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"Making Mathematics Accessible to All"

Editors: Christine Walker & Danielle Divis

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CALL FOR ARTICLES

The Utah Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Danielle Divis at danielledivis21@gmail.com. A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included.

UTAH MATHEMATICS TEACHER

Volume 12, Fall 2019

"Making Mathematics Accessible to All"

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UCTM President’s Message

Amy Kinder, President, Jordan Mathematics Administrator K-12

Do you remember why you starting loving mathematics? Do you remember when? I bet it didn’t involve worksheets or rote memorization of facts. It was probably a time when **learning sparked joy in you**. Maybe, like me, you used mathematics for a purpose, like to launch a rocket or solve an intriguing real-world problem. As mathematics educators, we are charged with being ambassadors to our content. We must use research-based strategies and truly promote your love for mathematics and share the beauty of mathematics with all. As educators, we must hold each other to high standards.

As President, these are my call to action.

1. **Honor our profession by sharing the joy of mathematics.**
2. **Shift the conversation to focus on the STRENGTHS, not deficits.**

It doesn’t stop at high standards; we must spread the **love and joy** of learning our subject. When was the last time you laughed in mathematics class? Share a delightful problem with students? Couldn’t wait to find out the many different strategies students used to solve problems? It is time to abandon tracking, math packets, rote memorization of facts, and solving trivial, contrived problems that have no connection to the real world or the students. If you have fallen into a rut, it is never too late to change. We can always do better for our students. Students **love** mathematics when their teacher loves teaching students mathematics. Spend a moment each day being delighted by a student’s justification or a small group discussion or simply talking with a student about their work. Honor our profession by sharing the joy of mathematics.

The second idea is built on the first and is equally important, but slightly more complicated. It is essential that we learn to focus on mathematical strengths with our students. Many times in mathematics, the focus gets placed on students who have falling behind, students with huge gapes, students who need Tier 2 interventions, and nonproficient students.   This focus has led to a culture in school when teachers focus heavily on deficits during collaboration and professional development. My challenge is for the next two weeks, look for strengths in students throughout the lesson: during the starter, task work, and exit tickets. Think about: What can students do? What were students successful at doing? Build on those strengths. I promise the more you learn to look for strengths, the more you tend to see. When we see what students can do, we begin to see a clearer path to strengthening their understanding by building off of their strengths.

REMEMBER

FIND JOY

and

LOOK FOR STRENGTHS!

Letter from the Editor

Christine Walker, Utah Valley University

As we open this fall season and consider our students, we need to recognize that there are many students, with and without disabilities who struggle with mathematics. We know that every student can achieve in mathematics, but it is up to us to help them find the beauty of math, and to find ways to make mathematics engaging and accessible. To achieve this objective the theme for the journal is making mathematics accessible for all students, with several strong articles that will help you and your colleagues achieve this earnest goal.

In the article *Do the Common Core State Standards for Mathematics provide a framework to access mathematical knowledge through opportunities of conflict and development of emancipatory knowledge?*, the author found that “the standards for mathematical practice provide opportunity for cognitive conflict and development of emancipatory knowledge.” In addition, it was noted that the standards for mathematical practice, as utilized in classroom activities, helps improve student engagement for all.

As we turn our attention to the article *Money Makes Sense: Understanding Standard Division Algorithm*, we learn that “students who have a conceptual understanding of the mathematics, but do not learn a way to organize their understanding into an efficient strategy are in danger of giving up on math, perceiving it to be too hard or take too much time.” From this article we learn a powerful lesson that in some cases hands on manipulatives and the realistic application of math engaged students on a level of fun where students were enjoying the learning process.

Interdisciplinary Mathematics and Music Instruction: A Review of the Literature, we find that the integration of music into mathematics classrooms has an impact on student beliefs and dispositions, most especially mathematics attitude, confidence, and motivation.

I encourage you to spend some time reading over the articles as you will find valuable ideas and points of discussion that you can share with your colleagues. Finally, a very sincere thanks to the Assistant Journal Editor, Danielle Divis who did the production of this journal. Her tireless work and endless hours made this all possible.

