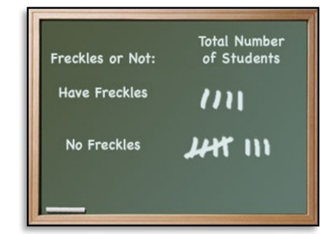
Apple Harvest

Of 6000 apples harvested, every third apple was too small, every fourth apple was too green, and every tenth apple was bruised. The remaining apples were perfect. How many perfect apples were harvested?

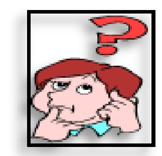
Utah Mathematics Teacher

Fall 2009

Volume 2, Issue 1



What does it **say?**



What does it **mean**?

How can I **apply** it?

http://www.uctmonline.org/

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UCTM 2008-2009 BOARD OF DIRECTORS

Call for Articles

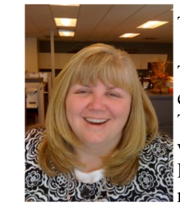
The ***Utah Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Some of the features are: UCTM Leader Spotlight; Letter from the NCTM President; Letter from the UCTM President; Voices from the Classroom; Mathematics for English Language Learners; Puzzle Corner; Recommended Readings and Resources; the Utah Elementary Mathematics Curriculum and Implementation; College and University Research; and others.

Teachers are especially encouraged to submit articles for ***Voices From the Classroom***, including inspirational stories, exemplary lessons, beginning teacher ideas; or managements tools. Sample ideas are (but not limited to) focused on the CMI framework, teachers or districts who have successfully implemented the CMI, Inquiry based calculus, new math programs K-12, the U of U/ Jordan District partnership master's degree program, Cross-district Algebra assessments, and many others.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Christine Walker (Christine.Walker@uvu.edu). A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included. Items for ***Beehive Math News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

UCTM Awards '08-09

Karl Jones Award

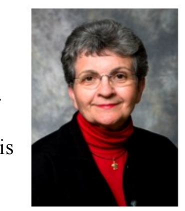


Tamara Bird

Tami Bird has greatly enjoyed teaching mathematics to elementary students in Murray and Jordan School Districts. Tami is National Board Certified, holds a Masters Degree with emphasis in Literacy and endorsements in Elementary Math and ESL. Tami currently serves as an elementary mathematics teacher specialist in Jordan School District. In this role, she also enjoys teaching mathematics classes for teachers. Tami attended the Mt. Holyoke Summer Math program for Teachers in Mt. Holyoke, Massachusetts where she prepared to facilitate professional development classes from DMI (Developing Mathematical Ideas). Tami helps oversee the elementary mathematics endorsement program for elementary teachers in Jordan School District. Most importantly, Tami and Jim, her husband of 26 years, have two daughters, a son, and three dachshund dogs.

Don Clark Award

Eula Monroe



Eula Monroe has been in education for fifty years. Her reason for going into the profession was to make a difference in the lives of others. She believes good teachers made a difference in her life, and she wanted to pass that on to others. Eula's favorite thing about teaching is the people, both teachers and students. She has high expectations for her students, but she is willing to do whatever it takes to help them meet those expectations.

Eighteen years ago, she left her home state of Kentucky to come teach in the Department of Teacher Education at BYU. She has authored or co-authored several books including a series on problem solving, three basals, and a math dictionary. She has also written numerous articles.

For the past four years, Eula has been teaching courses for the state's Math Endorsement for Alpine School District. Eula is a great example, thank you Eula Monroe!

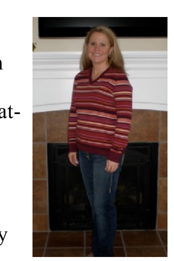
George Shell Award

Donna DerDerian Hall

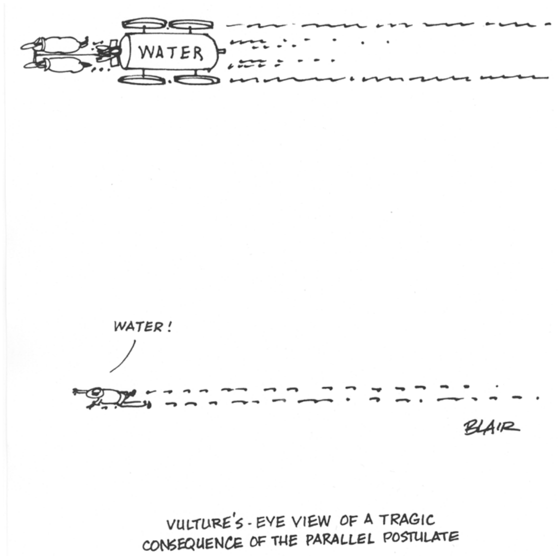
Donna knows that it is important to understand how children learn mathematics from the "ground up" so she has worked toward degrees in mathematics and education resulting in certification in grades k-12. Her first teaching job was as a kindergarten teacher in New York State. She was amazed at how much those little people could learn in one year and how easily they took to math-oriented activities. After that year personal circumstances brought Donna to Utah where she began working at Park City High School in 1997. Donna's career in Park City has run the gambit - from teaching Alternative Education in Mathematics to Calculus and Concurrent Enrollment classes. As department chairperson Donna has been influential in creating specialized classes for the subgroup populations including a specialized Geometry and Math Skills class.

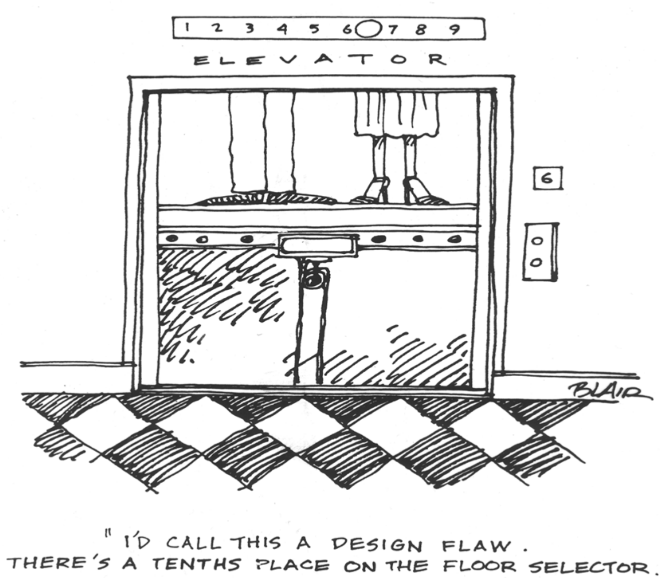
Muffet Reeves Award

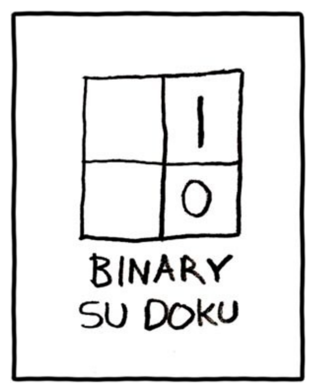
Heidi Kunzler

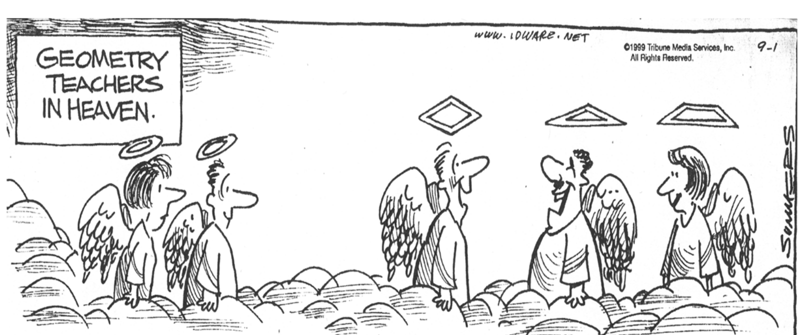


Heidi Kunzler served as the Alpine School District Elementary Math Specialist for 4 years (2002-2005). During that time she was instrumental in guiding teachers to rethink the way children learn mathematics. Heidi was very influential in her role of supervising of Professional Development that focused on teaching through a three-part inquiry-based model of launch, explore, and debrief. This model allowed for many students' needs to be met as they engaged in worthwhile mathematical tasks. Heidi helped build a foundation for quality mathematics instruction in Alpine district. She now works as an elementary school principal in Salt Lake City School district where she continues to influence teachers and provide them with professional development opportunities in mathematics.









Utah Mathematics Teacher

Table of Contents

Fall 2009 **Volume 2, Issue 1**

NOTES:

Presidential Award Elementary Awarded

Natalie Robinson

We congratulate Natalie, she was unable to join us today but as the recipient of this award she will visit Washington D.C. and receive a $10,000 honorarium.
This school year the Presidential award program is accepting nominations for elementary level teachers. We encourage you to nominate a colleague. For those at the secondary level nominations and the application process will occur next year.

Presidential Award Secondary State Finalists

Joy Coates

Joy Coates currently teaches math and physics at Cedar High School and serves as the secondary math specialist for Iron County School District. She received her associate's degree from Dixie State College, her bachelor's degree from Southern Utah University, and is working on her master's degree at Southern Utah University. She has enjoyed participating on the U.C.T.M. board this past year. Joy really enjoys getting students actively engaged in learning experiences and believes that all students can thrive in a challenging environment when they use mathematics in a meaningful setting.
Joy is married to fifth-grade teacher, Brandon Coates. They enjoy spending time together with their five children in the mountains, at the lake, and working in their garden

Troy Jones.

Troy is currently in his 19th year of teaching. He has taught math at the middle school, high school, and university level. He currently teaches math at Westlake High School in Saratoga Springs. He has been a member of NCTM since 1993, attending and presenting at many regional and national conferences. Troy served for 8 years as treasurer for UCTM.
Troy finds pleasure in recognizing mathematical patterns in the world and modeling these patterns with dynamic geometry software and graphing utilities. Some of his favorite investigations have been the mathematics of rugby, modeling projectile motion, the geometry of piles of salt, as well as points of concurrency in triangles and tetrahedra. He enjoys sharing these insights and opening up mathematics to his students.

Janet Sutorius

Janet Sutorius has been a teacher for 23 years. She is a graduate of Brigham Young University, where she earned a bachelor's and master's degree in math education. She teaches fulltime at Juab High School, while also serving as the district's math specialist. In the last three years, she has worked to help develop the state professional development workshops, *Focus on Functions* and *All Things Rational*.

Janet has been an instructor for math endorsement classes for Southern Utah University and an adjunct instructor for Snow College. Janet also enjoys reviewing transcripts for The National Council of Teachers of Mathematics.

Janet has been very instrumental in making it possible for teachers in rural Utah to participate in master's degree programs and earn their degrees in education with an emphasis in elementary mathematics. Last summer, 25 teachers graduated with their masters degrees as a result of Janet's efforts.

Probability Isn't Crap

Lindsey Cracroft

As a senior in high school, I took AP Statistics. I had never taken a stats class before. The extent of my knowledge was how to find averages and some basic probabilities. My AP Statistics teacher was outstanding and can be credited with furthering my knowledge in statistics and probability. One specific lesson in particular has stuck with me over the years.

I had class first period which began at 7:30 in the morning. Not the most ideal time to do math since I wasn't fully awake yet. Although it was early, she kept me focused and excited by her cheerfulness and enthusiasm throughout the different activities she proposed. One particular morning, she informed us that we were going to play Craps.

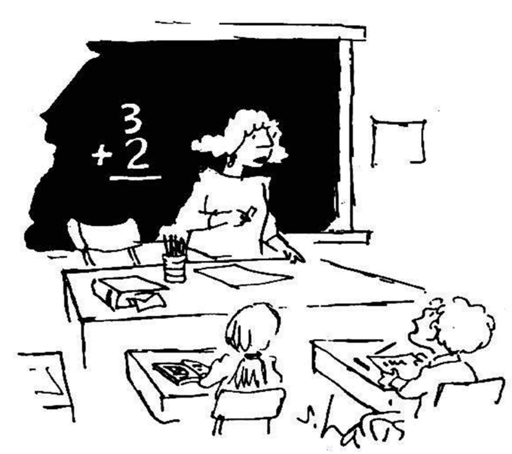
We were bewildered and had to confirm with each other to make sure we'd heard right. "Craps? Like what they do at casinos? Is that allowed in school?" She assured us it was fine since it was applicable to the curriculum, and we wouldn't be gambling.

She explained the basics and how probability affects the outcomes and proceeded to hand out a worksheet to be filled out during the game. The worksheet contained two calculations to compute. The first was a calculation of the probability was of different totals of a roll, such as 2, 3, 4, etc. To roll a 7 or 11 were winning rolls and we had to calculate the probability of rolling a win.

Working off these probabilities, the teacher then divided us into groups of four with a total of two dice per group. As a group we found a spot in the room and commenced playing. The second calculation was a record of our actual outcomes from the rolls.

After playing for about 20 minutes, we then had to compare our findings with our previous calculations. In comparing predictions to recorded results, we found that probability is not always exact, but gives a good general idea of outcomes.

A valuable lesson was learned that day. I recognized that probability is applicable in a real world situation. My teacher found a very effective method to teach us that probability isn't crap, but Craps is probability.



"Do we need this even if we're not planning to go to college?"

The "Socratic Method" and additional thoughts

Christine Walker

It is my privilege to introduce the second volume of the *Utah Mathematics Teacher*. Launched in 2008 as a general interest public school teacher journal to showcase the talent of our very own teachers, the journal reflects the depth and diversity of thought and academic excellence that Utah teachers strive to achieve.

The journal's publication is the culmination of a yearlong editorial process, carried out by the UCTM board and the dedicated reviewers. I would like to take this opportunity to formally thank each of them for their committed service in this ongoing work.

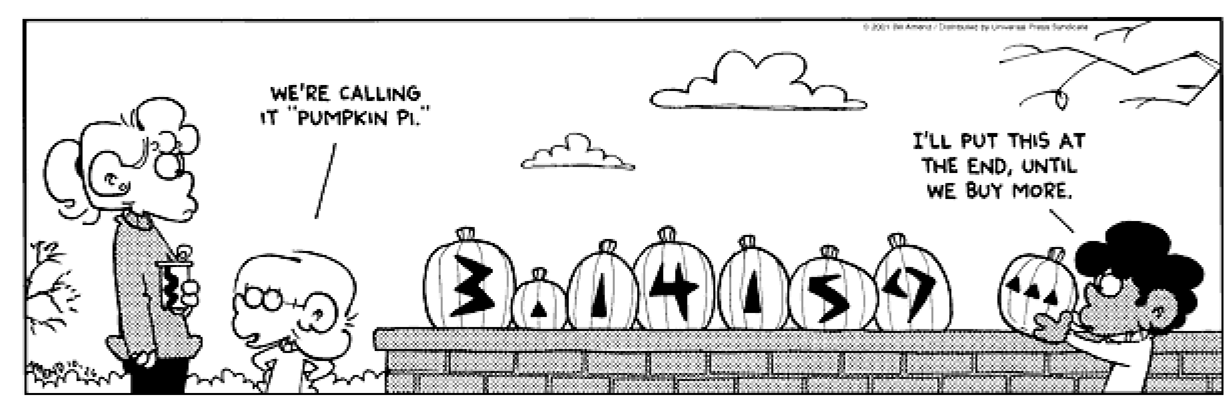
The fall 2009 theme, *Making a Difference: the 21st Century Mathematics Classroom*, focuses on a core question of "why" not "how." For Socrates, he readily acknowledged his own ignorance and recognized this as a first step in acquiring wisdom. He pursued wisdom by means of a process we now call the "Socratic method," which involves questioning answers more than answering questions.

Oftentimes we see our students solve word problems, such as interest rate, where an individual invests $1000 compounded quarterly at 7% interest rate, and concludes that after one year the individual will have accumulated $93,621. Wow…sign me up! How do we as educators shift the focus in our classrooms from "how" (procedural understanding) to "why" (conceptual understanding)? Considering the sample problem, why would $93,621 be an unreasonable amount to accumulate, and how can we get our students to reason out "why" the solution is illogical? These are the issues you face each day. It is a life-time struggle with the goal that within these pages of the 2nd Volume you will find some ideas that will inspire you to continue in this great work that each of you do.

It is a joy to serve as your president and I look forward to another year of working with and alongside many of you as you go about your task of educating the next generation of Utah leaders. Thank you each for your dedicated service to the children in our great state.

Sincerely,

Christine Walker, UCTM President



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Making a Difference:
The 21st Century Math Classroom

As you and your students are settling into a routine, I welcome you to what promises to be an exciting year. Congratulations on participating in one of the most valuable activities you can undertake as a teacher – continuing to learn from your colleagues and your students – as you participate in activities of the Utah Council. Teaching is a love of learning – first, so that you can continue to expand your knowledge of the mathematics your students are responsible to know, and second, so you can expand your strategies in engaging those students in significant mathematical reasoning and applications. The National Council of Teachers of Mathematics encourages and actively supports opportunities for teachers to work with colleagues and experienced leaders in reflecting on how to actively engage your students in the mathematics they are responsible to know.
I hope you will be attending **Making a Difference: The 21st Century Math Classroom** this fall. Throughout the conference, keep thinking of what you have learned from your students so far this year. What were the surprising, exciting thinking strategies they brought to your mathematical assignments? What were the challenging ones that made you struggle with how to help a student revise a learned idea? In each case, remember how these enriched your preparation to teach mathematics as well as to see the mathematics in a slightly new way.

Throughout your conference, or communications with colleagues, be prepared to examine the mathematics from multiple perspectives and to use these views to engage students in doing mathematics and explaining their work. Throughout the conference, reflect on your students' mathematical work – their written work and oral explanations in class.

If this is your first conference, get to know at least three new colleagues whom you may keep in contact with professionally. If you are an alumni, renew professional contacts and reinforce your commitment to sharing throughout the year. Record three new mathematical ideas, instructional strategies, or resources that you will take back to your school and classroom to benefit your students' learning.

I am sorry that I won't be able to join you, but I hope to meet you at an NCTM or Utah professional development event in the near future. I look forward to what you have learned from listening to your students' reasoning as they grow mathematically.

Sincerely,

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Students who learn by doing should be asked to perform, create, or do something to demonstrate their learning. Such assessments are more difficult to grade than multiple-choice exams because they require the use of rubrics or other grading standards. They cannot simply be run through a grading machine, but they are better indicators of the learning that students have experienced.

In conclusion, I contest that authentic (valuable and meaningful) assessment is much more than mere testing. Authentic assessment aligns with the principles and practices we value in education. It is coupled with learning objectives, and it seeks to measure them in the best way possible. It is designed with the specific needs of individual learners in mind, and it is not widely implemented across large populations of students. No effective "one size fits all" assessment program has yet been created. Authentic assessments are more time-consuming, expensive, and difficult to administer than traditional testing, and they do not easily yield comparative information about school performance. However, when authentic assessments are used, students like Alex and the rest of my *Math Interventions* class have a chance at showing what they really know.

I call upon math educators and policy-makers everywhere to be courageous enough to establish a better assessment system. Throw out the ancient, biased, and ineffective, and adopt a more authentic assessment program – one that actually allows students to truly demonstrate what they know and can do. If we are successful in establishing better assessments, the public image of schools will improve, teaching practices will be stronger, and student achievement will increase.

