

We are at this time seeking articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Some of the features are: UCTM Leader Spotlight; Letter from the UCTM President; Voices from the Classroom; Mathematics for English Language Learners; Puzzle Corner; Recommended Readings and Resources; the Utah Elementary Mathematics Curriculum and Implementation; College and University Research; and others.

Teachers are especially encouraged to submit articles for ***Voices From the Classroom***, including inspirational stories, exemplary lessons, beginning teacher ideas; or managements tools. Sample ideas are (but not limited to) focused on the CMI framework, teachers or districts who have successfully implemented the DMI, Inquiry based calculus, the new Alpine Math program K-12, the U of U/Jordan District partnership master's degree program, Cross-district Algebra assessments, and many others.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Christine Walker (Christine.Walker@uvu.edu) with a copy to Valerie Chambers (vchambers@alpine.k12.ut.us) . A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included. Submission deadline is **September 30th, 2009 by midnight**.

Any questions or comments please e-mail or call the address or numbers below. We look forward to your response.

Editor: Christine Walker
Christine.walker@uvu.edu

Co-Editor: Valerie Chambers
vchambers@alpine.k12.ut.us

Kudos Anyone?

The Utah Council of Teachers of Mathematics requests your assistance in recognizing the outstanding mathematics educators in the State of Utah. Please take a few minutes to look over the award descriptions below and nominate a math educator you feel deserves one of these awards. For a description of the four awards, please read below:

Teachers nominated for the **Karl Jones Award** (Elementary) and the **George Shell Award** (Secondary) should meet the following criteria:

I. Encourage student achievement by:

- Providing activities, which enhance understanding of math concepts.
- Generating enthusiasm for math among students.
- Providing opportunities to apply and extend mathematics skills.

II. Serve as a role model for other teachers and for students by:

- Demonstrating interest in acquiring and maintaining skills in mathematics, including instructional skills.
- Implementing effective teaching techniques.
- Demonstrating a willingness to share skills and ideas with other teachers.

The **Don Clark Award for Lifetime Contributions to Mathematics Education.** This award is for the educator who has impacted mathematics education in Utah over the course of several years.

The **Muffet Reeves Award** is for the teacher who has shown outstanding work in the area of professional development related to mathematics and is a great motivator of teachers.

To nominate someone, please go to the Award Nomination Form page on the UCTM website www.uctmonline.org.

Utah Council of Teachers of Mathematics

Executive Board 2008-2009

Award Recipients

Muffet Reeves Award

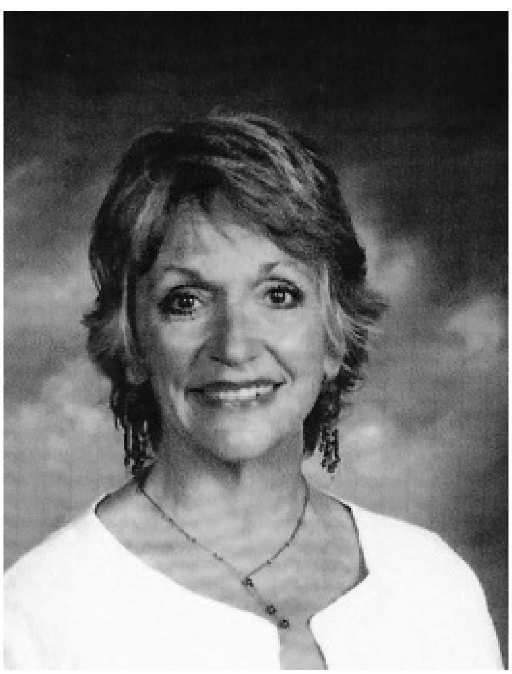
The Muffet Reeves Award is for the teacher who has shown outstanding work in the area of professional development related to mathematics and is a great motivator of teachers. This year's recipient is Kris Cunningham. Kris is a graduate of Brigham Young University and Southern Utah University earning her B.S. and M. Ed. consecutively. She has taught for:

1. ★PROVO SCHOOL DISTRICT
2. ★DEPARTMENT OF DEFENSE
3. ★SPECTRUM ACADEMY FOR GIFTED
4. ★SOCORRO INDEPENDENT SCHOOL DISTRICT
5. ★WASHINGTON COUNTY SCHOOL DISTRICT

She has 29 years total teaching experience with 19 of those years at Dixie Middle School. She has received the Huntsman Award for Excellence in Education, the George Shell Award for Secondary Mathematics Teachers, the DMS Teacher of the Year Award (on two different occasions), the Wal-Mart Teacher of the Year Award, and the Superintendent's Award.