Note: Any mistakes are the sole responsibility of the editor and assistant editor and will be remedied. Please send corrections to Christine.walker@uvu.edu and danielledivis21@gmail.com.

Money Makes Sense: Understanding the Standard Division Algorithm

Dr. Kristy Lister, Valdosta State University

B.J. Wright, Sage Creek Elementary

Confucius once said “He who learns but does not think is lost. He who thinks but does not learn is in great danger.” This is especially true in mathematics where students who learn rote mathematics procedures, but do not have a conceptual understanding are often at a loss for when to use the procedure or prone to errors. Figure 1 provides a sample of four common errors students may make when they have learned the standard division algorithm, but no not have a firm conceptual understanding of place value.

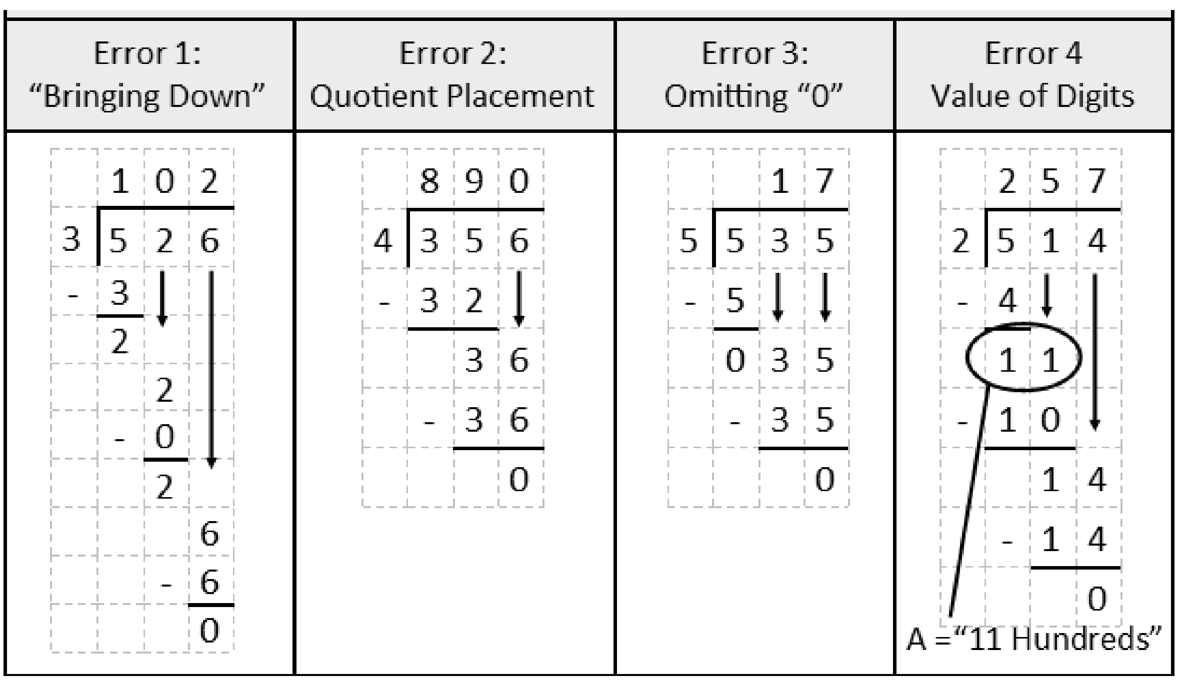


Figure 1. Four common errors in student work.