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WHAT WE VALUE VS. WHAT WE DO

There is a jarring conflict between the things that educators value, and the way that we assess our students. We value active learning yet we ask students to quietly sit in their seats during 3 hour testing blocks. We value cooperation and group work yet we quickly pounce on students who seek help from others during tests. We value holistic tasks, process learning, problem-solving, and reasoning yet we give our students multiple choice exams where there is only one right answer to every question. We test our students on factual knowledge and algorithms. We ask them to regurgitate memorized information, and we ask them to do it in a way that a computer can easily interpret their responses for us. No wonder Alex thinks these tests are “lame;” they do not align with the things we, as educators, say we value. We as math educators need to assess better. And we need the backing of policy makers to support us in that change.

There has been much debate about the purpose of public schooling, and I do not dare to approach that topic in this paper. However there are overarching goals of public education that are commonly accepted (at least to some degree). Society believes that schools should prepare students for careers and opportunities in higher education. Society believes that schools should foster the development of social and communicative skills. Society wants students who are prepared to contribute to government, business, and academics so that prosperity (both individual and collective) may continually be afforded to all of its members. Society wants students who understand principles of gender, economic, and racial equality. Society wants schools to produce students who can make the world a better place.

Members of society expect much of public schools and policy-makers are generally willing to invest in society’s expectations (especially if it increases their chances of re-election). They rightfully seek evidence that public money is being well-spent, that schools are fulfilling their charge to do all that is expected of them. I find it interesting however, that policy-makers have selected assessment systems that in no way attempt to measure the ability of schools to accomplish the goals identified above. They select the cheapest and easiest method for measuring something … and a narrowly defined ‘something’ at that. They seem to be satisfied with any kind of data that can be normalized and compared regardless of whether it indicates that schools are meeting their many varied expectations or not.

Perhaps the values we have as educators and the expectations society has on public schools are things merely said and not truly believed. If we truly believed in valid assessments that align with our educational values, would we not invest the extra time and effort required to implement them? If we truly believed that success of schools is broader than can be represented by a score on a multiple-choice test, would we not invest more time and money in developing better ways of measuring school success?

ALTERNATIVE WAYS OF ASSESSING

Since the primary purpose of assessment is to measure student learning, there must be some connection between the two. In order for assessment programs to give more valid and meaningful information on student and school performance, selected assessments need to be more closely aligned with accepted theories of learning. Some of the most popular theories of learning are that knowledge is constructed by experience and that it can be manifested on many different levels (Skinner, 1968 & Bloom et. al., 1956). Assessments should therefore be grounded on the experience by which knowledge is developed and should emphasize the various levels of understanding.

For example: A student who has learned the relationship between volume and surface area of three-dimensional objects by measuring, exploring, and practicing with formulas should not merely be asked to select one of five possible answers on a multiple-choice question. The student should be given an opportunity to discuss the learning process and demonstrate their understanding of the concept without being limited to the task of selecting of a single right answer. They should be encouraged to discuss concepts verbally or in writing, explaining how and why the algorithms used for finding surface area and volume work. This kind of assessment requires that teachers (or other assessors) have access to each student for at least a few minutes. It is time-consuming, expensive, and difficult to ensure objectivity. However, it is authentic assessment that measures the learning process and root knowledge of learning objectives, not mere memorization of facts or formulas.

Another learning theory upon which assessment methods can be founded is that students learn by doing (Montessori, 1965). Imagine asking a student who has learned technical skills in music or theater to demonstrate their learning on a traditional paper/pencil exam. Imagine requiring students in a computer class to demonstrate their knowledge of webpage design on a pre-printed answer sheet full of bubbles.

Say/Mean/Apply (SMA): A Strategy for Teaching Mathematics Vocabulary

Eula Monroe, Beverly Boulware, and Michelle Baron

The nature of the relationships between thought and language has been a topic debated by linguists for many decades (e.g., Carroll, 1956; Piattelli-Palmarini, 1983). Yet, much like discussions about “Which comes first, the chicken or the egg?” debates about the extent to which language shapes thought and thought shapes language have not resolved the uncertainties. Nevertheless, we can be certain that important relationships between thought and language do exist and that we as teachers must teach the language of a content area in order for our students to comprehend the concepts being taught (Gee, 2004).

The recognition of the importance of learning the language of mathematics has come into its own in the last two decades. In its landmark *Standards* documents, the National Council of Teachers of Mathematics (NCTM) has emphasized the role of language in the teaching and learning of mathematics (e.g., 1989, 1991, 1995, 2000), and many state mathematics curricula reflect this emphasis. Even so, many teachers who are both adept at--and comfortable with--teaching language in the literacy classroom do not believe they know how to teach the language of mathematics.

This article is designed for you if you relate to either of the following perspectives.
Perspective 1: You are not sure how to begin teaching the language of mathematics.

If you are here in your thinking, we first encourage you to begin your language instruction in mathematics by teaching vocabulary. The vocabulary strategies you use in literacy instruction work well for mathematics, so you probably already have many from which to choose. We then describe a strategy that may be new to you but is so straightforward that it can be used immediately. Entitled Say/Mean/Apply (SMA), (Boulware & Monroe, 2008, p. 105), it is a comprehension strategy that we have found to be effective and easy to implement in vocabulary instruction. We provide examples to help you get started using this strategy in your mathematics classroom.
Perspective 2: You already are convinced of the importance of teaching the vocabulary of mathematics but are looking for additional strategies.

In this case, we encourage you to scan this article until you get to the section describing the SMA strategy. You may want to see what it has to offer you and your students.

Why Teach the Vocabulary of Mathematics?
As early as a quarter of a century ago, mathematics was recognized as the most difficult content material to comprehend, “with more concepts per word, per sentence, and per paragraph than any other area” (Schell, 1982, p. 544). Research in the field of literacy helps us recognize that comprehension is heavily influenced by vocabulary (word) knowledge, with the greatest amount of variance in comprehension accounted for by vocabulary knowledge (e.g., Ouellette, 2006; Stahl, 1999). If we accept Vacca and Vacca’s premise that “words are labels . . . for concepts” (2002, p. 163), then vocabulary (word) knowledge must indeed be important to mathematical thinking.

Perhaps, however, the following excerpt from Vygotsky’s (1934) seminal work on thought and language provides the most convincing rationale:
The relation between thought and word is a living process; thought is born through words. A word devoid of thought is a dead thing, and a thought unembodied in words remains a shadow. The connection between them, however, is not a preformed and constant one. It emerges in the course of development, and itself evolves. (Section VI, ¶ 10)

Convinced but still a bit fuzzy on what to do? Again we go to the field of literacy for guidance. Hoffman, Baumann, and Afflerbach (2000) offer us three clear findings from research on vocabulary:

1. Almost any kind of systematic attention to vocabulary development produces positive learning results.
2. Studying fewer words in depth typically produces better results than studying larger numbers of words superficially.
3. When the learner finds the words under study to be meaningful, he or she can learn them more readily.

Because these findings are not prescriptive, they also are not limiting. The first finding establishes the need for systematic attention to vocabulary and, at the same time, promises student learning as a result. Inherent in the second finding is the caution to select carefully the words you plan to teach and to teach them for deep understanding.

The third finding encourages the teacher to identify mathematics terms that students have a need to know. These three findings free you to use your professional judgment in choosing the strategies best suited to the words to be taught and to the students you are teaching.

Say-Mean-Apply (SMA): The Three Questions

Language has layers of meaning (Raphael, 1986; Beck, McKeown, & Kucan, 2002), and the SMA framework asks three questions that guide students through the layers to internalize depth of understanding regarding the mathematics concept addressed. First, they learn the literal use of the word, framed in the question *What does it **say**?* Next, they interpret the word by sharing thoughts on *What does it **mean**?* Finally, they relate the vocabulary to their individual experiences by responding to the following: *How can I **apply** it?*

What does it say?

Students should read the word, either orally or silently, within the context of the sentence, paragraph, or graphic in which it appears. Sometimes mathematics vocabulary is difficult to decode, so reading the word aloud in context helps learners clarify and then remember the correct pronunciation. You may also ask each student to write the term on an index card or in a math journal.

What does it mean?

The oral or silent reading in the previous question provides a literal understanding for use in initiating a discussion of students' thoughts on the meaning of the word. You may want to guide your students into meaning making by showing them pictures or objects that relate to the mathematics term. They may share the meanings they construct either orally or in writing, again using index cards or math journals.

How can I apply it?

After students have discussed the meaning of the word, they can now connect their new knowledge with other mathematical concepts, ideas in other content areas, or their real-world experiences (NCTM, 2000). Depending on the age of the learners and the word being studied, objects can be located, pictures drawn, or personal or practical uses explained. These connections can again be discussed aloud and written on their index cards or in their math journals.

Teachers may apply the three SMA questions to any area of mathematics. Examples to use for teaching the strategy may be derived from many different resources. The following are three examples of the use of SMA in reading printed mathematics text.

THE ONE SIZE FITS ALL APPROACH TO ASSESSMENT

One of the lessons that teachers must learn early in their career is that there is no such thing as a "one size fits all" approach to education. Educators work in an environment of individual circumstances, personal preferences of style, and challenging new problems that require constant changes in practice. Consider again the example of Alex and his peers in my math remediation class. In the class of about 18 struggling students, I had some who were so introverted that it was difficult to engage them in any way. There were a few who were so talkative that they had difficulty focusing on any task for more than a few minutes, and I had 2 or 3 with severe behavioral and social problems that required most of my attention during class.

Having a group of students with such diverse needs, if I had stood up at the front of the room and didactically delivered a 45-minute lecture on basic math skills, I would have been unable to engage a single one. Each student had individual circumstances that required me to provide some kind of differentiated instruction in order for learning to take place. My class was not an anomaly with regard to the need for differentiated instructional strategies, and most agree that differentiated instruction is good practice in every classroom. Why then is it so difficult for policy-makers to see the value of differentiated assessment? Why are we using a "one size fits all" method of assessing students? Instruction and assessment are not separate elements of a student's education but are intimately connected. Students who require differentiated instruction in order to learn will surely require differentiated assessment in order to fully demonstrate their learning.

Please note that in this discussion, when I refer to assessment practices, I do not speak of teacher-directed classroom assessment. I speak of policy-driven state assessment programs, often based on commercially developed, multiple choice exams. Proponents of multiple choice assessments argue that they are the most practical way to assess large numbers of students. I disagree. It is true that current practice is one of the most inexpensive ways to assess large numbers of students, but in order for something to be termed "practical," it must also be of good quality and beneficial. I don't think that Alex and his peers would classify the current assessment system as either.

There is other evidence that assessment programs are failing certain groups of students. Passing rates of high stakes tests are significantly lower for economically disadvantaged and racial or language minority students. A look at state end-of-level test scores in one Utah district shows the severity of the discrepancy. The average pass rate of African American students is nearly 25% lower than that of Caucasian students (Davis School District, 2007). In the same district, economically disadvantaged students, and those with limited English proficiency had significantly lower pass rates as well.

For further illustration of the inequities that result from current assessment practices, I will once again return to my experience with Alex and his peers. They were enrolled in a remedial math course called *Math Interventions*. The course was designed for students who had already failed the UBSCT – Math test. In a school where over 90% of students were Caucasian (Davis School District, 2007), 7 of the 18 students in my class were Hispanic or African American. All but 2 or 3 were economically disadvantaged, and over half were legally classified as special education students (working under an Individualized Education Plan). The demographic makeup of my *Math Interventions* class did not match that of the whole school. It seems that at some point in the math education of racial minority, economically disadvantaged, and special education students we're missing something important.

Perhaps we are failing these students in the instructional phase of teaching, but in my experience the greater problem is with the assessment itself. I watched as my *Math Interventions* students worked hard in class to learn math concepts. I watched as they experienced success and helped each other develop basic skills. Although they had repeatedly demonstrated to me that they had mastered the skills that would be measured on the UBSCT, I watched them worry and stress as test day approached. I watched as they took the test. A few gave up and began filling in random bubbles on their answer sheets. Most did not finish in the allotted time. All left the room feeling frustrated and disappointed – convinced that they had surely failed the test again.

Often parents and teachers say, "he doesn't do well on tests," or "she's not a good test-taker." The problem is not that we have students who don't do well on tests. The problem is that we have students who don't do well on tests *like the ones we keep giving them*.

Although high-stakes testing comes in many forms, one of the most popular is the use of a graduation exam – one which students must pass prior to receiving their high school diploma. In 1980, North Carolina became the first state to implement a policy requiring students to pass a test before graduation (Amrein & Berliner, 2002). By the year 2000 there were 18 states that had adopted a similar test. With the passage of the No Child Left Behind (NCLB) Act of 2001, many states were forced to re-write their testing policies and included graduation exams in them. Only two years after the passage of the NCLB Act, Amrein and Berliner (2003) reported that there were 28 states that had adopted the use of graduation exams into their testing policies.

The increase in the use of graduation exams has not come without opposition. Ravitch (2002) pointed out that many professional educators oppose high-stakes testing in order to maintain professional discretion. A group of parents in Texas filed a lawsuit in federal court claiming that graduation tests were unfair to African Americans and Hispanics. The court did not rule in their favor (Branch, 2000). Despite opposition, policymakers and courts have consistently upheld graduation exams as a valid means of determining student achievement and holding schools accountable for student learning.

In order to fully understand the effect that these exams have on students, policymakers should develop a solid understanding of the nature of such exams. As explained by Burger (2002), the federal No Child Left Behind Act requires that a state's accountability system (including exams) be aligned with content standards established for each grade level. Although graduation exams are taken during the high school years, a brief overview of Utah's exam framework reveals that nearly all of the standards and objectives tested on the UBSCT-Math come from the curricula of Pre-Algebra, Elementary Algebra, and Geometry – courses recommended for students in grades 6 through 9 (USOE, 2002a, 2002b, 2002c, 2005). Utah is not the only state that uses basic skills as the foundation of graduation exams; another example is Alabama. Most of the content of Alabama's graduation exam is taught in the Jr. High grades as well (Alabama State Department of Education [ASDE], 1999). One could rightly expect basic skills to come from courses in lower grades, but a problem arises when students are in high school and are no longer exposed to basic skills in their regular course work (especially if they never truly mastered it when they were in Jr. High).

For the students who fail to master the basic skills in the early grades, the misalignment between the content of high school courses and the graduation exams makes it even more difficult to pass the exams. In a discussion about alignment between assessment content and classroom instruction, Burger (2002) said, "standardized … tests, used traditionally for accountability, have only partially aligned with curricular materials and classroom instruction. These conditions obviously result in poor test scores" (p. 2).

Another problematic characteristic of high-stakes exams across the nation is that they are typically formatted to contain multiple-choice questions that can quickly and easily be graded by a computer. There have been countless discussions about how poorly multiple choice questions reflect student learning – especially in content areas such as writing and mathematics where process and reasoning are most indicative of student achievement. Multiple-choice tests are often selected because of the ease and cost-effectiveness of their implementation, but they are not good measures of student achievement (National Center for Fair and Open Testing, 2007).

In addition to content misalignment and the problematic question structure (multiple choice) discussed above, there are many other factors that contribute to poor test scores on graduation exams; poverty, low family support, and poor school attendance are just a few (Nichols, 2003). However, schools prefer to focus on addressing factors over which they have control. Providing additional instruction on test material during the high school years is one way for educators to positively influence test scores. Such a practice is commonly called remediation, and it has become a priority for many school districts across the nation.

Example 1: Children's Textbook
Equal Parts

What does it **say?**

"A fraction can name <u>equal parts</u> of a whole shape" (Charles, Crown, & Fennell, 2004, p. 271).

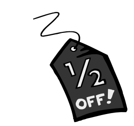
What does it **mean**?

Each part of something is the same size as another part of the same thing.

How can I **apply** it?

My dad baked a cake for my birthday. He cut it into equal parts so that we could each have the same amount.

Example 2: Math Dictionary
Discount



What does it **say?**

"A <u>discount</u> is an amount by which the regular price of an item has been reduced" (Monroe, 2006, p. 44).

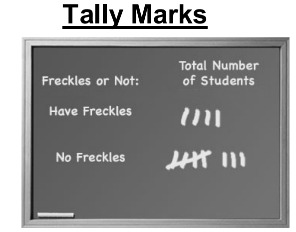
What does it **mean**?

The item costs less than the regular price.

How can I **apply** it?

If I wait to buy a new bicycle until it is on sale, I can get a discount off the regular price of the item.

Example 3: Graphic
<u>Tally Marks</u>



(Illuminations, n.d.)

What does it **say?**

"Tally marks."

What does it **mean**?

A mark to show how many [_________] have been counted.

How can I **apply** it?

I want to know how many toys I have. I can draw one tally mark for each toy. Then I can count all the tallies.

Conclusion

In vocabulary instruction, an important role of the teacher is to help students access the background knowledge they have acquired both in and out of school as they connect what they already know with what they are learning. As illustrated in the examples in this article, SMA provides a systematic framework to guide students in connecting mathematics concepts presented in print or graphic contexts to their prior knowledge and to real-world contexts. Students who internalize the meaning of the term through the *"say"* and *"mean"* steps, then anchor and extend that meaning as they respond to the *apply* question, will probably have little reason to ask their mathematics teachers that most irksome question: "When are we ever going to use this anyway?"

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ASSESSMENT FOR THE MASSES: A MATHEMATICS EXAMPLE OF HOW MODERN ASSESSMENT PRACTICES AND POLICIES ARE FAILING PUBLIC SCHOOLS

Logan T. Toone

INTRODUCTION

The day the test results were reported to the school, I spoke with Alex in the hallway. His junior year of high school was nearing its end, and he was upset with his performance on the test. Alex had been enrolled in my math remediation class for three consecutive semesters, and he was sick of "not being good enough" to pass a test that "everyone else" had passed a year and a half ago. Since 2004, students in Utah have been required to pass the Utah Basic Skills Competency Test (UBSCT) as a requirement for receiving a basic high school diploma. Alex had just received the news that he had once again failed to pass the math section of the UBSCT. With his senior year ahead, he faced another semester of my remedial math class or as he called it "dumb-kid math." He doubted that he could ever pass such a "lame test," and he was ready to give up and stop trying altogether.

Alex was a victim of high stakes testing, and despite his feeling that he was the only one who had not passed the UBSCT, there were about 15 others in the same predicament. I had worked with Alex and his peers for nearly two years, and I would not categorize them as "dumb kids" or even as kids who "couldn't do math." They are students who struggled with high-pressure testing environments. They are students who were not fully able to demonstrate their knowledge of math concepts on a multiple-choice answer sheet. They are students for whom the UBSCT was a poor method of measuring competency in basic math skills. In my classroom, they had repeatedly shown that they knew the concepts they would be tested on, but for some reason on test day, they froze. They did not perform well enough to qualify for graduation.

In the past 30 years, public education in the United States has been subject to increased scrutiny by federal, state, and local government and the general public. Prior to the 1980's, there were critics of education, but they were typically educators expressing their views on a better way of educating the nation's youth. Never before has there been so much media exposure and public discussion about the policies and practices of public education.

Government, civic, and business leaders, as well as parents and other members of society (most of whom are not educators) have increased their involvement in lobbying and petitioning for changes in educational policy. Their recent mantra is one of failing public schools and a call for increased accountability. They rightfully argue that as taxpayers, they fund public schools, and thus they should be able to see that their money is being well-spent. The problem lies in defining what "well-spent" means. What constitutes success in a public school? What goals should schools seek to accomplish? How can a school demonstrate that it is doing a good job? How do you truly define and measure success in public schools?