She is a trained facilitator for *Seven Habits of Highly Effective People*, is a graduate of the Utah Mentor Academy and a Verizon Grant Recipient. Currently she is a DSC Adjunct Mathematics Professor and the WCSD Mathematics Coordinator.

Her philosophy towards mathematics is: "Students have different abilities, needs, and interests…yet everyone needs to be able to use mathematics in his/her personal life, in the work place, and in further study. All students deserve an opportunity to understand the power and beauty of mathematics. Students need to learn a new set of mathematics basics that enable them to compute fluently and to solve problems."



Karl Jones Award (Elementary)

Laurie Murdock (6th Grade, Terra Linda Elementary) loves teaching. It's evident in her smile when she's with her students and her pride in their learning.

Laurie hit the ground running beginning her first year of teaching. She rarely passed up an opportunity to learn regardless of the subject matter. She's been involved in the Jordan History Academy, Balanced Literacy training, REACH Training, Video Journaling, ESL training, Writing Workshop, Core Academy, Jordan math courses, and is currently the Engaged Classroom teacher for her school. She has developed a well-rounded understanding of teaching and implements strong research based practices in her instruction. Her varied interests strengthen her math instruction because of her ability to see correlations between content areas. Studying Ancient Egypt will also be an exercise in thinking about geometric solids, surface area, angles, and other geometry concepts. Each new topic is a new opportunity to explore the mathematics of that domain. That's why children in her classroom feel lucky to have her for a teacher.

Last year, Laurie completed her Utah State Elementary Mathematics Endorsement. She gravitated toward the algebra classes and thrills to share activities from these courses with her students. She firmly believes that her students can be proficient with reasoning algebraically or creating and solving linear equations.

Laurie applies her passion for algebra in her role as facilitator for district courses. She is viewed as knowledgeable and as a teacher who understands both teaching and learning. Laurie is full of energy and life, and is respected by her colleagues whether in the school or district. She is one of those teachers that children remember years later. While she might not recognize this just yet, she is creating a legacy of learning in her classroom.

George Shell Award (Secondary)

Michelle Jones

I started my teaching career when I was a junior in high school. I struggled to understand the concepts when my teacher was explaining them, so I would read the textbook and teach myself. I would then teach a few of my fellow classmates. When I saw those students understand, it was so cool. I decided then and there that I wanted to become a math teacher. I officially started my teaching career at Timponogos High School, where I spent my first two years. I have been teaching at Pleasant Grove Jr. High School for the past nine years, minus one year that I took off to work at the district office as the district math specialist. I believe that all students can learn and that it is our responsibility to help them be partners with success.

Don Clark Award for Lifetime Contributions to Mathematics Education

This year, the UCTM board is proud to announce Barbara Kuehl as the recipient of the Don Clark Award. Barb exemplifies the qualities of this lifetime achievement award and has touched the lives of so many of us as a Mathematics educator in Utah.

Barb's background started in Secondary Mathematics which includes teaching at Churchill Jr. High, Cottonwood High School, and the University of Utah. At Cottonwood, she co-created and team-taught an integrated mathematics and physics program.

Barb's career took her to Jordan School District where she worked as a Secondary Mathematics Specialist and Mathematics Consultant. During this time she co-developed the IMPACT project. She is currently the Mathematics Supervisor in Salt Lake City School District.

Her work has consisted of:

- Taught *Developing Mathematical Ideas* courses for elementary teachers in Jordan and Salt Lake School Districts, as well as the CITES Math Initiative.
- Served on the 2008 Core Revision Committee.
- Worked with the development team for the USOE Reading, Writing, and Technology in Mathematics professional development and the Focus on Functions professional development.
- Served as local arrangements co-chair for the NCTM Annual Conference in Salt Lake City.
- Facilitates courses for teachers and administrators in the Mathematics Leadership Program at Mt. Holyoke College in Massachusetts.