Error 1 in Figure 1 illustrates ignoring the “remainder” after follow the basic four procedures in the algorithm (dividing, multiplying, subtracting, & bringing down) for each of the digits in the dividend. In this example, the student essentially ignores 220 of the original 526. Error 2 illustrates the alignment of the quotient with the digit on the left and the need to have the same number of digits in the quotient and the dividend. In this example, the student found the correct answer of 89, and then added a "0" to fill the last place for the answer of 890, which is larger than the original dividend. Error 3 illustrates the omission of a "0" in the quotient. In this example, the student did not place "0" after the "1" in the quotient to show that "3" could not be divided evenly by "5", for a final answer of 17 rather than 107. Error 4 illustrates that even students who can correctly use the standard algorithm do not always understand the value of the digits at any point in the procedure. In this example, the student referred to point "A" as "11 hundreds" rather than "11 tens." Students may be able explain to explain that the "4" in the same problem was obtained by multiplying 2x2, but not how the 4 related to the 5 above it, nor that it has a value of 400, generally expressing it as "4 ones" or simply "it's just 4." Students who make these four types of errors may not understand that the standard algorithm for division works by sharing out individual place values and then breaking down and combining the remainder with the place value to the right.

Students who have a conceptual understanding of the mathematics, but do not learn a way to organize their understanding into an efficient strategy are in danger of giving up on math, perceiving it to be too hard or take too much time. For example, a student may have a conceptual understanding that 356 divided by 4 can represented by sharing out 356 ones into 4 groups. One strategy to accomplish this is to draw 356 tally marks shared equally between 4 groups. This inefficient strategy, though mathematically correct, takes a lot of time. The standard algorithm for long division can be a great resource for students to divide efficiently. Although students are not expected to be fluent in the standard division algorithm until sixth grade (CCSS 6.NS.B.2), students are often introduced to this algorithm in fourth or fifth grade along with other approaches. If learned on its own as one of several procedures, student may fall into a habit of learning, but not thinking. Instead, "students can profit from making sense of standard algorithms just as they should be able to reason about other approaches" (Van De Walle, Karp, Williams, Wray, 2015, p. 257).

The NCTM Principals to Action (NCTM, 2014), Mathematics Teaching Practices outline eight practices that teachers can use to help students make sense of standard algorithms. This article looks at how two teachers used four of these standards to help their fifth-grade students make sense of the long division algorithm: 1) Implement tasks that promote reasoning and problem solving, 2) Use and connect mathematical representations, 3) Facilitate mathematical discourse, and 4) Pose purposeful questions.

Implement Tasks that Promote Reasoning and Problem Solving

Tasks that promote reasoning and problem solving actively engage students in the task and allow for multiple entry points or solution strategies. Real-world contexts can increase student motivation to engage with the tasks. Many students today love money and video games. As such, two teachers designed the Money Makes Sense activity to involve both. The teachers introduced themselves as taking on the role of co-directors for a video game company who was in charge of paying all the employees who helped design a new game.

One teacher asked the students to help solve a problem she was having. The two directors were paid $500 for setting up the convention with five $100 bills – How can we divide an odd number of bills equally between two people? The students proposed that two bills could be given to each director. The teachers then asked if they should throw away the last $100 bill or rip it in half, at which point there was a collective “NO!” from the students. Students proposed that the directors should take the last $100 bill to a bank and trade it in for $10 bills that could be shared equally.

Using a real-world context allowed the students to bring forward their own personal experiences with money to immediately catch what would be a grave error in throwing away part of the money (Figure 1, Error 1) as well as introduce the idea of decomposing one $100 bill into ten $10 bills, similar to the standard division algorithm. It also motivated students to engage with the next part of the task, helping pay all the employees at the convention (i.e. game designers, programmers, graphic designers, beta testers.

Use and Connect Mathematical Representations

Tasks that engage students in making connections between mathematical representations help “deepen understanding of mathematics concepts and procedures” (NCTM, 2014, p. 10). In the Money Makes Sense activity, the teachers wanted students to make connections between the concrete representation of dividing money and abstract representation for the standard algorithm. To accomplish this goal, they first provided each group of students with four envelopes containing preset amounts of money to concretely share with different numbers of employees. Additionally, the four envelopes were designed to move student groups through different levels of complexity and address common errors in division (see Table 1).