The connection between the call for accountability and a system of assessing school and student performance is not a difficult one to make. It seems logical that assessment would be a reasonable mechanism for holding schools and students accountable to stakeholders, but policy-makers have selected an assessment system that is failing many students like Alex. In this paper, I will first analyze the prevailing philosophy of "one size fits all" assessment systems identifying specific inequities that result from such an approach. Following the analysis, I will discuss the inconsistencies between current assessment practices and the qualities we value most in education. I will conclude with a brief discussion of assessment techniques which could be considered as alternatives to the current system.

THE EXISTENCE AND NATURE OF HIGH-STAKES EXAMS

Although testing was not traditionally used as a means for holding schools accountable, testing and accountability are intimately connected in today's education world (Ravitch, 2002). High-stakes tests have been used more and more to determine the success of schools, teachers, and students. According to Amrein and Berliner (2002), a high-stakes test is one from which results are used to make important educational decisions about schools, teachers, administrators, or students. The consequences that have traditionally been attached to high-stakes tests include publication of test scores in local newspapers, school closures, disciplinary proceedings for teachers, and denial of grade advancement or graduation (Amrein & Berliner).

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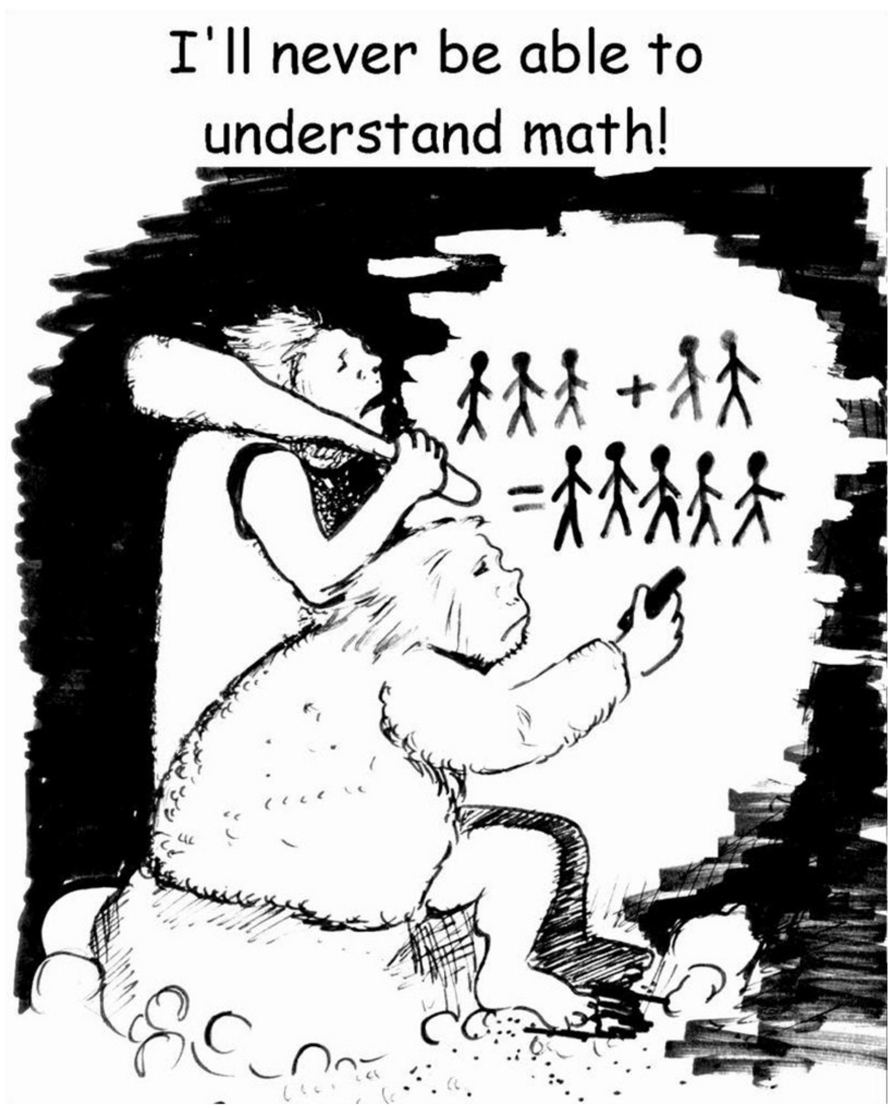

Using Video Recordings to Collect Data about Student Thinking
Teach by listening and learn by talking

Jessica Munns, and Brynja R. Kohler

INTRODUCTION

The math methods course at Utah State University is unique in its extensive field component. Prior to student teaching, preservice teachers have the opportunity to work in middle school and high school classrooms designing and testing lessons. The entire methods class meets out in the schools for eight weeks of the semester, and collaborative groups of preservice teachers from the university work with specific inservice teachers in their classrooms. During the meetings in the schools, the groups have one hour of collaboration time to develop or revise a lesson followed by two hours of instructional time when a preservice teacher has the chance to lead the lesson with middle school and high school students in their regular classroom setting. The methods course professor facilitates the collaborations, lesson development and evaluation, and observes and supports the classes. This program has been successful for many years due to the enthusiastic participation of teachers, and the diligent work on the part of students of the methods course.

We spent a semester (Fall 2008) videotaping this collaboration to get a better understanding of the components of the program that lead preservice teachers to succeed in their student teaching and careers as teachers, to find ways we can improve the program for the inservice and preservice teachers involved, and to provide a more sustained support for teachers and the work that they do. This semester's research, which involved additional collaboration time for inservice teachers and a graduate student researcher doing the videotaping, was possible due to a Mathematics Education Trust grant and funding from the Park City Mathematics Institute.

In trying to observe the effects or variables of the collaboration between inservice and preservice teachers, we decided to videotape as many different elements of the process as possible. In the collaborative sessions, we observed many discussions involving how to gauge student understanding during a lesson. Teachers shifted their focus while developing lessons from themselves (detailing the teachers' instructional procedures), to their students (i.e. how to check for student understanding, how to generate student involvement, how to encourage deeper thinking). This caused us as researchers to reflect on how we could collect data on the effects of this collaboration on student understanding. In an attempt to do so, during group or individual work time we shifted from videotaping the preservice teachers to videotaping students working. Instead of positioning the camera in a stationary location at the back of the classroom with the focus on the front of the room, we began to circulate around the room and capture what students were doing and saying. As we began to do so, we found that students saw us as another source for help. This allowed us to be activist researchers and begin posing questions and interviewing students about their thinking in the middle of the class period. In a sense, this made us as researchers inseparable from the environment we were researching since we took on an active role in the teaching of the lesson. But this also enabled us to collect more relevant and useful data for the collaborative team involved in lesson development and for us in our goals to improve the program for the inservice and preservice teacher participants. Ultimately, we found that students seemed to be learning mathematics by talking to us about their thinking.

Over the course of videotaping student conversations we observed students' mathematical thinking related to lesson topics as well as their broader perceptions of mathematics as a discipline in their schooling experience. The camera in the classroom also served to motivate students to engage in lessons. We found that using videotaping was a good way for us to be involved in teaching, in particular, in developing questioning strategies to help students talk to learn.

STUDENTS REVEAL THINKING RELATED TO THE LESSONS

Once we began to circulate around the class and interview students, we found many opportunities to record various types of conversations. On one occasion, while students were working in groups to discover properties of isosceles triangles, we found evidence of student-to-student reasoning and debate. The following is what two students said to the camera:

Student A: On number 8, it asks the question, "Is an equilateral triangle an isosceles triangle?" And I say no because an isosceles triangle…
Student B: Yea, there are two sides that are the same length and then there is one side that is shorter…
Student A: Or longer.
Student B: Yea, or longer. But on an equilateral triangle …
Student A and B (in unison): All three sides have to be the same.

When students faced the camera to talk about their ideas, this often lead them to discover mathematical relationships and make connections. False assumptions, however, could also lead to the development of false conclusions. One student told us she was certain that adding the same amount to each side of a triangle resulted in a triangle similar to the original. She went on to assert that she had actually proven this by setting up proportions. But when students continued to talk, they would think through their reasoning and often be able to correct their own mistakes. This provided a great opportunity for deeper questioning in an attempt to cause deeper thinking on the part of the student.

Later in this particular lesson, the teacher brought the whole class's attention to the debate of whether or not an equilateral triangle is considered an isosceles triangle. After the whole class had the opportunity to weigh in on this discussion, the teacher asked a student to look up and read definitions from the textbook to settle the debate.

STUDENTS REVEAL ATTITUDES ABOUT MATHEMATICS AND SCHOOL

Even before the whole class discussion on this topic, the conversation above continued, if somewhat off topic, as follows:

Researcher: Do you think that a square is a parallelogram? And why or why not?
Student A: Because uh…
Student B: I don't know because a parallelogram …. Well… yes actually it could be because a parallelogram … what it defines is: two pairs of parallel sides. And a square has those. And a square is a parallelogram but a parallelogram is not a square.
Student A: But on a parallelogram the diagonals are not equal, but in a square they are equal. They're sort of different in a way but they can be similar in a way. Because let's say you draw diagonals in it, and … the diagonals in the parallelogram will be different. But the squares' diagonals will be the same. So I say not really but it could be in a necessary way.
Researcher: So it depends on the definitions is what I'm getting out of it?
Student A and B: Yes
Researcher: And you're not entirely sure what the definitions are, or are you sure about them?
Student B: Different people could define it different ways I guess.
Student A: And a lot of people have different opinions.
Researcher: That would be very confusing if mathematics were like that.
Student B: Well it's kind of like zero, some people say its positive and some people say its negative.
Student A: Math is a world of unknown things, you know, like you could go like forever on without knowing one thing that you're trying to find.
Researcher: Ok, I'm going to end this movie by saying that I think we need to establish some clear definitions.
Student A and B: Yea.

Although very few parents agreed with the three gender-stereotyping questions, when the means were compared between the gender-stereotyping statements, all the parents of girls more strongly disagreed than the parents of boys. Parents of girls more strongly disagreed that it is more important for boys to learn math. They also more strongly disagreed that math is harder for girls to learn than it is for boys. In this study the parents of girls more adamantly disagreed than the parents of boys against the gender-stereotyping questions.

The results of this study indicate that it would be beneficial to educate and encourage parents on how they discuss their attitudes and beliefs about math with their students. They need to be positive and encouraging regardless of whether they had a good experience in math. Schools should also communicate with parents about the importance of parent involvement as it relates to a student's math achievement.

Parents are a predominant factor in how their child believes and the attitude their child holds toward math. Parents need to value math and instill these same values in their students so their students can have increased math success. Parents need to make sure that their beliefs and attitudes are not impeding their student's attitudes and beliefs about mathematics.

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To investigate the difference between the attitudes and beliefs of parents about the importance of math learning for girls versus boys, the gender-stereotyping questions from the parent survey were evaluated. This study found that the majority of parents disagreed with the gender-stereotyping questions which indicate that very few parents hold gender- stereotypical beliefs. The results for each of the three questions are found in table 3.

Table 3

Frequency and Percent of Parent Responses to Gender-Stereotyping Questions

	n	%
Math is more important for boys		
Strongly Agree/Agree	10	5.6
Strongly Disagree/ Disagree	160	94.1
Math is harder for girls to learn		
Strongly Agree/Agree	13	7.6
Strongly Disagree/ Disagree	154	90.5
There are higher expectations for a boy's math learning		
Strongly Agree/Agree	11	6.4
Strongly Disagree/ Disagree	158	92.9

To further investigate these gender specific attitudes, the data from parents of girls was analyzed separately from parents of boys and their means and standard deviations were totaled separately. A mean equal to 1 indicated that a parent strongly agreed with the statement. A mean equal to 4 indicated that a parent strongly disagreed with the statement. Table 4 presents these means and standard deviations, along with the means of the students that answered similar questions.

Table 4

Frequency, Mean, and Standard Deviation of Parents of Boys, Parents of Girls, and Student Beliefs on Gender-Stereotyp

	n	*M*	*SD*
Math is more important for boys			
Parents of Boys	69	3.16	.678
Parents of Girls	101	3.59	.513
Students	170	3.40	.686
Math is harder for girls to learn			
Parents of Boys	69	3.12	.565
Parents of Girls	101	3.48	.611
Students	170	3.38	.689
There are higher expectations for a boy's math learning			
Parents of Boys	69	3.18	.622
Parents of Girls	101	3.53	.558
Students	170	3.34	.697

A *t* test was performed to evaluate if parents of boys as compared to parents of girls felt differently about the gender-stereotyping questions. Parents of boys less likely to strongly disagree with the statement "Math is more important for boys" ($t(168)=-4.753$, $p<.01$). Parents of boys were less likely to strongly disagree with the statement "Math is harder for girls to learn" ($t(165)=-3.852$, $p<.01$). Parents of boys were also less likely to strongly disagree with the statement "There are higher expectations for a boy's math learning" ($t(167)=-3.908$, $p<.01$).

Figure 1. Student A and B tell the camera about isosceles triangles, equilateral triangles, and mathematical definitions.

The initial part of the conversation shows what each student understands about parallelograms. One makes an argument from the standard definition, while the other makes an assertion about the diagonals. Later the conversation seems to reveal that these students had both an appreciation and a tolerance for mathematics that did not make sense to them. The view of the researcher and teachers involved in the lesson design was that mathematical precision and definitions needed stress. The researcher was attempting to reinforce this aspect of the lesson through the conversation, but also found what students felt about mathematics: perhaps mathematics as a subject is not so clearly defined for students.

STUDENTS STRIVE TO BE ON-TASK OR AT LEAST APPEAR THAT WAY

The students' reactions to being on videotape were different depending on the student. Some students were more reserved and didn't like to be on video but others were excited and anxious to share their ideas. They wanted to "impress the camera" or have their discovery documented. This next exchange deals with a student testing a hypothesis dealing with isosceles triangles. The student waved down the researcher and told her that he found something he thinks to be true.

Student: The two angles that are near the two congruent sides [points to angles] there at the bottom, they are always the same and I think that an isosceles…I think it has something to do with an isosceles triangle.
Researcher: Ok so you've tried a couple examples and you think it's always true?
Student: Yea, if it wasn't then it wouldn't be an isosceles triangle would it?
Researcher: You think so?
Student: Uh huh!
Researcher: Thanks.

We observed that often as we were videotaping, off task students did not want to appear off task. Simply by walking around the room students would become on task.

Also, those at the middle school were more willing to be on camera than those at the high school. Consequently, we needed a strategy for getting their mathematical thinking recorded.

ENGAGING THE RELUCTANT STUDENTS

It appeared to us that those students who were less confident about their work did not want to be on camera. To try and get these students to describe their thoughts we tested two strategies. One was to tell the student that they could record the researcher doing one problem if the researcher could then record them in turn. This would serve two purposes, (1) the student would get one more explanation of the type of problem they were dealing with, and (2) the student would have a chance to work the problem while explaining their thinking out loud. The student's verbalization of his thinking helped the researcher to pinpoint and address the students' misunderstanding or lack of confidence related to the problem. The second strategy was to engage the students by having a peer record the researcher and another student discussing a mathematics problem. We found that they were more willing to be taped in this manner. This also engaged two students in the mathematics of the lesson because the one videotaping would pipe in on the topic.

Figure 2. Creating a safe way for students to express their mathematical thinking by putting the researcher in the center of the camera's view.

We observed that once students realized that we would help them while we recorded, they were more willing to be on camera. This also opened a door for questioning strategies that allowed us to not only answer the question directly but also to attempt to cause deeper thinking.

REFLECTIONS ON OUR QUESTIONING STRATEGIES

Our challenge as researchers was to collect appropriate data about student understanding and mathematical thinking. But students do not naturally just reveal their thinking to a camera in the back of the class. This caused us to reflect on our own questioning strategies and helped us realize the parallels in collecting this data to actually instructing or at least supporting the instructional activities lead by the teachers.

As researchers we were challenged to improve ourselves as teachers. Thus, the enhancement of the verbalization of student thinking through video recording is not restricted to a research project. The videotaping gave students a natural forum for explaining their mathematical thinking. Teachers could implement this idea in their classes by having students make recordings of each other or by directly making a movie about the lesson as it takes place. The resulting videos could further be used to enhance discussion in subsequent classes.

Results and Discussion

To investigate the relationship between the level of parent education and the level of math a student is taking, a correlation was performed between the parent education level, the parent highest math taken, and the student math course (see Table 1).

Table 1

Correlation Table between Parent Education Level, Parent Highest Math Class, and Student Math Course

	Student Math Course
Parent Education Level	.239(**)
Parent Highest Math	.205(**)

**p<.01

The correlation between parent education level and the student math course was statistically significant (r=.239, p<.01) which means if the parent had a higher education level then the student was more likely to be enrolled in a higher level of math. Additionally the relationship between the parent highest math taken and the student math course (r=.205, p<.01) was also statistically significant. If the parent had completed a higher level of math then the student was more likely to be taking a higher level of math.

To investigate the relationship between parental attitudes and beliefs about mathematics and student attitudes and beliefs about math learning, survey questions about attitudes and beliefs were summed for a total Attitudes and Beliefs score for both parents and students. A higher score indicated a more positive feeling towards mathematics. The means of the parent attitudes and beliefs along with the means of the student attitude and beliefs were correlated along with student math course, parent highest math level completed and parent education level. The results are shown in table 2.

Table 2

Relationship between Parent Attitudes and Beliefs and Student Attitudes and Beliefs

	Student Attitudes Beliefs	Student Math Course	Parent Highest Math	Parent Education Level
Parent Attitudes and Beliefs	.440(**)	.108	.327(**)	.103
Student Attitudes and Beliefs		.202(**)	.134	.004

**p<.01

The correlation between parent attitudes and beliefs and student attitudes and beliefs was statistically significant (r=.440, p<.01) This indicates that is the parent felt more positively about math, the student was more likely to have more positive math beliefs. Both parent (r=.327, p<.01) and student (r=.202, p<.01) beliefs were related to their math level with higher levels of math related to more positive math beliefs. was the relationship between parent attitudes and beliefs and parent highest math.

If there is a positive attitude about math from parents it is going to help students feel more positive about their math learning. In contrast a negative attitude from parents about math can have a similar effect on students' resulting in negative attitudes about math. Parents need to be very mindful of their attitudes expressed in the home about math and math learning and need to understand the benefits of increasing their own understanding of math and having a positive attitude towards math.

when faced with a math problem. Adults face this same anxiousness, even after they are out of school (Stodolshy). views, beliefs, and attitudes

This inability of some parents to help students with math has not gone unnoticed in the schools. Parents' ability to help their child with their homework falls as the math level becomes higher. Chavkin (1993) reported that 71.8% of parents with an elementary student strongly agreed that they could help their student with their math homework while only 55.4% of parents with middle school age children agreed when asked this same question.

Some parents encourage children to develop skills which are assumed to be natural or appropriate based on the gender of the child (Parsons, Adler & Kaczala, 1982). Accordingly, boys and girls have often had very different towards math. Eccles (1986) found that boys and girls of approximately equivalent math ability have a different perception of the causes of success and failures in the math class they are taking. This perception leads them to have a different perception of the causes of successes and failures in math. If boys and girls are crediting their successes and failures differently they will make different decisions regarding future prospects for success in mathematics courses.

Tocci and Engelhard (1991) found that there were two areas where gender differences occurred. These areas were their attitudes toward mathematics and the perceived usefulness of mathematics within society. They also found large differences between the genders on whether or not they see mathematics as a predominantly male area. Tocci and Englehard said that "females believe more strongly than males do, that studying mathematics is as appropriate for them as it is for their male peers" (p. 284). Girls also reported lower self-perceptions of math ability and were less likely to choose careers in physical science and math (Bleeker & Jacobs, 2004). Boys tended to evaluate their math performance more favorably than girls (Chen, 2003). Another difference between the genders is that male students were more likely than female students to attribute ability and effort as reasons for their math grades (Singer, Beasley & Bauer, 1997).

The different perceptions may stem from parental attitudes and beliefs. Eccles (1986) found that both mothers and fathers thought that math was harder for their daughters than for their sons, despite the fact that the boys and girls had earned equivalent math grades, test scores, and teacher ratings. Eccles also found many parents rated advanced math courses as less important and English and history courses as more important for daughters than for sons. This resulted in girls enrolling in approximately one semester less mathematics than boys.

Parental beliefs, attitudes, and gender-stereotyping, along with the inability to help their children in higher level math classes, is affecting their students' attitudes and beliefs in math and impeding their learning. The purpose of this study was to investigate parents' attitudes and beliefs about mathematics including the impact on their student's attitudes and beliefs about mathematics, including differing beliefs for gender. Additionally, the study sought to find how parents' mathematics level affects the level of math a student takes.

Method

A survey was completed by 170 parent/student pairs at a suburban high school in northern Utah. Participants were chosen from classes covering each level of math taught at the high school. Two classes of each of the following levels of math were chosen: Elementary Algebra, Geometry, Intermediate Algebra, Pre-caclulus/1050 concurrent enrollment, and A.P. Calculus/A.P. Statistics.

The survey consisted of two sections: a parent survey and a student survey. Both surveys asked students and parents about their beliefs and attitudes toward mathematics. Demographic information was asked on each survey and questions regarding their completed level of mathematics were also addressed.

First Year Strategies That Work

Graycee Memmott

Warning... may cause dizziness, nausea, thoughts of regret, and frequent thoughts consisting of "What am I doing here? What am I getting myself into? and "What made me choose this major?" This is the label that should be on the back of all of our acceptance letters into the education program, but it isn't. Standing in front of a room full of teenagers nearly the same age as me was the first time that the shock of these warnings filled my body and mind. In August of 2008, I was able to start my year long internship at Spanish Fork High School at the young age of 20. Along with the feelings of the previously mentioned warnings, I felt under-qualified and terrified that the last two years of cramming through math and education classes would all be wasted if I did not end up enjoying teaching as much as I thought that I would.

From the get go I knew that I had to have a plan; a plan that could keep me calm and organized on both the bad days and the good. I developed multiple strategies to get myself through each day. I have given a lot of thought on which of those strategies I would give to a new intern or first year teacher.

Start out strong! Explain to your class the rules and enforce them on the first day.

Consistency is key! I quickly learned that all students have heard my rules before and very seldom have they seen them enforced. Prove to them that you are serious about your class rules.

It is ok if the only thing you completed on any given day is that which you are going to teach tomorrow. My downfall in the beginning was that I thought that I had to have everything completed everyday. I would stay after school for hours and hours trying to get ahead. Although staying ahead is essential, don't kill yourself doing it.

This strategy is specific to a math class. **Always list the steps when you teach new material.** For example:

Solve for x in the following problem.

2x + 3 = 4 The answer is x= 1/2

Every time I teach something like this I list my steps off to the side.

Step 1: Subtract 3 from both sides 2x +3 - 3 = 4 - 3
Step 2: Rewrite the problem. 2x = 1
Step 3: Divide each side by 2. 2x/2 = 1/2
Step 4: Simplify. x = 1/2

Often times students go home and when they work on their math homework they cannot remember what they learned that day. Also, most students will ask their parents for help and their parents have no clue what they are doing. The steps help the students to retrace their steps with a similar problem to the one that they are doing. That way the problem suddenly becomes possible instead of impossible. It also reinforces the language of math. I write it, I speak it, and then I model the action.

Have fun. Show the students that there are reasons for why you chose this major. Continue to act enthused about your subject. The students may act like they think that you are crazy, but it intrigues them. Although, some may still think that you are crazy and always will.

Always stay updated on new materials, teaching strategies and games for your subject. It is a lot of work, but it will keep you from landing in a rut. Things in this world evolve and change and we have to learn how to evolve and change with it. During college I came up with a fun way to review concepts right before a test. I call it my Relay Race Review. The day before I test, I split the students into groups of four or five and tell them all to move their desks together and pull out a piece of paper. There are a couple of rules:

1. Everyone has to show their work.
2. Everyone must have the right answer to move on to the next question.

I tell them to pick one person in their group to be the runner. The runner is the person that will gather up all of their groups' papers and run them back to me to check. I write the first question on the board and tell them to start. As each group gets done, they run the papers back to me and I check them. If they are right I hand them a little piece of paper that has the next problem on it. The game continues in the same manner for about 12 problems and I usually give treats to the first and second place winners. I have found that this game is a nice break from lecturing and it is a way to trick the students into studying without them realizing it.

Don't assume that you know everything. Take help and criticism with open arms. I invited my mentors and advisors into my classes often. I would always ask them to take evaluations of my teaching. I wanted their opinions and advice because I considered it to be very valuable and beneficial. I kept in mind each day that I was an intern and although I felt completely capable, I was not afraid to run my ideas by my fellow teachers or my advisor. Taking criticism is certainly not easy, but is well worth it. A teacher once told me, "The minute you think you know it all about teaching, is the exact minute you should retire!" We should constantly be learning new things; whether it be new things with classroom management or with our own subject. The stage of being a student never ends.

Above all else be their teacher. Be the person that shows up each day prepared and ready to fill their minds with materials necessary for life. For some students, the fact that we are there each day is more structure and consistency than they experience at home. We have to be careful to keep that imaginary line clear though. That line that lies between being appropriate and inappropriate. I feel that it is instinct to us to reach out to each student and be their friend. I do think that it is necessary to be kind and approachable, but I always remind myself and my students throughout the year that I do not have to be their friend in order to be a good teacher. So again, above all else be their teacher!

Throughout my intern year I learned many things. Although the warnings listed at the beginning of my article were brought to my attention basically the first day of my first year, it was also brought to my attention that I love to teach math. It is the most rewarding career that I could have ever picked. I am thankful for each day that I struggled with my math classes in high school and in college. I was never one of those brilliant math students that just instantly got everything. I had to work hard at everything that I have learned in math and even harder in everything else. It has helped me to truly understand some of the difficulties that my students face with the concepts that I am teaching them. I can relate to them on a level that helps them succeed.

How Parent Attitudes and Beliefs about Math Affect Student Attitudes and Beliefs

Heather Johnson & Kristin M. Hadley

Mathematics is vital for students to learn in today's world. Learning the inquiry-driven process underlying math principles helps develop the human intellect and assists in the learning of other subjects (Committee on Science, 1999). Math arms students with the knowledge and skills to interpret scientific information so that as adults they will be prepared to intelligently participate in society. In a mathematics classroom, students can learn to reason, communicate, and practice skills of sophisticated reasoning that many view as critical for the workforce.

Math is present in many other disciplines and areas of study. "Math is the foundation stone of biological science and medicine. Without it, there would be no measuring, dosing, and risk assessing-let alone formal statistics and epidemiology." (McNamee, 2005, p. 108) Another example of a diverse discipline is software engineers who "make use of their college mathematics education every day" (Devlin, 2001, p. 21). Devlin found that completion of a rigorous course in mathematics appears to be an excellent means of sharpening the mind and developing mental skills that are of general benefit. Sanders (2004) stated

> America's competitive edge in the global economy, the strength and versatility of its labor force, its capacity to nourish resource and innovation-all are increasingly dependent on an education system capable of producing a steady supply of young people well prepared in science and mathematics. (p. 3)

In our schools today more students are taking more higher level math courses than ever before due to these increasing job demands and increased math graduation requirements. During the year 2005, males and females completed advanced mathematics courses at about equal rates, except for pre-calculus/analysis, where females had a slightly higher rate than males (National Science Board, 2008). Students should be encouraged to take upper level math courses to learn valuable skills that will prepare them for life and will allow them to have greater earning potential in their careers. Teachers, peers, and mentors can all be a source of encouragement for students, but one of the best sources of encouragement is by parents in the home.

Parents play a critical role in helping to build their child's confidence in mathematics. Parental involvement and a parent's role in helping a child value math are both very important (Furner & Berman, 2004). Parents' values regarding math and science play a role in shaping their children's values and affect the children's later choices (Bleeker & Jacobs, 2004). The importance the parent puts on a subject area has a profound impact on how a child perceives that subject area, which affects the amount of encouragement and opportunities a parent provides to the child (Andre, Whigham, Hendrickson, & Chambers, 1999).

In a parent survey by White (2001), a majority of parents felt that learning mathematics would help their child earn a living, and that mathematics was a necessary subject which would be used in adulthood. Unfortunately some students enrolled in advanced math courses often have little parental support. The parents of these students are often unable to help with their homework because the mathematics is too advanced. Many parents' beliefs, attitudes, including gender-stereotyped beliefs, along with their inability to help their children in higher level math classes, are in turn affecting their students' attitudes and beliefs in math and may impede their learning (Tocci & Engelhard, 1991). "Adolescents' perceptions of their parents' reactions to mathematics and ability to do mathematics, along with the amount of encouragement to study the subject and to do well at it, may affect the students' attitudes toward mathematics" (Tocci &Engelhard, p. 285). Parents' beliefs and perceptions of their child's math ability can be a more powerful indicator than past performance on a child's perception of their own math ability (Jacobs & Eccles, 1992).

Sometimes the perceptions of parents can be distorted by their own biases. Different perceptions from parents were given between a child who is succeeding in math and one who is not (Yee & Eccles, 1988). Stodolshy (1985) found that many students perceive math learning as a problem area and ceased studying math as soon as they were given the choice. Stodolshy also found that math was not viewed favorably by a substantial number of adults. Many high school students and adults in the United States do not like mathematics and perceive it as too difficult. Some students become anxious we are satisfied with the schooling that Utah students receive, then there may be no hope of reversing the current trend.

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Utah's Largest Math Event

Utah's Largest Math Event (ULME) provides an opportunity for Utah teachers and students to join together in meaningful mathematics. The ULME activities are designed to encourage students to work together creatively as they solve rich and challenging problems.

Teachers conduct the event's activities in their classroom, and submit up to 3 samples of student work to compete for prizes. The winning students (and their teachers) in each grade level receive a monetary award. Those students whose work is considered for competition at the state level receive certificates of Honorable Mention, and each student in the class receives a certificate of Participation.

Creativity with Examples

Jim Brandt

Examples have played a role in the teaching and learning of mathematics for thousands of years. Examples are often used to illustrate computational methods in a specific situation. Going back to Egyptian mathematics, for instance, problem 26 on the Rhind Papyrus involves finding a quantity so that, when it is added to a quarter of itself, we get 15. The solution, written by an Egyptian scribe almost 3,000 years ago, is to "Assume 4. Then 1¼ of 4 is 5. Multiply 5 so as to get 15. The answer is 3. Multiply 3 by 4. The answer is 12" (Katz, 2004, p. 7). The method, called the method of false position, involves making an adjustment to an initial guess for the solution in order to find the correct solution. In this case, we multiply our initial guess of 4 by 3 in order to get the desired value. Presumably, students facing a similar problem would follow a similar procedure to find the solution. The scribe's example presents a proportional reasoning problem in a concrete and specific way, without referring to general principles or processes.

While the method of false position may not be familiar, the presentation using a concrete example has the same flavor as many modern mathematics textbooks and classrooms. Namely, students are presented with a concrete example illustrating a computational procedure, and then are expected to mimic this procedure in solving similar problems. While more general principles may be discussed, understanding these general principles is not really necessary when solving computational exercises. Presumably, students will come to understand these general principles and underlying concepts through repetition of the solution process. Unfortunately, repetition does not always lead to conceptual understanding. In terms of reforming mathematics education, a main focus of the NCTM Standards was an increased emphasis on conceptual understanding, with less emphasis on memorizing procedures and processes. Since the publication of the Standards, this change in emphasis has been reflected in a greater focus on problem-solving, communication, reasoning, and mathematical connections. Similarly, there has been a greater focus on group projects, explorations with manipulatives, and visualization using technology. Given this shift, it might seem that the traditional use of examples in teaching mathematics should also receive decreased emphasis. However, with a small twist, I believe examples can play an important role in promoting conceptual understanding in a wide variety of mathematics settings.

Student Generated Examples

Traditionally, the instructor presents an example and the students solve exercises following this method. We can turn this model upside down and have students create examples of various concepts. As an illustration, consider the following geometry problem:

Geometry
Give an example of two quadrilaterals with equal area but different perimeters.
Give an example of two quadrilaterals with different areas but the same perimeter.
Is it possible to have polygons with the same area and the same perimeter? Ex-

Instead of presenting a geometric object and asking the student to mimic a previous calculation, this problem forces the student to think about the relationship between the concepts of area and perimeter. While students must be able to calculate areas and perimeters using standard formulas, the focus is not on the calculation. Instead, the focus is on the concepts of area and perimeter and how they are related (or not related). Rather than mimicking procedures, generating examples forces students to think and explore ideas.

In this article, I will share some of my experiences in using student generated examples in a variety of mathematics classrooms. In addition, I will recount a few of the suggestions presented in Watson and Mason (2005) where these types of problems are explored in detail.

Student Tasks

Consider the following example generation task:

> *Sequences*
> Make up a sequence of integers.
> Make up another.
> And another.

Notice that the task involving sequences is quite open-ended. Students are only limited by their own imaginations and the fact that the numbers must be integers. I have found this to be a useful task in introducing terminology. That is, after students share their solutions, we can discuss similarities and differences among their sequences, i.e., which ones are arithmetic, geometric, increasing, decreasing, etc. On the other hand, in the earlier geometric task involving perimeters and areas, students were limited to considering only quadrilaterals. Other options might be to open the problem up to general polygons or limit the problem further to rectangles. When opening the problem up to general polygons, some students invariably guess that the solution lies in considering different shapes. That is, they believe that for certain shapes the perimeter will always be larger than the area, and they just find one of those shapes. This can lead them to unnecessarily complicated figures where calculating the area and perimeter is quite difficult. While simplifying the problem to only consider rectangles eliminates this difficulty, it makes it such a simple problem that many students stumble across a solution almost immediately. Thus, when creating an example generation task, one thing to consider is the appropriate limitations to place on the problem. These limitations will vary depending on the topic, the students, and your goals for the activity.

Turning to fractions, I have used the following task to point out a common cancellation error:

> *Cancellation*
> Give an example of three natural numbers *a*, *b*, and *c* so that $\frac{a+c}{b+c} \neq \frac{a}{b}$
> Give another example.
> And another.
> Can you find three numbers so that $\frac{a+c}{b+c}$ is equal to $\frac{a}{b}$?

course taking are very strong indicators of success in college and in the labor market.

Conclusion

Is Utah living up to the resources available for education? The drop in ranking of mathematics score is a central reason that Utah has dropped from having the 15th rated educational system in 1998 to the 27th rated educational system for 2006-2007 as put forth by the American Legislative Exchange Council (ALEC) chaired by the Secretary of Education. The rankings take into account educational inputs (spending, teacher per pupil ratio, etc.) and educational outputs (NAEP, SAT, and ACT scores). This adds to the large body of evidence that Utah's scores are dropping, particularly in mathematics. Utah was once considered a top ten state when it came to education, and some even considered it a top five state. But we have fallen to about the national average, and in some measures, below the national average. There may be a very different story in some particular schools or districts in Utah, but that is beyond the scope of this article. There are probably schools or districts in Utah that could compete against the best schools or districts in the country, but as a whole Utah is falling behind.

It is interesting and perhaps important to also consider aspects of Utah demographics and culture and how they relate to education. How did Utah maintain a top ten educational system with bottom ten funding? We did not set out to answer this question and have not gathered specific data to argue an answer, but one popular hypothesis is that Utah parents had strong traditional values that emphasized the importance of succeeding in school, working hard, and going to college. The social environment in Utah has been a great support to the school system. But the culture in Utah, for whatever the reason(s), is slowly changing and the schools and teachers are working in a social environment that is much more like that nation as a whole, and thus the scores are going in that direction, towards the national average. The Utah schools may not be doing much different than what they have done in the past, but to get better results they need to improve. This, of course, is just one hypothesis about the drop. More data and more research would be needed to go further.

For whatever reason, Utah is falling behind other states. Some changes will be needed to improve the achievement of our students to be competitive with the top states. But students in top achieving states are still behind the students in top achieving countries, but both top achieving states and especially top achieving countries have lessons to offer about how to develop an effective education system, curricula, school, and instruction.

There are indicators that Utah has some positive social resources that are usually associated with higher student achievement. For example, Utah students watch the least amount of television (Snyder et al., 2004), and tend to be healthier, happier, and safer than in most states. Utah also has low rates of teen pregnancy, drug use, gang activity, and single parent households (Morgan & Morgan, 2007). These indicators tend to be associated with better schools and greater student learning. These positive indicators, as well as other economic and social indicators, got Utah ranked as the fourth most livable state in the country in 2007 (Morgan & Morgan, 2007). Five or six (depending on the score used) of the top ten most livable states are also in the top ten for achievement. So in the past it seemed that Utah was doing more with less – high achieving students with few monetary and physical resources. Now the argument could be made that we are doing less with more, at least compared to other states with similar economic and social resources.

What can we do to reverse the trend of dropping mathematics scores in Utah? These results do not suggest any immediate responses other than we must do something if we want to reverse these trends. One thing we do not want to do is to make changes just to make changes. The changes should be well thought out and based on evidence from other high achieving countries and states. Many of the policies and strategies hastily implemented to make improvements often have no effect or even the opposite effect the policy intended. Many popular reasons attributed to the low success of school in the US are revealed to be myths when compared to high achieving districts and high achieving countries. Many OECD countries that outperform the US have a much larger immigrant population, have teachers with less or equal compensation, have a large variation of SES and racial diversity. Yet many of these OECD countries still produce higher average achievement than the US and more equitable achievement as well (OECD, 2007).

Perhaps one of the most striking results of the latest international studies is the strong correlation between how satisfied parents are with their student's schooling and the actual achievement of the student. Readers may not be surprised at this fact, but what is surprising is that the direction of the correlation is ***negative***. Countries where parents are more satisfied tend to have students that achieve less. If costs for public institutions, $8,348 compared to a national average of $11,441 and a high of $16,349 in New Jersey (Morgan & Morgan, 2007).

Legislatures may want to consider moving some of the resources spent on higher education and moving them to the K-12 education system. This may demand a raise in tuition prices, but this may be justified as our tuition is so low compared to other states. It also means that taxpayers would be subsidizing the education of all students in k-12 public institutions rather than the smaller portion of students in state institutions of higher education, which seems a much more equitable way to spend money.

Fortunately for Utah elementary and secondary school students there is a weak association between the amount of money spent on education and the amount that students learn. It is how effectively the resources are used that impacts student learning. Those that give general calls for more money in education as a fix need only to look at Utah history or US history to find adequate counter examples. Utah has been spending the least amount per student for more than ten years, yet in 1997 we ranked 13th in the country on eighth grade NAEP scores. We still rank last in spending, but have dropped to 31st in eight grade mathematics NAEP scores. The rank of educational funding has not changed yet with the same low funds we used to be able to maintain higher scores than we currently are maintaining. The US as a whole has more than doubled the amount of spending *in real dollars* in the last 30 years but scores have not seen a dramatic change with the dramatic increase in funds.

Money and resources, of course, can be used effectively to improve learning and maintain high educational standards and other desirable outcomes. There are two principles to keep in mind about effective use of resources. First, efforts to improve student learning *must* change what happens in the classroom because that is where students learn material in school. This may seem tautological but many policies or funding practices don't take this simple fact into account, or anticipate that classroom changes are easily made and maintained. Many of the most dramatic improvements in US schools have come about because they take this principle. KIPP charter schools for example, focus efforts on the highest quality instructional environment in school and usually show dramatic improvements in achievement and college attendance.

The second principle of effective educational reform is to have a coordinated effort with many policies and strategies to accomplish high levels of student learning. The effects of policies are often in the opposite direction of their intended effect, and sometimes different policies work against each other, rather than with each other, to increase learning. Coordinated changes that support each other can make substantial changes in outcomes.

Class size.

Utah has some of the highest class sizes in the country. The average class size in our secondary schools ranks third and the average class size in elementary school ranks fourth (Morgan & Morgan, 2007). However, this is not necessarily something to worry about in and of itself. Some states ranking in the top ten in achievement also rank in the top ten in class size. Research studies have shown some benefit to significantly decreasing class size, but the benefits are small, especially compared to the costs involved. Most high achieving countries have average class sizes much larger than the US, with typical class sizes from 38 to 50 (Stephenson & Stigler, 1992; NCES, 2003). Although it seems counter intuitive, having large class sizes can actually be used to greatly improve student learning. Great teachers are often adept at handling a large number of students. Consider the following extreme experiment as an illustration. Suppose Utah were to double the number of students in a class, then we would only need half as many teachers. Keeping the most effective teachers and letting the less effective teachers go would ensure that all students would have a "better than average" teacher. This would also free up funds to increase teacher salaries, teacher training, and implement effective practices. This simple example, which we don't necessarily recommend, shows how increasing class size can provide all students with better teachers and increased resources for learning.

Students' View of Mathematics

A national survey in 2003 asked students if they agreed with this statement: Mathematics is useful for solving everyday problems. Of all states, Utah had the lowest percentage of students that answered agree or strongly agree to this statement (Snyder et al., 2004). We could not find a more recent survey to see if this has changed since 2003. Apart from the dropping test scores this is also an indicator that we could be doing much better in helping our students learn and appreciate mathematics. Students that enjoy mathematics and view it as important tend to learn more and take more mathematics classes in high school. Both of these, greater knowledge of mathematics and high school mathematics

When working with symbolic expressions, many students believe that they can cancel the *c* in $\frac{a+c}{b+c}$. In this example generation task, students almost immediately find many examples of numbers where this is not the case. In fact, it takes a bit of thinking to come up with numbers where $\frac{a+c}{b+c}=\frac{a}{b}$. Rather than watching me demonstrate this, students can easily discover this pattern. I also wanted to point out that the last two tasks involved finding one example, then another, and another. This is a tactic recommended by Watson and Mason to encourage greater creativity. After finding one example, many people simply make small adjustments to find another. However, the third example often sends them "searching for something rather different. They find themselves asking 'what else is possible?'" (Watson and Mason, 2005, p. 120).

Consider the different restrictions on the parabolas in the following tasks:

Quadratics
Using an algebraic formula,
1. Give an example of a quadratic function with vertex at $x=3$
2. Give an example of a quadratic function that never intersects the x-axis
3. Give an example of a quadratic function with y-intercept at $y=4$
4. Find a quadratic that satisfies all three of the conditions above.

I have used this problem after discussing transformations of graphs of functions via translations, reflections, etc. The first three problems are fairly simple. The last question is more difficult for many students, as they have to combine three different ideas. A purely algebraic solution is not too difficult. If $y=ax^2+bx+c$, the vertex must be at $x=\frac{-b}{2a}=3$, the discriminant b^2-4ac must be less than zero, and the y-intercept *c* must be 4. On the other hand, it is simple to draw a picture of a parabola satisfying all three conditions. The challenge for students that do not understand transformations of graphs is how to take that picture and come up with the corresponding equation. I like this problem because there are multiple solution paths. When working in groups, these different methods often come up, giving students the opportunity to discuss the strengths and weaknesses of algebraic and graphical approaches.

I have used the following task as a test question:

Remembering that $n(A)$ refers to the number of elements in the set *A*
give an example of two sets *A* and *B* so that $n(A\cup B)=n(A)+n(B)$
give an example of two sets *C* and *D* so that $n(C\cup D)\neq n(C)+n(D)$

Students that understand the union concept and are willing to try a few different sets are quite successful in solving this problem. If they begin with two sets that have no elements in common, they have a solution to (a). If they begin with two sets that share at least one element, they have a solution to (b). Either way, they have solved part of the problem and, after some more exploration, this usually helps them in solving the other part. Moreover, although student answers vary significantly, it is a very simple problem to grade. In particular, for (a), check to see if $A \cap B = \varnothing$ and, for (b), check to see if $C \cap D \neq \varnothing$. Rather than giving students two sets and asking them to list the elements in the union, this problem allows students to be a little creative in finding their own sets and exploring how their sets are related.

> Give an example of an experiment with six different outcomes where the probability of any individual outcome is *not* one sixth.

For six equally likely outcomes, the probability of any individual outcome will be one sixth. To make it *not* one sixth, we must think of a situation where the outcomes are not equally likely. We might be throwing a dart at a board with six different sized regions, or recording the color of an object drawn from a bag containing differing quantities of six colored objects. Making sure a condition is not true is a slightly different question and requires a slightly different thought process. Watson and Mason describe this type of problem as "this and not that" (2005, p. 66), and point out that searching for these types of nonexamples can help students to refine their understanding of a concept. In this case, while counting up and dividing is a common tactic in calculating probabilities, that tactic is quite different than the concept of measuring the likelihood of an outcome.

Conclusions

In this article, I have tried to share some of my experiences in using student generated examples to enhance student understanding of mathematical ideas. These types of tasks offer learning opportunities for students across the mathematics curriculum. They can be used to introduce a topic, as problem solving activities involving groups or individuals, in formal assessments or in informal explorations. The only limitations are your own imagination. Instead of presenting examples and hoping students understand the underlying concepts, these type of tasks force students to think about those underlying concepts and to be flexible in exploring mathematical objects and ideas. Moreover, they are a lot of fun. Students get to express their creativity, and instructors get to be surprised by that creativity.

Examples have played a role in mathematics education for thousands of years because concrete examples help us to understand abstract ideas. Examples are powerful and meaningful. Tall and Vinner use the term concept image to "describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures." (1981, p. 152). They also point out that a limited set of experiences may lead to an incomplete or inadequate concept image. Experiences in constructing examples help us to refine our understanding of a mathematical concept, providing opportunities to focus our attention on properties and connections rather than processes. As Paul Halmos points out, examples "are to me of paramount importance. Every time I learn a new concept, I look for examples … and non-examples." (1983, p. 62) We should expect the same of our students.

References

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Notice that all Japanese students have finished geometry and algebra II level material by the time they have finished ninth grade. About ten percent of Utah students finish Algebra II by then. Nearly all of the Japanese students take multiple math courses so nearly all students get some calculus (because basic calculus concepts are taught in the second course). This is true even for students in humanities or vocational tracks, not just students on the science/math track. In Utah only about 14 percent of students take calculus by the time they graduate from high school. The high school mathematics courses in Japan may seem very extreme to the typical US parent or student but aggressive mathematics course sequences where all students take pre-calculus and calculus-level material are the norm among high achieving countries. This is one reason why US students score almost dead last on international mathematics assessments by 12th grade when in eighth grade the US scores at about the international average (Glenn, 2000).

While students in other countries become more academically focused in high school a large percentage of US students are toning out academics from their lives. Forty four percent of high school seniors do less than three hours of homework a week (Rosenbaum, 2001). This is a shocking result considering the average US elementary school student spends four hours a week on homework (Stevenson & Stigler, 1992).

ACT scores.

As mentioned earlier there is little data available to compare how Utah's high school students or high school graduates do compared to other states. ACT and SAT scores can give some indication but there are problems comparing across states because the ACT or SAT test is not required in most states. Students take them at different rates in different states. States where few students take these tests have an easier chance of making a high average score because the few students that take them are often the highest achieving, college bound students. It is not fair to compare a state like Michigan, which requires all states to take ACT test, to Maine, where only about 9 percent of students take the ACT (www.act.org).

With this in mind, Utah ranks 23rd among all states in ACT score with an average composite score of 21.7. On the mathematics section of the ACT Utah ranks 33rd with an average score of 21.1. How does Utah compare to other states that have about the same percentage of students taking the exam? Utah has about 68% of the students take the ACT. Minnesota had 69% of students take the exam and scored an average of 22.6 on the mathematics section, one of the highest math scores in the nation. Wisconsin had 67% take the ACT test and scores 22.3 points on average on the mathematics section. New Mexico and West Virginia had 63% and 64% of their students take the ACT test, respectively, but scored quite low on the mathematics sections with scores of 19.8 and 19.6 respectively. So it appears that Utah students fall near middle of states on the mathematics section of the ACT but slightly higher than the national average on the ACT composite score.

Utah Education Resources

Educational Spending.

Families in Utah tend to have a large number of children compared to other states and the national average. Utah ranks first in the percentage of population under five years old, 9.5% compared to the national average of 6.8% (Morgan & Morgan, 2007). Utah ranks second in the percentage of population between the ages of 5 and 17, 20.5% compared to the national average of 17.9% (Morgan & Morgan, 2007). Utah's demographics leads to a high ratio of students-to-taxpayers which results in low spending per pupil. Utah spends the lowest amount per pupil in the country, $5,347 compared to a national average of $9,022 and a high in New Jersey of $13,781 (Morgan & Morgan, 2007). A consequence of the low spending is lower teacher salaries for Utah teachers. Salaries rank 43rd but after accounting for how much other bachelor-degree-holders earn in each state the ranking rises to 38th. The low amount of money in the system also leads to a high pupil per teacher ratio. Utah ranks fourth in elementary school class size with an average of 23.7 students and ranks fourth in secondary school class size with an average of 27.1 students.

The fact that Utah spends the least amount per student in elementary and secondary education is well known. But less well known is that Utah spends the most money per capita on higher education, $921 compared to a national average of $589. Utah also ranks first in *higher* education spending as a percent of all state and local government expenditures, 16% compared to a national average of 9.1% and a low of 5% in New York. Incidentally Utah college students pay the second lowest average tuition

White NFRL students are the highest scoring population nationwide next to Asian/Pacific Islander NFRL students. Utah, however, does only slightly better with this population than with Hispanic FRL students. Utah places 39th out of the fifty states (neither District of Columbia or Department of Defense schools had a large enough population for reliable scores). White NFRL students in Utah score six points behind the national average for White NFRL students and 19 points behind the highest scoring state for this population, Massachusetts.

Results for these focus populations show that Utah is not doing well helping the most disadvantaged students (Hispanic FRL) or the most advantaged students (White NFRL), at least for sub-populations large enough in Utah for reliable measures. Some readers may wonder how we can be ranked 31st in the NAEP if both of these focus populations fall below the thirty first ranking. It is because of the only large student subgroup in Utah that does better than the national average. White FRL students score three points above the national average for this population of students.

Mathematics Education in 8-12 Grade

There is not a good database of scores to compare how twelfth graders do in mathematics between states so assessing how Utah schools are doing in high school is difficult. There are some indicators that we are doing very well in getting students to take mathematics classes as compared to other states, but achievement on SAT/ACT or AP exams is not as high as in other states (although these measures of achievement are problematic because the students who take these are not representative of a state's population).

Utah is top in the country at having eighth graders take algebra. The *Measuring Up* study of 2006 showed that Utah had 60% of students taking algebra in the eighth grade, well above the national average of 35% (NCPPHE, 2006). Utah also leads the country in the percent of students taking upper-level mathematics classes in 9-12 grade. An upper level mathematics class is defined as Algebra II or higher. These are laudable accomplishments by themselves but especially because mathematics course taking in high school is one of the strongest predictors of success in college. Figure 2 shows the probability of a student finishing a bachelor's degree based on the highest math course taken in high school. A student that takes pre-calculus is more than three times as likely to graduate with a bachelors degree than a student that has only taken Geometry as their highest mathematics course (Rosenbaum, 2001).

Getting a four-year college degree depends a lot on how far you go in high school math.
79.8% OF HIGH SCHOOL STUDENTS WHO TAKE **CALCULUS** GET A B.A.
PRE CALCULUS: 74.3%
TRIGONOMETRY: 62.2%
ALGEBRA II: 39.5%
GEOMETRY: 23.1%
ALGEBRA I: 7.8%
Percentage of high school graduates earning a B.A. by highest level math course taken in high school.

Figure 2. Mathematics courses and receiving a Bachelors degree.

How do the course taking results of Utah (or US) students compare to those of other countries? In many other countries practically all students take Pre-calculus and Calculus. In Japan, for example, there are six high school courses (10-12 grade) with only the first one being required. Students intending to pursue a college degree usually take all six. The content of the first required course covers topics usually reserved for pre-calculus and trigonometry classes in the US. The second course finishes the topics usually covered in a US pre-calculus course and begins introducing the ideas of limits, derivatives and the definite integral. The latter courses address advanced single and multi-variable calculus concepts, linear algebra, the complex plane, numerical computation, and some calculus-based statistics (Judson, 1999).

So You Want to Improve Your AP Calculus Enrollment

Kenley Brown

In the spring of 2003, American Fork High School gave 16 AP Calculus AB exams and 10 AP Statistics exams for a total of 26 AP Math exams. In 2008, just five years later, those numbers had grown to 85 AP Calculus AB exams, 24 AP Calculus BC exams, and 26 AP Statistics exams for a total of 135 AP Math exams. That is a growth of over 400%. Also during that time, the number of female students taking AP Math grew from six or just 23% of the students taking AP Math to 50 or 37% of the students taking AP Math. Did this happen by chance, or were there some organizational changes that took place?

Here is just a little background before we explore the changes that allowed for the growth. In the spring 2003, American Fork High School had just six math teachers, three of which were in their first three years of teaching, the other three were veterans, two remaining from the Lone Peak High School split a few years before and the other teacher transferred in just after the split. There were a handful of Precalculus classes being offered at the time, but there was only two Precalculus Honors classes being offered, which is what students had to take if they wanted to take AP Calculus. The same teacher taught these two Precalculus Honors classes, along with AP Calculus AB and AP Statistics. If students wanted variety, they could take one of two non-AP Calculus classes, which is what many students did. In addition, Algebra 2 Honors was a prerequisite for Precalculus Honors.

The first major change occurred the following year as students were given a second teacher and a third class to choose from when it came to Precalculus Honors. This administrative decision proved to be a good one as all three classes filled up. However, this did not do much to improve the AP Math enrollment the following year as many of these students opted to take the non-AP Calculus course. Why were these students settling for non-AP Calculus? Was it because they were not ready for AP Math or because they were being given a chance to opt out of it?

In the spring of 2005, the administration made another major change. They canceled the non-AP Calculus course. This upset many students and their parents, but once again, this decision proved to be a good one as many of the students enrolled in AP Calculus. This increased the enrollment enough to necessitate a second AP Calculus AB teacher.

The AP Calculus numbers were now increasing every year. Many of these students were juniors, so they would take AP Statistics their senior year, which also increased the AP Statistics enrollment. The Precalculus Honors numbers had continued to grow as well, but there were starting to be some issues with the students coming out of Algebra 2 into Precalculus Honors. The rule requiring students to take Algebra 2 Honors had been relaxed, so now students from multiple Algebra 2 teachers and two different junior highs were feeding the Precalculus Honors classes. Even though there was a state core for Algebra 2, there was not a CRT for Algebra 2 and as such, there was not a common curriculum for Algebra 2.

At this time, the school district had started a new collaboration program. Every Monday, students would get out of school one hour early and teachers would use this time to collaborate. The American Fork High School Algebra 2 teachers used this time to align their Algebra 2 curriculum with the core and to create common assessments. Time was also spent with Algebra 2 teachers from the junior highs trying to do the same thing. This did not change things overnight, but it did help to improve the Algebra 2 curriculum and thereby helped more students come into Precalculus Honors better prepared. The collaboration time also gave non-Honors Precalculus teachers an opportunity to improve their curriculum allowing their students the opportunity to take AP Calculus as well.

By the time the spring of 2006 rolled around, there was enough demand to offer an AP Calculus BC class for the first time, three AP Calculus AB classes and AP Statistics. This also allowed a third teacher to teach AP Math and it allowed for multiple time slots for AP Math. The AP Math program was

was more noticeable than the overall number of students was the number of female students taking these advanced math courses and specifically Precalculus Honors. Previously to this year, the number of male student to female students in Precalculus Honors was typically 2:1, 3:1, or sometimes even 4:1, but this year, there were more female students than male students.

Some of the theories as to why more students, and especially more female students are taking advanced math range from American Fork High School is doing a better job of teaching all students to the elementary schools and junior highs and the investigations and CMP programs they are using are working. One thing is for sure, that something is working and it needs to continue.
These numbers have continued to increase. It is true that part of the increases are due to the fact that American Fork High School is growing, but much of it has to do with other factors. Some have been identified, but there are likely other reasonable explanations as well. As the 2007-2008 school year wound down and preparations were being made for the 2008-2009 school year, sophomore and junior students in Precalculus Honors and AP Calculus were asked what their plans were for the next year. Nearly 240 students responded with the desire to take an AP Math course during the 2008-2009 school year.

American Fork High School had now grown its advanced math program bigger than it could handle. The school did not have enough willing and eligible math teachers to teach the eight AP Math classes and the six Precalculus Honors classes that students were requesting. For this reason, and the fact that seniors get to register before juniors, many juniors who had planned to take an AP Math class were not able to take AP Math in 2009. This setback will hopefully be remedied in the future.

The plot also begins to reveal some discouraging news. Although Utah scores are going up, we have not grown as much as the national average. Utah went from being above the national average in 1992 to being right at the national average in 2007 in both the fourth and eighth grade. How does the growth of Utah scores compares to other states over the same period from 1992 to 2007? There were only three states that had fourth grade scores with a smaller gain over this period. In 1992 our fourth grade scores ranked (tied) 12th of 42 states or jurisdictions (District of Columbia or Department of Defense schools) and were only eight points behind the top state. In 2007 our fourth grade scores ranked (tied) 29th (out of 52) and were 13 points behind the top state.

In eighth grade the results were worse. Again, Utah only had larger growth than three other states. However, we fell from being ranked 10th to 31st (tie) and went from scoring nine points behind the top state to seventeen points behind the top state.

So what has caused the drop in rankings? Some people have pointed to the changing demographics of the state, for example the increasing number of Hispanics, to explain the drop. Hispanic students usually don't do as well in academic tests, for example, on the latest NAEP exam Hispanic students scored an average of 264 while white students scored an average of 290. It is true that the proportion of Hispanic students enrolled in Utah public schools more than doubled from six percent of the student population in Fall 1996 to 13.2 percent of the student population in Fall of 2006. But we will see that the problem extends beyond the ability of Utah schools to help Hispanic students to achieve at an acceptable level.

Scores for Specific Student Subpopulations

Comparing state scores can be misleading because the populations of states vary immensely. Massachusetts, the highest achieving state on the 2008 NAEP mathematics results in both fourth and eighth grade, has a large population of high-income, well educated parents which places it high among states on Socioeconomic Status and low in the proportion of students that qualify for free or reduced lunch (Snyder et al., 2004). Is it fair to compare them to a state like New Mexico or Mississippi which have very different populations than Massachusetts, with high proportions of minority families living below the poverty line? Controlling for the effects of family income and parent education on NAEP mathematics scores changes the results dramatically with Massachusetts dropping to 9th in the country. California drops from 46th to last after controlling for family income and parent education. Utah raises a little, from 31st to 25th.

Some of the complexity of state scores can be simplified by looking at populations of students that are largely similar across states. There are two populations in the state of Utah that are of particular interest because they are significant portions of the student population and represent two extremes of the educational spectrum: White students who do not qualify for free or reduced lunch, and Hispanic students that qualify for free or reduced lunch. These two groups of students can help reveal how the Utah schools deal with the most advantaged students (rich majority) and the most disadvantaged (poor minority). Of course these are not the only important sub-populations in Utah schools but they do capture a majority of the students in Utah schools. Another reason to focus on these two subgroups is that some of the racial subgroups for which NAEP collects data (Asian/Pacific Island, Black, or American Indian/Alaskan Native) are not available for Utah because the populations of these subgroups are too small to obtain a reliable state estimate of their performance.

How well do Hispanic students that qualify for free or reduced lunch (FRL) learn mathematics in Utah schools? Not very well compared to how well those students do in other states. Only three other states or jurisdictions with a substantial population of students in this category score lower then Utah on the 2007 eighth grade NAEP: Connecticut, Rhode Island, and the District of Columbia. Seventeen states or jurisdictions did not have a large enough population to get a reliable average score. Hispanic FRL students in Utah score nine points lower on the eighth grade NAEP than the national average for Hispanic FRL students and 22 points behind the highest achieving state for this population, Texas.

It is possible that many of our students are doing well in mathematics and it is this growing population of Hispanic that is drawing down Utah's ranking. If this is the case than it could provide a focus for mathematics improvement in the state, and in particular districts that serve a large number of Hispanic FRL students. However, before jumping to this conclusion let's look at our other focal population, white students that do not qualify for free or reduced lunch (NFRL).

Trends of Mathematics Achievement in Utah

Douglas Corey

Mathematics education debates have been a hot topic in Utah in the last decade, more so than in all but a few states. Although this article does not address these debates directly it does attempt to lay out where Utah students place right now compared to other states as well as where Utah students used to place. I also add a few caveats about how Utah is different from other states, adding resources other states might not have, or a lack of resources that may cause difficulties that need to be overcome. This information is important to know when deciding what to do about mathematics education in Utah.

State-to-state comparisons can be misleading. Some states have a high concentration of rich, well educated families (like Massachusetts). Students in these families traditionally do well on standardized tests. Other states have a large population of poor minority families (like Mississippi or New Mexico). These students have traditionally fallen behind their white peers from well educated families. I try to account for these differences by also doing analysis on sub-populations of students.

The evidence reported in this article supports the claim that Utah students, although doing slightly better than in the past, are falling behind compared to students in other states. This is especially true for two of the three categories of students that make up the bulk of the Utah student body: white students that do not qualify for free or reduced lunch and Hispanic students that do qualify for free or reduced lunch. Where as Utah students used to place in the top ten on national exams, our students now rank thirty first by the time they reach the end of eighth grade.

There is always some hesitation to put importance on standardized tests because they do not tap all of the important things in knowing and understanding elementary and secondary mathematics. The tests usually err on the side of testing basic mathematics and problem solving skills. But they are a good indicator of success in subsequent math classes, success in college, and success in the workforce. So students that do poorly on these exams are at a disadvantage. Certainly there are other things that are important, but the NAEP, which is where most of the data comes from for this article, taps skills and problem solving abilities that all students should be able to do.

Unless otherwise cited, the results reported here were pulled from websites maintained by the National Center for Educational Statistics. The main source for results is the website http://nationsreportcard.gov/math_2007 which reports the latest results from the NAEP exam.

Historical Trends of Scores up to 8th Grade

The National Assessment of Educational Progress (NAEP), sometimes nicknamed the nation's report card, provides the best data available to see how Utah has faired in the recent past and where we stand against other states. Are Utah's scores going up on the NAEP? The answer to this question is actually encouraging. NAEP scores for the US and Utah averages for both eighth grade and fourth grade are shown below in Figure 1. There is a clear trend that scores are going up.

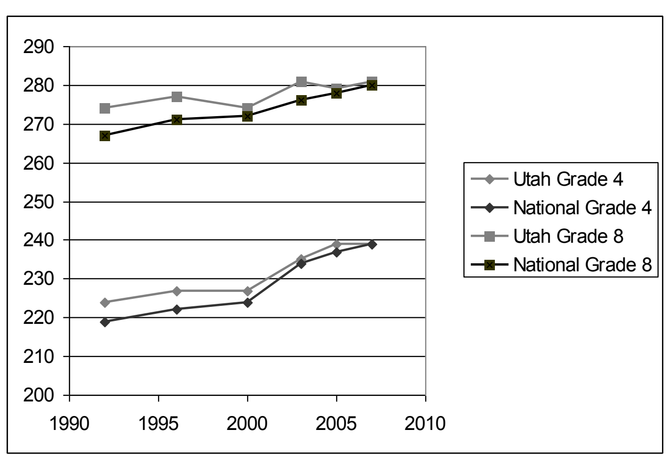


Figure 1. NAEP Mathematics Fourth-grade and Eighth-grade scores for Utah and the US from 1992 to 2007.

Using the *Comprehensive Mathematics Instruction* (CMI) Framework to Analyze a Mathematics Teaching Episode

Scott Hendrickson, Sterling C. Hilton and Damon Bahr

Introduction

Teaching and learning are processes that are so interdependent, it is difficult to study one without considering the other. Ideally, not only do teachers plan their instruction with their specific students in mind, but they also assess their students' learning in the act of instruction and modify their actions as needed to help their students achieve the desired learning outcomes. One purpose of the *Comprehensive Math Instruction* (CMI) Framework is to highlight this interconnection between teaching and learning in order to help teachers teach for deep mathematical understanding. Specifically, the CMI Framework can be used by the classroom teacher as a pedagogical tool before, during, and after teaching.

Prior to teaching, the Framework provides a planning model for designing lessons to meet intended purposes and desired learning outcomes. In the act of teaching, the Framework provides access for teachers to see order within the chaos of overwhelming and seemingly spontaneous classroom events, as well as a means to analyze the ideas that are present in the classroom and to plan productive and appropriate responses. When reviewing a teaching episode, the Framework creates opportunities for reflection by providing a lens through which to view the instruction. The Framework also focuses teachers on future work—where they can go next with student thinking and suggesting possible paths for how to get there.

While the CMI Framework consists of three major components: a *Teaching Cycle*, a *Learning Cycle*, and a *Continuum of Mathematical Understanding*, in this paper we will focus only on the Teaching and Learning Cycles.

The Teaching Cycle

Successful inquiry-based teaching moves through stages of a *Teaching Cycle* (Figure 1) that begins by engaging students in a worthwhile mathematical task (*Launch*), allows students time to grapple with the mathematics of the task (*Explore*), and concludes with a class discussion in which student thinking is examined and exploited for its potential learning opportunities (*Discuss*).

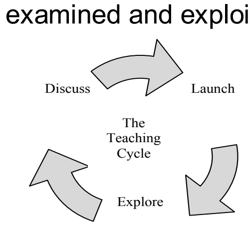


Figure 1: The Teaching Cycle

The Learning Cycle

We conceptualize that student learning progresses through phases of a *Learning Cycle* (Figure 2) that first surfaces students' thinking relative to a selected mathematical purpose (*Develop Understanding*), then extends and solidifies correct and relevant thinking (*Solidify Understanding*), and finally refines thinking in order to acquire fluency consistent with the mathematical community of practice both inside and outside the classroom (*Practice Understanding*).

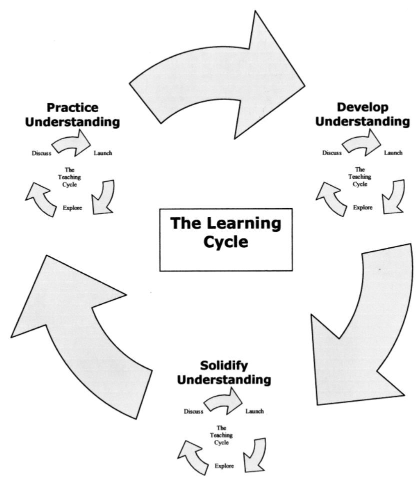


Figure 2: The Learning

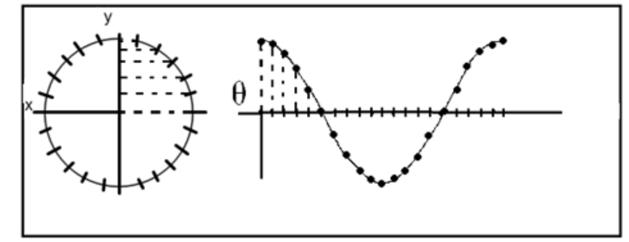
Interactions Between the Teaching and Learning Cycles: An Example

In the CMI Framework each phase of the *Learning Cycle* is fleshed out with detailed descriptions of the *Launch, Explore* and *Discuss* stages of the *Teaching Cycle*. These descriptions include three components: a purpose statement, the teacher role, and the student role. While the three stages of the *Teaching Cycle* are modified by each phase of the *Learning Cycle*, in this paper we will focus on the *Develop Understanding* phase of the *Learning Cycle* to illustrate how the CMI Framework can be useful to a teacher in planning and guiding instruction.

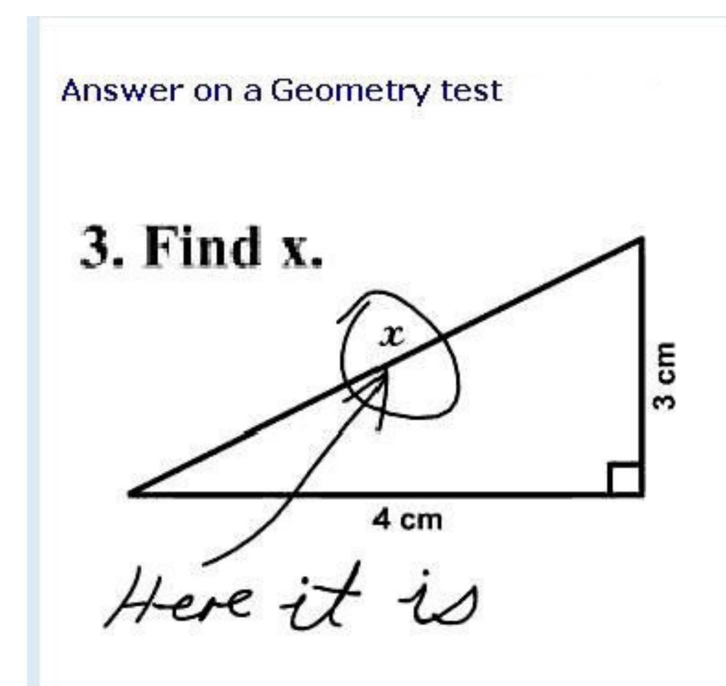
When planning a *Develop Understanding* lesson, the teacher should identify a mathematical purpose for the lesson aligned with state or national standards and select or design an appropriate task to surface student thinking relative to the chosen mathematical purpose. The teacher should also anticipate possible student thinking generated by the task for two reasons: 1) to prepare possible questions that will promote student exploration and discourse during the *Explore* stage of the lesson, and 2) to plan a possible structure and flow of the whole group discussion during the *Discuss* stage. Finally, when planning a lesson, the teacher should determine which specific student groupings (individuals, pairs, or small groups) will best promote learning of the mathematical purpose during the exploration.

During the *Launch* stage of instruction, the teacher activates students' background knowledge and clarifies the task. During the *Explore* stage, the teacher facilitates student exploration and discourse by asking questions to engage students in the task, to prompt student exploration, and to clarify and deepen mathematical thinking. Also, during the *Explore* stage, the teacher is formatively assessing the student work in order to select ideas, strategies and/or representations to share during the *Discuss* stage of the lesson. This selection may include incorrect examples of student work in order to illustrate common misconceptions among students within the class.

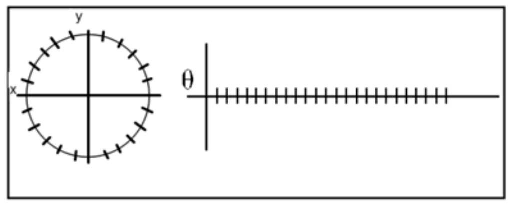
The purpose of the Discuss stage is to develop student understanding of emerging ideas, strategies, and representations by having students communicate, explain and support their own thinking and interact with the thinking of their peers. During this stage the teacher orchestrates a discussion by purposefully selecting relevant examples of student work related to the mathematical purpose of the lesson. This type of discourse is more challenging than simply inviting all students to share their work. Instead the teacher helps students understand criteria for judging the emerging mathematics and helps them clarify the mathematical reasoning behind it. The teacher also helps students compare and connect the various ideas, strategies and/or representations under discussion, providing appropriate mathematical vocabulary as needed.



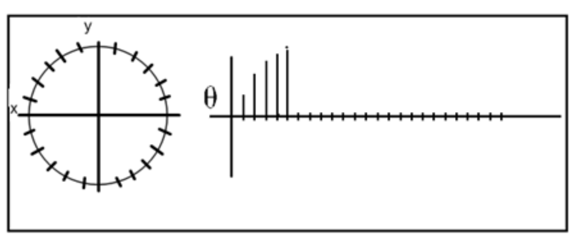
Transformations: One of the most powerful lessons to be learned from this lesson is the idea of transformation. For instance, ask the students what y = sin θ + 1 really means. Some may try to revert back to the rules they attempted to memorize previously. Help them think about what that '+1' means in the context of the spaghetti. Some student will realize that it means to take the broken piece of spaghetti and add an additional full length of spaghetti; thus the graph will be similar to the original but it will be one spaghetti higher. In order to graph y = 2sin θ, each piece of spaghetti will be doubled; thus the graph will be twice as high and twice as low. y = 1/2cos θ – 2 will have the shape of the cosine curve but it will be half as tall and shifted down two full spaghetti lengths. The other transformations such as y = sin(θ-π/4) or y = -sin θ can also be placed in context but I will leave them to be discovered by the reader.

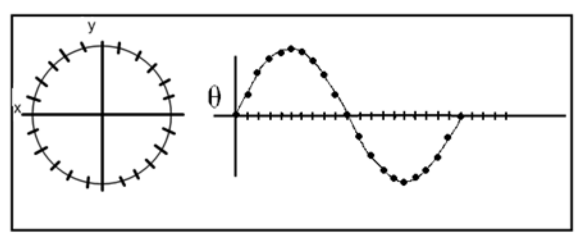


makes the unit circle so wonderful!) Also, when we defined a radian, we wanted an angle measure that translated nicely into a length. (All of this information is invoked from the students using careful questioning. I try very hard not to tell them much. It helps them understand both the construction and the motivators behind it.) Using the string, we are able to transfer our domain (in radians) to our new θ axis. Have the students wrap the string around the unit circles start at 0 radians and transfer the 24 tick marks on the long axis and label that axis the θ axis.



The students are then asked what the range or the 'Y' values will be. Somebody will remember that on the unit circle, sin θ = y. Visually, that corresponds the distance from the y axis to the corresponding tick mark. Thus, the distance around the unit circle is the input (domain) and the distance from the tick mark to the y axis is the output (range). This is where the spaghetti comes in. Instruct the students to break a piece to the exact length from the tick mark to the y axis. Then transfer those length onto the θ, y axes. The first spaghetti is of course length 0 and the 6th is a full length of spaghetti; the other four are somewhere in between.
The students will quickly discover that the second quadrant and the subsequent two will use the same pieces of spaghetti. It will not take them long to finish the graph.





It should be obvious that the graph would continue in both directions if the students were to continue around the unit circle. Some items for discussion include: Why will the pieces of spaghetti from the first quadrant work for all of the other quadrants? What does that tell us about the sine function? Why did the graph dip below the θ axis? How long does it take for the graph to start over? Is that true for any part of the function? Is the graph continuous?

Cosine Curve
It is also very valuable to discuss the graph of the cosine curve. It will not take much questioning before the students realize that the inputs are the same but the outputs are now the distance from the tick mark to the x axis (x = cos θ). They should also realize that they can use the same pieces of spaghetti that they use for the sin θ.

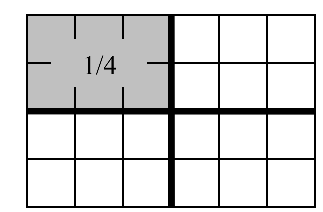
Analysis of a *Develop Understanding* Lesson

For the remainder of this paper, we analyze a teaching episode using the Framework as a tool for making sense of the work of the students and the teacher in the classroom. Monte, a fourth grade teacher, is beginning a series of lessons focused on using fractions to name a portion of a set of objects and has designed this lesson using the Framework as a planning guide. Prior to this lesson, his students have solidified the area concept of fraction and have practiced creating representations of fractions by shading a portion of a region (such as, identifying ¼ of a rectangle by dividing the rectangle into four congruent parts and shading one of them). This *solidified concept* and *practiced representation* become the "inputs" into a new cycle of lessons designed to surface, solidify and practice a new way of thinking about fractions—that a fraction can be used to name a portion of a set of objects.

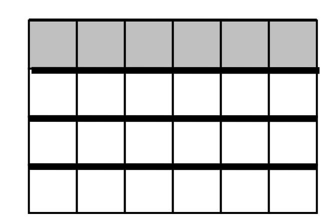
In this lesson, Monte expects students will see new ways of shading a rectangular array to represent a fractional amount and begin to notice that fractions can also be used to name a portion of a set. Intending that this lesson will surface new ideas about fractions and new ways of representing fractional parts of a region, Monte has characterized his class as being in the *Developing Understanding* phase of learning. Because Monte is aware that his goal during this phase of the learning cycle is to develop understanding, we note that he does not attempt to solidify all the ideas that arise during the discussion, knowing that an attempt to do so might "shut down" the broad perspectives he hopes students will bring to the work. Instead, we observe that during the implementation of the lesson Monte makes note of ideas that can be re-examined and solidified in future lessons.

Monte launched his lesson by talking about how hungry he was the previous day and that he had planned to eat ¼ of a cake when he got home. However, when he arrived home he found that his wife had already cut the cake into smaller pieces, forming a 4 ´ 6 rectangular array, as shown in an image which Monte projected onto the whiteboard. Since the cake is already cut into smaller pieces, Monte wonders how he can decide how much to eat so he will still eat ¼ of the cake.

At this point Monte distributed a sheet with several 4 ´ 6 rectangular arrays on it to each student and gave them the task of illustrating different ways he could eat ¼ of the cake. While this representation of fraction as area has the potential to develop students' ideas about fraction as objects within a set, within a matter of just a few seconds all students drew the following representation on the paper, and most seemed content that they had finished the task.



A few students made a second drawing that looked like this.



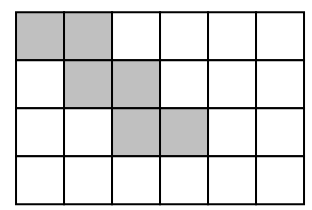
1/4

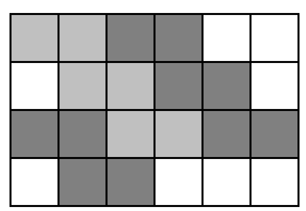
Monte realized that the lack of variety in student responses could have been a result of at least two different reasons. First, the specific phrasing of the question used to launch the task may not have prompted the students to explore multiple solutions, or second, the students' conceptualization of fractions may be so closely tied to area, that they couldn't see noncontiguous pieces combining to form a fraction of the set. Determining the reason behind the lack of variety in student solutions will inform Monte's instructional decisions regarding how to move his students towards the mathematical purpose of the lesson.

Even though the *Explore* stage only lasted a few minutes, the students were definitely finished. In order to determine the reason for the limited responses, Monte decided to move to the *Discuss* phase to have students present their drawings and explain how they knew that their shaded regions represented ¼ of the cake. Monte is aware that one of the students, Lars, has a unique representation that could move the class towards the mathematical purpose; however, Monte first selects the most common representations to give students an opportunity to explain their thinking.

Andrew described his work on the first diagram. "Use imaginary lines to make 4 big parts. Then eat one of the big parts." Andrew's language suggests that he recognizes that ¼ of the cake was made up of smaller pieces. "Or," said Megan, "you can divide the cake up into parts like this," as she pointed to the second diagram. Despite Andrew's use of the phrase "big parts", it was difficult for students to notice that there was anything new to pay attention to here. Recognizing that many students weren't seeing the smaller pieces of cake as objects of a set, Monte concluded that the lack of variety in solutions was most likely linked to the students' rigid conceptualization of fraction as area; therefore, he kept pointing out that "the cake is divided up into much smaller pieces."

Lars and several other students were really anxious to share their drawings, and because Monte hoped that Lars' drawing might help students see the smaller pieces, Monte asked him to share his drawing next. Note how Lars' presentation surfaces a new set of ideas and misconceptions among his classmates. Immediately, most of the students complained that this didn't work, that the shaded region was <u>not</u> ¼ of the cake. When asked to explain why it didn't work, Leticia said that she didn't know, but it just wasn't right. McKenzie, however, thought that she could show that it was ¼ by shading additional similar shapes on Lars' diagram. She came to the whiteboard and added the following:





But then she said, "Oh, I can't make four of them, so it's not ¼. I thought it was, but it's not." Apparently, her inability to find four "cookie cutter" shapes exactly like the one Lars had drawn had convinced her that the shape could not be replicated four times, so Lars' shaded region was not ¼ of the rectangle. McKenzie has surfaced a common misconception that fractions of a region must consist of congruent pieces.

Jacob disagreed with the conclusion that Lars' shaded region was not ¼ of the rectangle. He asked if he could add something to Lars's drawing and then outlined regions on the array to look like this:

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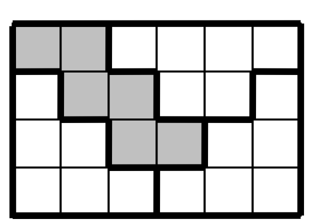


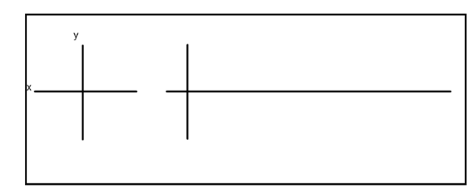
Sine Curves and Spaghetti

Josh White

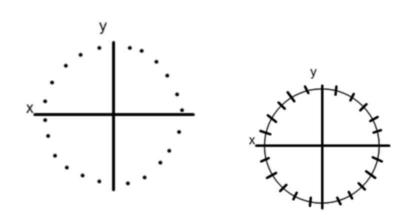
Note: I do not hand this sheet out to the students. The task is driven through conversation and careful questioning. This sheet is only for preparation; it is not intended as an instruction sheet for the students. I use the first part of this task to teach the unit circle. My students' first view of a unit circle is on this large piece of paper. A day or so later, after the students have worked problems dealing with the unit circle, we pull out these papers again and graph the actual sine and cosine curves.

Materials: Butcher paper – 8 ft long, 1 per group
Markers, 1 per student – the more colors the better
Spaghetti, 20 pieces per group (only 7 are really needed but they are typically broken or eaten)
String – 6 ft long, 1 per group

Preparation: Ideally, the butcher paper should be cut and the two axes drawn on it before the student enter the classroom. The length of the unit circle axes should be a little more than double the length of a piece of spaghetti. The sine curve axis should take up the rest of the paper



Unit Circle
The students need to create a unit circle. The unit we want to choose is the length of a piece of spaghetti. Have the students place one end of a piece of spaghetti on the origin and make a mark at the other end. Next, instruct them to rotate the spaghetti and continue making marks until they have enough to draw a circle with radius one spaghetti. Next, have the students draw a circle to the best of their ability and divide that circle into 24 equal pieces using tick marks, 5 marks in each quadrant. Ask the students to label each tick mark in radians. Each mark is π/12 radians. The students are then asked to label each tick mark and then enter into a conversation about the unit circle and its uses. Each student is given a blank unit circle and asked to transfer the information from the group unit circle to their individual unit circles. The class then spends adequate time exploring the unit circle and using it to find trig values.



Sine Curve
At the beginning of a new day, the students are told that they will be graphing the sine function. The students already know that to graph a function they must have a domain and a range. From previous lessons they understand that the domain of a sine function is an angle and the range of a sine function is a ratio. In our discussions of the unit circle, they are also aware that when the hypotenuse is equal to 1, the ratio of the opposite side to the hypotenuse is equal to the length of the opposite side. (That's what

Example Subtraction: Subtract 89 from 267 and check the result by Casting Out Nine's (CO9).

267	Minuend Remainder: (2+7=~~9~~) + 6 = 6	← discard digits that sum to 9
– 89	Subtrahend Remainder: 8 + ~~9~~ = 8	← discard digits = 9
178	Difference Remainder: (1 + 8 = ~~9~~) + 7 = 7	← discard digits that sum to 9

Minuend – Subtrahend Remainder: 6 – 8 = – 2 ≠ – 2 + 9 = 7

Add ↑ + 9 when the remainder is negative

Difference Remainder: 7, Remainders: 7 = 7 CO9's checks, the result: 178 is correct.

Section 3.1) Checking Procedure Examples:

C 1) Add:

89	CO9 Rule: 8 + ~~9~~ = 8 ← discard digits = 9	
+67	CO9 Rule: 7 + 6 = 13 ≠ 1 + 3 = 4	
156	CO9 Rule: 1 + 5 + 6 = 12 ≠ 1 + 2 = 3	

Rem: 4 + 8 = 12 ≠ 1 + 2 = 3
Rem: = 3, Remainders: 3 = 3 CO9's checks; 156 is correct.

C 2) Subtract:

4567	CO9 Rule: 6 + 7 = 13 ≠ 1 + 3 = 4
– 789	CO9 Rule: 7 + 8 + ~~9~~ = 15 ≠ 1 + 5 = 6
3778	CO9 Rule: (3 + 7 + 7 + 8) = 25 ≠ 2 + 5 = 7

CO9 4 – 6 = – 2 ≠ – 2 + 9 = 7 ← Add + 9, when the remainder is negative
Rem = 7, Remainders: 7 = 7 CO9's checks; 3778 is correct.

C 3) Multiply:

89	CO9 Rule: 8 + ~~9~~ = 8 ← discard digits = 9
x 67	CO9 Rule: 6 + 7 = 13 ≠ 1 + 3 = 4
5963	CO9 Rule: 5 + 9 + (6 + 3 = ~~9~~) = 5 ← discard digits = 9 and sum to 9

CO9 Rem: 4 * 8 = 32 ≠ 3 + 2 = 5
Rem = 5, Remainders: 5 = 5 CO9's checks; 5963 is correct.

C 4) Divide: 4567 ÷ 123 = 37, Remainder: 16

Equation: Divisor * Quotient + Remainder = Dividend

123	1 + 2 + 3 =	6	CO9 Rule
x 37	3 + 7 = 10 ≠ 1 + 0 =	1	CO9 Rule: Multiply 1 * 6
4567 – 16		6	Result

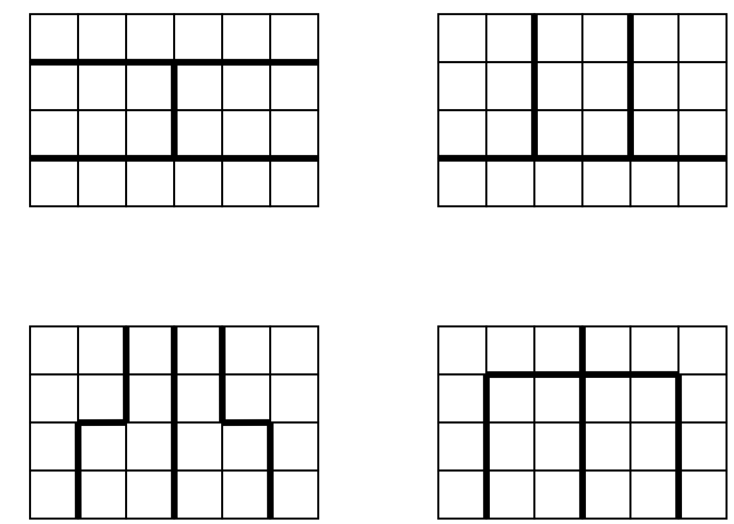
4567	~~4~~ + ~~5~~ + 6 + 7	= 13	CO9 Rule: cast out 4 + 5 = 9
– 16	1 + 6 = 7	= – 7	CO9 Rule: subtract 7 from 13
		Rem = 6	Result

Remainders: 6 = 6 CO9's checks; 4567 ÷ 123 = Dividend: 37Rem16

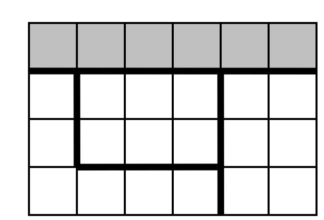
Jacob wrote a big 6 inside each of the four regions, and as he sat down made the claim, "Each of the big portions has 6 pieces, so they are each ¼ of the cake."

Recognizing that Jacob's suggestion could move the class towards seeing fraction as objects within a set, Monte wanted to give students an opportunity to think about Jacob's work. He launched a revised task by posing the following question to the class. "Andrew, Megan and Lars have all suggested different ways that I can eat ¼ of the cake. Will I eat the same amount using each of their strategies?" With this revised task, students will have the opportunity to grapple with the necessity of congruent shapes and contiguous pieces in representing the amount ¼ .

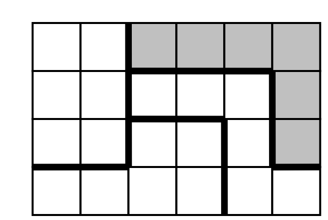
As students thought about this question they realized that each serving contained 6 small pieces of cake, and therefore represented the same amount of cake. Many teachers might assume that students at this point have solidified the idea that if we divide the cake into four portions, each with an equal number of pieces, that each portion is ¼ of the cake. Monte recognizes that this is an emerging idea—somewhat fragile and tenuous—therefore, he allowed students to continue to discuss this idea. It became apparent from student comments that there was much disagreement as to whether this amount should be called ¼ of the cake. At this point, Monte asked students to generate other ways that the cake could be divided into 4 equal servings. As students generated the following diagrams Monte noted that some, but not all students, labeled the 4 portions of the cake as each being ¼.



Most students continued to use some congruent pieces in their diagrams, until Annie and Oskar presented the following:



Annie's drawing



Oskar's drawing

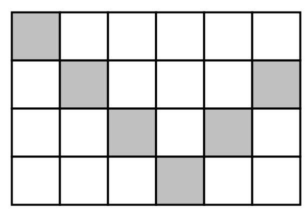
Since Annie and Oskar labeled the shaded portions as ¼ of the cake, Monte asked them how they would convince someone else that the regions they had shaded were actually ¼ of the cake.

Harrison could hardly contain himself as he blurted out, “You have to have four even groups, and all of the groups have to have six pieces in them.” (In the moment, Monte assumed that Harrison was using the term “even” to denote “groups of equal size”; however, Monte wondered later if he should have allowed Harrison to clarify his thinking on this point.) Monte decided to pursue Harrison’s claim that each portion had to contain six pieces—a claim that had been made much earlier by Jacob, but which seemed to now be more accessible to students. “So what do the rest of you think about Harrison’s claim that each group has to have six pieces in them?”

At this point Martin commented, “You can move the 6 shaded pieces in Oskar’s drawing to make it look like ¼.” Monte let Martin come to the board to show how he would rearrange the six small pieces of cake to form the arrangement that everyone had agreed upon at the beginning of the class period to be ¼ of the cake.

“So, can we call the shaded portion in Oskar’s drawing ¼ of the cake?” Monte asked. Most students seemed to be in agreement until Oskar raised another concern. “Yeah, but the pieces have to be connected.”

At this point, Monte asked Lars to present another one of his drawings.



Lars’ drawing #2

“If I eat Lars’ shaded pieces, have I eaten ¼ of the cake?” Monte asked. Monte has carefully orchestrated the discussion to elicit, examine, and begin to dispel two misconceptions that his students had about shading ¼ of an area: that portions need to be congruent and contiguous. Implicit in the work of the students is that the area has to be divided into <u>four</u> portions, and that each portion has to contain the same number of smaller, equal-sized pieces—in this case, six of the twenty-four.

Conclusion

There is still much that needs to be solidified for these students before they fully understand fraction as a portion of a set of objects. For example, will they still see the same relationship if the *pieces* are not *pieces of area,* or when amount means quantity not area? That is, will they see 6 pennies out of 24 as ¼ of the pennies, or 6 students in a class of 24 as ¼ of the class? How will they record the amount using fractions? Will they record $^6/_{24}$ and will they recognize that $^6/_{24}$ is equivalent to ¼? These are some of the issues that will need to be explored and solidified in future lessons to help these students more fully understand fraction as a portion of a set of objects. And, while all students seemed to follow the flow of ideas as they occurred in this lesson, for at least some of them their understanding is still fragile as evidenced by this summary response from Brooke’s paper: *“1/4 is always one group out of four groups, so there will always be four even, never odd groups.”*

As evidenced by this teaching episode, teaching and learning can be very intertwined and interdependent. The CMI Framework provided Monte with principles and tools that helped him understand and exploit this interdependence. Specifically, the framework helped him plan and guide his instruction. It helped him analyze and make sense of his students’ work which in turn helped him guide their thinking and focus their learning. Teaching for full and genuine mathematical understanding is challenging. Teachers often need to change their paradigms of mathematics and mathematics teaching, as well as change their instructional practices in order to accomplish this. The CMI Framework provides teachers with key principles, language and a general yet flexible structure to guide and support these changes.

Comparing the two algorithms: (**SA**) in Example (3) and the Textbook Algorithm (**TA**).

The textbook algorithm (**TA**), Example (7), is the most difficult way possible to do subtraction. The method requires extensive minuend modification, 3 borrows and 6 number transformations: 3 to 13, 5 to 4 to 14, 0 to 10 to 9 and 4 to 3, and the subtraction of 5 from 13, 6 from 14, 3 from 9 and 2 from 3. These are much more difficult operations than transforming the subtrahend into its 9’s complement and adding that to the minuend.

Subtraction by Addition, Algorithm (**SA**) also applies to mixed numbers.

$$\text{Example (7) Subtract } 5\frac{3}{8} \text{ from } 7\frac{1}{4} \quad \text{Nine's compliment of} : 5\frac{3}{8}\ is \Rightarrow 3\frac{5}{8}\ because,\ 3\frac{5}{8}+5\frac{3}{8}=9$$

$$7\frac{1}{4}+3\frac{5}{8}=7\frac{1}{4}\left(\frac{2}{2}\right)+3\frac{5}{8} \Rightarrow 10\frac{7}{8} \text{ Add the left digit 1, to the right digit 0, (mental step)}$$

$$7\frac{1}{4}-5\frac{3}{8}=1\frac{7}{8} \text{ Difference obtained by Addition}$$

Section 2.1) Conclusion:

Subtraction by Algorithms (SA), (SWB) and (RMS), Examples (3), (5) and (6), appears to be the easiest methods to do the operation of subtraction. Algorithms (SA), (SWB) and (RMS) avoid all the borrowing, the carries and the extensive minuend modifications. Which method of subtraction should be taught to elementary school students? All known methods and those that will be invented in the future; the students should be permitted to choose which method is within their procedural and conceptual understanding.

<u>Section 3.0) Checking Procedures</u>:

Acronyms: CO9 = Casting Out 9’s, Rem = Remainder

Casting Out 9’s is a method for checking the results of the arithmetic operations without having to do the procedure over again. The methods enable the students to check their work for accuracy without the tedium of repeating the operations. The simplest method uses the remainders of division by 9, labeled: Casting Out Nine’s (CO9), which can be readily found by operations that negate the need for division. The following sequence shows the constant remainder of 1 when 9 divides into powers of 10.

10 ¸ 9 = 1, Remainder 1
100 ¸ 9 = 11, Remainder 1
1000 ¸ 9 = 111, Remainder 1
10000 ¸ 9 = 1111, Remainder 1
100000 ¸ 9 = 11111, Remainder 1

Casting Out 9’s (CO9):

The above sequence indicates that the remainder (Rem) obtained by dividing a number by 9 can be found by summing the digits in the number repeatedly until one digit remains. This digit is the remainder that would result if the number were divided by 9. Alternative method: discard all digits in a number whose sum is 9. If all digits are discarded, the remainder is zero, that is, the number is divisible by 9.

Examples: CO9. 123456 ¸ 9 = Dividend 13717, Remainder 3

123456 à 1+2+3+4+5+6 = 21 à 2+1 = 3 Remainder: Rem = 3.

123456; discard 4+5 = 9 and 3+6 = 9, Remainder is 1+2 = 3, Remainder: Rem = 3

Note: The operation on the remainders corresponds to the operation being performed; when Casting Out Nine’s, any negative remainder is made positive by adding + 9.

Comparing Examples (3) and (4); in (3), the 9's complement is found by addition not subtraction, that is, what number added to 2 = 9, what number added to 3 = 9, and so on.

Example (5) Invent Algorithm (SWB) such that the Subtraction is Without Borrowing

Subtract 2365 from 4053

```
54 +  3999   ← Expand & Add 0 to the Minuend: 4053 = 53 + 1 – 1 + 4000 = 54 + 3999
   –  2365   ← Subtract the Subtrahend
      1634   ← Result
        54   ← Add 54; amount that must be added to obtain the original Minuend 4053
      1688   ← Difference obtained by (4) Subtractions without borrowing and 1 Addition
```

Example (6) Invent Algorithm (RMS) to obtain a Reduced Minuend and Subtrahend

Subtract 2365 from 4053

```
4   0   5   3      ← Write the Minuend digits in expanded form
2   3   6   5      ← Write the Subtrahend digits in expanded form
2  (–3)(–1)(–2)    ← Subtract the smaller from the larger digit
```

If the Subtrahend digit is greater than the Minuend digit, the sign is (–) negative

```
1 + 1999   ← Write the thousands place digit (+2) as 0 + 2000 = 1 ---- 1 + 2000 = 1 + 1999
   – 312   ← Subtract the hundreds, tens and ones place digits: (–3)(–1)(–2) = – 312
    1687   ← Add 1 to the right digit 7, the amount subtracted from the Minuend = 2000
    1688   ← Difference obtained by (7) Subtractions without borrowing
```

Example (7): Exercise from *Basic Mathematical Skills*, by Streeter et al, 4th Ed, p 62.

Subtract 2365 from 4053, using the Textbook Algorithm (TA)

```
            41
Step 1    4053
        – 2365
             8
```

In the first step we borrow 1 ten. This is written as ten ones and combined with the original 3 ones. We can then subtract the ones column.

```
          3141
Step 2    4053
        – 2365
             8
```

We must borrow again to subtract in the tens column. There are no hundreds, and so we move to the thousands column.

```
             1
          3941
Step 3    4053
        – 2365
             8
```

The minuend is now renamed as 3 thousands, 9 hundreds, 14 tens and 13 ones

```
             1
          3941
Step 4    4053
        – 2365
          1688
```

The subtraction can now be completed.

To check our subtraction: 1688 + 2365 = 4053

College Prep

Brady Rowley

In 2007 Lehi High followed suit with other high schools in Alpine School District as they ventured to create a new mathematics course titled "College Prep". This course was designed to bridge the gap between Algebra 2 and Precalculus courses in the Vertical Alignment of mathematic courses in the state of Utah.

The group of students we targeted was the students exiting Algebra 2 with average grades at as they entered the new school year in Precalculus. The majority of these students would struggle remembering the basic algebra concepts that were required as a mathematical foundation. When many of our students struggled they would exit the class and some would never attempt another math course until they entered college. This mass exodus was a concern to many of our Precalculus teachers, administrators, and patrons.

After watching this cycle for many years it became apparent that the students were not building the strong foundation of algebra skills that were necessary to be successful in Precalculus or at the University level. In conjunction with our feeder schools we meet as a group of Algebra 2 teachers and mapped the Algebra 2 curriculum and then created end of the semester exams to improve our chances of helping the Algebra 2 students be prepared and succeed at the next level. Even with this improved curriculum we felt that many of our students were not prepared for Precalculus or University courses.

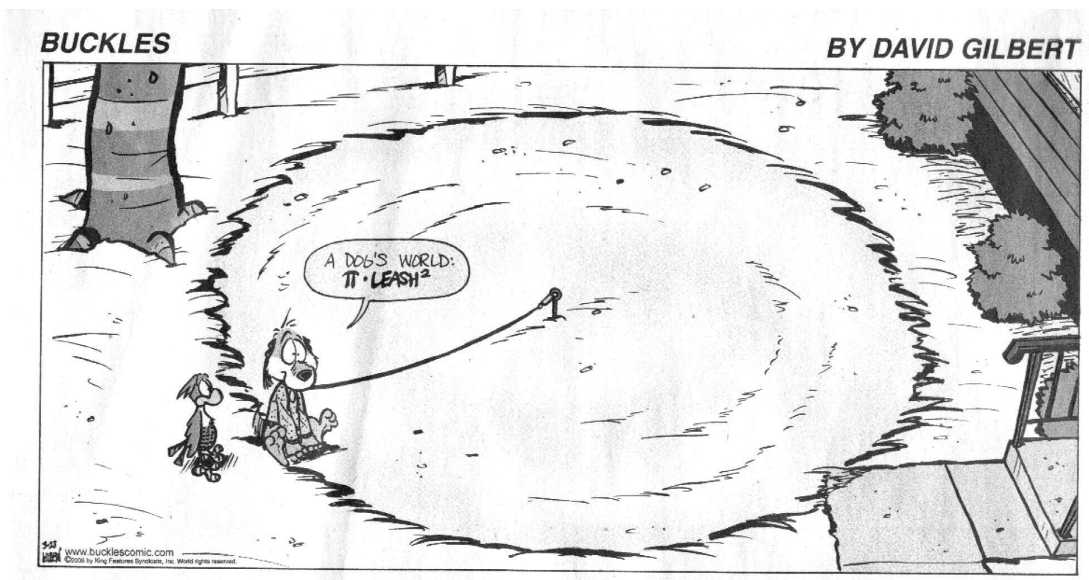
When I moved to Lehi High back in 2001 one of the goals that our department set was to increase our enrollment numbers of students taking advanced math courses. (We had very low enrollment in advanced math courses in relation to our school wide enrollment). With this desire to increase enrollment we not only increased the number of prepared students, but also unprepared students to take advanced math. After time our course became watered down and we were not effectively teaching the core for Precalculus and Non-AP Calculus. We had achieved our enrollment goal but now we had to increase the rigor of the courses. To do so meant that we must also increase the opportunities for our students to build their algebra foundation. This had been achieved when we watered down the Precalculus curriculum to meet their needs, but prevented many of them from having a strong base for AP- Calculus or be successful in University math courses. Thus we found that our Precalculus numbers increased and our Non-AP Calculus numbers drastically increased, but our ability to prepare students for AP-Calculus and University courses were compromised. To increase our enrollment at the AP- Calculus level we needed to teach a "True Precalculus" course which in turn required that we meet the needs of the exiting Algebra 2 students with another opportunity to build their foundation, thus "College Prep" became the vehicle to meet the needs of these students so they could succeed at Precalculus and continue on either to AP-Calculus or into a University level (Math 1050) course once they had graduated.

Part of the "College Prep" course's birth had to do with the philosophy that algebra is the foundation for most advanced math courses including Calculus. If our students can be successful in building strong algebra skills they can comprehend the Calculus concepts and master them more effectively & efficiently. If we are pushing our students into taking Precalculus when they have not mastered the fundamental algebra concepts then we are sentencing them to a life of mathematical frustration and agony. We compound their learning by adding new concepts of Precalculus on top of algebra concepts the students are only becoming slightly familiar with from Algebra 2. The College Prep course allows students to master their algebra skills and then carefully and systematically add the new Precalculus concepts into their "tool-belt of mathematical understanding" the following year.

The process for creating the "College Prep" course began in Alpine School District by organizing a math steering committee, who in conjunction with UVU Math 1010 instructors, outlined a curriculum that aligned with UVU's Math 1010 and the suggested curriculum as designated on the USOE website. Next a textbook was selected and many sample worksheets and assessments were developed. Much of what was created was in conjunction with UVU's math department due the high volume of Alpine graduates who attend UVU directly out of High School. Also integrated into the curriculum was a sizable dose of ACT prep work.

Our original goal was twofold: 1. Enable more of our students to be prepared to pass UVU's "Compass Exam," they now use the ACUPLACER, (which is their entrance exam for mathematics placement). 2. Prepare more of our students for the ACT exam with the hope that students would score atleast a 23 on the math portion of the ACT thereby qualifying them to take Math 1050. By achieving these two goals we would in turn also prepare more students to continue their high school mathematics courses of Precalculus and AP Calculus. Another benefit that emerged from this College Prep course is that students in Utah are being required to have three math credits for graduation, one of which can be College Prep.

Since 2007 when the College Prep course was developed there have been some significant changes to the course. We are working towards having our end of the year assessment be the "Accuplacer" exam which is the replacement of the "Compass" exam used by many of the surrounding Universities to determine University math course placement. This will give us a better read on how our students are exiting the course and how well they are prepared for University courses. Also, Westlake High School is now offering "Concurrent Enrollment" credit as an option for students who are taking the course if they so desire, pay the fees, and pass the corresponding exams with acceptable marks.



MISSION: To promote quality teaching and learning of mathematics in Utah, UCTM will:

Provide essential opportunities for professional collaboration among mathematics educators and advocates.
Communicate and publicly advocate for mathematics education in Utah.
Recognize and honor exemplary performance in mathematics by Utah students and teachers.

Reformulate Equation (A) by re-arranging terms to obtain Algorithm (**SA**):

Algorithm (SA): Minuend – Subtrahend = Minuend + (99 – Subtrahend) – 100 + 1

Example (2) Subtract 7 from 15; using steps in Algorithm (**SA**) and the **9C** table above

15   15 ← The subtrahend must equal the minuend digits; subtract 07
- 07   +92 ← The 9's complement, **9C**, of the subtrahend 07 is: (0 + **9**), (7 + **2**) = 92
107 ← Intermediate result; transpose and add the left-most digit 1, to the right digit 7
8 ← Difference obtained by 1 Addition, the 9's complement of the subtrahend

The above is the **Final Form** of **Algorithm (SA)**. The invention of Algorithm (**SA**) was obtained by defining the 9's complement of the subtrahend, evaluating the term (99 – subtrahend) by addition and adding a zero to the identity equation (A).

Acronyms: **min**: minuend, **sub**: subtrahend, **9C**: 9's complement of the subtrahend

Minuend Digits	Algorithm (SA) Table
1	**min – sub = min** + (9 – **sub)** – 10 + 1
2	**min – sub = min** + (99 – **sub)** – 100 + 1
3	**min – sub = min** + (999 – **sub)** – 1000 + 1
4	**min – sub = min** + (9999 – **sub)** – 10000 + 1

Example (3): Subtract 2365 from 4053, using the **Final Form** of Algorithm (**SA**)

4053   4053 ← Subtrahend digits are equal to the Minuend digits; subtract 2365
- 2365   7634 ← The 9's complement, **9C** of 2365 is: (2 + **7**),(3 + **6**),(6 + **3**),(5 + **4**) = 7634
11687 ← Addition of 9C; transpose and add the left-most digit 1, to the right digit 7
1688 ← Difference obtained by 1 Addition, the 9's complement of the subtrahend

Example (4) Using Equation (A) simplified such that the last 4 steps are done mentally

Subtract 2365 from 4053

9999 ← Subtract from Minuend 9999 first; the 9's equal to the (4) minuend digits
– 2365 ← Subtract the Subtrahend from 9999, not from the original Minuend 4053
7634 ← Result is the 9's complement of the subtrahend (see Example (3) above)
4053 ← Add the original Minuend
11687 ← Result; transpose and add the left-most digit 1, to the right digit 7
1688 ← Difference obtained by (4) Subtractions without borrowing and 1 Addition

Note-1: The Special Case Algorithm (**SC**) is valid in the ranges, Minuend: 10 to 19 and Subtrahend: 0 to 9. The students will memorize the basic facts of subtraction by the repetitive use of this algorithm and, at the same time, verify the number facts in the subtraction table.

20 = 11 + 9	2 + 0 =+ 2	← Add the minuend digits 2 + 0
– 5 = 4 – 9	+ **4**	← **4** is the number that when added to subtrahend 5 equals 9.
15 = 15 + 0	+ 6	← Result: Algorithm (SC) Fails 6 ≠ 15; 20 is out of range, see **Note-1**

Section 2.1) Subtraction by Addition: Invent an Algorithm that is valid for all numbers:

Acronyms: SA: Subtraction by Addition, **9C:** 9's complement of the subtrahend
Consider the identity Equation (A) with a 2-digit minuend:
(A) Minuend – Subtrahend = Minuend – Subtrahend

Adding a zero: 0 = 99 – 100 + 1 to the right hand side of (A) does not change the identity:

(A) Minuend – Subtrahend = Minuend – Subtrahend + 99 – 100 + 1

Reformulate (A) by re-arranging and grouping terms:

Equation (A): Minuend – Subtrahend = (99 – Subtrahend) + Minuend – 100 + 1

Example (1) Subtract 7 from 15 using Equation (A):

15	99	← 99; the 1st term in Equation (A)
- 7	– 7	← Subtract the subtrahend; the 2nd term in Equation (A)
	92	← This term is the 9's complement, **9C**, of the subtrahend (see **9C Table**)
	15	← Addition of the minuend; the 3rd term in Equation (A)
	107	← Intermediate result
	– 100	← Subtract 100
	+7	← Intermediate result
	+1	← Addition of 1
	8	← Difference

These 4 steps will be done mentally hereafter. Transposing the left-most digit 1 in 107 and adding 1 to the right digit 7 is equivalent to subtracting 100 and adding 1. The number to be transposed and added will always equal 1.

9C Table: the nine's complement of the Subtrahend with:

digits	are	because
0 & 9	9 & 0	0 + 9 = 9 + 0 = 9
1 & 8	8 & 1	1 + 8 = 8 + 1 = 9
2 & 7	7 & 2	2 + 7 = 7 + 2 = 9
3 & 6	6 & 3	3 + 6 = 6 + 3 = 9
4 & 5	5 & 4	4 + 5 = 5 + 4 = 9

Simplified Methods In Arithmetic

George Mondras

The experimental evidence that verifies that all human abilities are normally distributed is substantial. The center point mean, of a normal distribution, implies that one-half of the student population will be below average in mathematical ability. This is the existential condition that a K-12 teacher is confronted with when they attempt to teach any level of mathematics.

The data from the article, *Grades and Distributions*, by Norman E. Rutt, published in a journal of the Mathematical Association of America, depicts a normal distribution of grades that, in turn, implies a measure of mathematical ability.

A	B	C	D	F	No. of cases
12.2%	22.9%	32.4%	18.7%	13.8%	8319

The object of this article is to help all students but especially those whose circumstance places them in the left tail of the distribution. One strategy that may help is to augment the current pedagogy with algorithms that simplify the procedures as much as possible; to that end this article was written.

The simplified algorithms avoid, as much as possible, the mental and computational demands of borrowings and carries that are prevalent in the basic arithmetic operations, especially subtraction. To that end, subtraction through nine's complement addition is formulated and discussed. A second level of difficulty in learning subtraction is encountered when the subtrahend digit is greater than the corresponding minuend digit; a special case of subtraction by addition is formulated for these numbers.

The special case of subtraction by addition will enable students that are initially learning integer addition to be learning integer subtraction by default.

The paper includes a checking procedure, casting out nine's, that will enable the students to verify the results of the operations without having to do the procedure over again.

The methods are intended to augment and be taught in parallel with the textbook algorithms that are currently in use in the early grades.

Page Section & Contents

Section 1.0) Addition, Multiplication, and Division:

Writing the numbers in expanded form is a simplification of these operations.

A 1) Addition:	89	80 + 9	Write 89 in expanded form
	+47	40 + 7	Write 47 in expanded form
		16	Add 9 + 7, without the need to carry
		120	Add 8 + 4, append 1 zero
		136	Result

M 1) Multiplication:

```
  89      80 + 9     Write 89 in expanded form
x 47      40 + 7     Write 47 in expanded form
4183          63     Multiply 7 * 9, without the need to carry
             560     Multiply 7 * 8; append 1 zero
             360     Multiply 4 * 9; append 1 zero
            3200     Multiply 4 * 8; append 2 zeros
            4183     Result
```

D 1) Division: Divide: 4567 by 123

```
                 40 - 3                  Quotient is 40 – 3 = 37, Remainder: 16
100 + 20 + 3 ) 4000 + 500 +  60 + 7      Write 123 and 4567 in expanded form
               4000 + 800 +120           Multiplication by 40
                    – 300  – 60 + 7      Subtract: 8 – 5 and 12 – 6; change signs
                    – 300  – 60 – 9      Multiplication by – 3 (see Note*)
                              + 16       Remainder = 16
```

Note*: In division, multiplication by a negative number is necessary to obtain the simplification.

Subtraction by Addition also applies to long division or to any operation that requires subtraction **(See Section 2.1 for details)**.

Example (8) Divide 4597 by 123, using Algorithm (**SA**) in section 2.1) for the subtractions.

```
        30 + 7  ß Multiply by a multiple of ten such that the product ≤ dividend
  123)4597

   6309  ←  30 * 123 = 3690 < 4597; 9C of 3690: (3 + 6),(6 + 3),(9 + 0),(0 + 9) = 6309
  10906  ←  Addition of the 9C; transpose left-most digit 1 and add to the right digit 6
    907  ←  Result
    138  ←  7 * 123 = 861 < 907; 9C of 861: (8 + 1), (6 + 3), (1 + 8) = 138
   1045  ←  Result; transpose left-most digit 1 and add to the right digit 5
     46  ←  Remainder is 46; Quotient is 37, Remainder 46
```

Section 2.0) Special Case of Subtraction by Addition:

The object of this section is to develop algorithms such that the operation of subtraction can be done by addition. Learning subtraction per se is difficult enough for many students but a second level of difficulty is encountered when the subtrahend digit is greater than the corresponding minuend digit and borrowing and carries are necessary.

Subtraction Table

Subtrahend digit greater than the corresponding minuend digit

	10	11	12	13	14	15	16	17	18
-1	9								
-2	8	9							
-3	7	8	9						
-4	6	7	8	9					
-5	5	6	7	8	9				
-6	4	5	6	7	8	9			
-7	3	4	5	6	7	8	9		
-8	2	3	4	5	6	7	8	9	
-9	1	2	3	4	5	6	7	8	9

Define Algorithm: a logical step-by-step procedure for solving a problem in mathematics.
Define Acronym: a word formed from the initials or other parts of several words.
Acronyms used in this section:
min: minuend, **sub**: subtrahend, **9C**: 9's complement of the subtrahend, **SC**: Special Case

Invent the Special Case Algorithm (**SC**) that will do Subtraction by Addition.
Consider the following Subtractions and Additions:

```
 18 = 9 + 9     1 + 8 =  + 9  ←Add the minuend digits 1 + 8
 – 9 = 0 – 9             + 0  ← 0 is the number that when added to subtrahend 9 equals 9.
  9 = 9 + 0              + 9  ← Result: Difference obtained by Addition

 16 = 7 + 9     1 + 6 =  + 7  ← Add the minuend digits 1 + 6
 – 8 = 1 – 9             + 1  ← 1 is the number that when added to subtrahend 8 equals 9.
  8 = 8 + 0              + 8  ← Result: Difference obtained by Addition

 13 = 4 + 9     1 + 3 =  + 4  ← Add the minuend digits 1 + 3
 – 7 = 2 – 9             + 2  ← 2 is the number that when added to subtrahend 7 equals 9.
  6 = 6 + 0              + 6  ← Result: Difference obtained by Addition

 11 = 2 + 9     1 + 1 =  + 2  ← Add the minuend digits 1 + 1
 – 6 = 3 – 9             + 3  ← 3 is the number that when added to subtrahend 6 equals 9.
  5 = 5 + 0              + 5  ← Result: Difference obtained by Addition

 12 = 3 + 9     1 + 2 =  + 3  ←  Add the minuend digits 1 + 2
 – 5 = 4 – 9             + 4  ←  4 is the number that when added to subtrahend 5 equals 9.
  7 = 7 + 0              + 7  ←  Result: Difference obtained by Addition, see Note-1
```