Table 1

Levels of Complexity and Common Errors Address by Four Leveled Tasks

Level	Quantity	Employees	Bank Exchange	Common Error Addressed
1	$240-260	2	Max 10 bills	Error 1 – Breaking down remainders
2	$300-400	3-4	Max 30 bills	Error 2 – “0” First digit in quotient

3	$700-999	6-7	Max 60 bills	Error 3 – "0" Second digit in quotient
4	$800-999	8-9	Max 80 bills	Error 4 – Add coins to focus on value

As seen in Table 1, although the quantities were different for each group, to allow for later comparisons across groups, the intent and complexity for groups were the same within each level. Increasing the number of employees in which to share the funds as well as the increased quantities, were designed to promote the use of multiplicative and divisive reasoning to share out bills quickly and easily. To focus students on sharing out the bills in a similar manner and order as the standard algorithm, students were asked to start with the highest denomination. Any remaining bills were exchanged with the banker for a smaller denomination that could be shared (see Figure 2).

Figure 2. Sharing out the bills.

In order to focus on connections within the base-10 system, the "bank" was only supplied with bills and coins that, when exchanged, would mimic the base-10 place value system ($100 bills, $10 bills, $1 bills, dimes, and pennies). While the money was being shared or exchanged, one student wrote down what was happening to the money at each phase and another student was verifying that the exchanges with the bank created equal values and each employee received the same pay.

After students had concretely engaged with dividing sums of money, the teacher directed students to open a "“top secret” envelope which they were told contained a formula that could revolutionize a faster sharing out of money. Inside the envelope was an answer key for all four problem levels using the standard algorithm for long division. To promote connections between the two representations, students were asked to compare the accountant's notes for any or all of the problems they performed during the first phase of the activity with the formula to determine what the different numbers in the formula represented (see Figure 3).

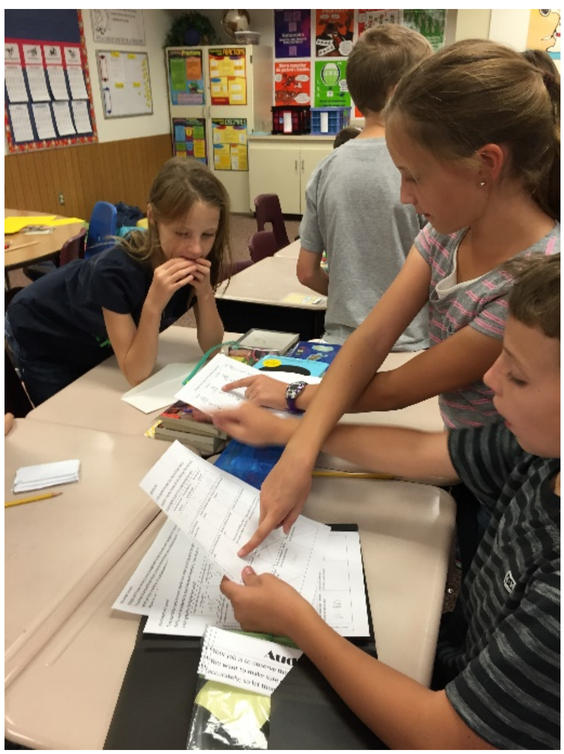


Figure 3. Students compare concrete and abstract representations.

Facilitate Mathematical Discourse

Facilitating opportunities for students to engage in discourse provides opportunities for students to make connections, engage in reasoning, and build a shared understanding of the mathematics (Litster, 2019; NCTM, 2014). Working in small groups facilitated one opportunity for students to share their ideas and make connections between their concrete experience dividing money and the standard division algorithm. The teacher facilitated a second opportunity for students to make connections across groups by bringing the whole class together to discuss their connections and unpack the standard algorithm (See Figure 4).

Figure 4. Whole class discussion.

During the whole class discussions, the first connection that students shared were observations relating to the three parts of a division problem (quotient, divisor, & dividend). Students were quick to point out that the number of employees was the divisor, dividend was the amount they started with and the quotient was the amount each employee received. Another group tentatively shared that they thought the numbers below the dividend matched the exchanges with the bank. This prompted a great discussion on place values and the value of the digits. For example, one student shared their observation that the “27” in the problem 627 ÷ 3 = 209 were the number of ones they had after exchanging two tens with the bank. (see Figure 5).

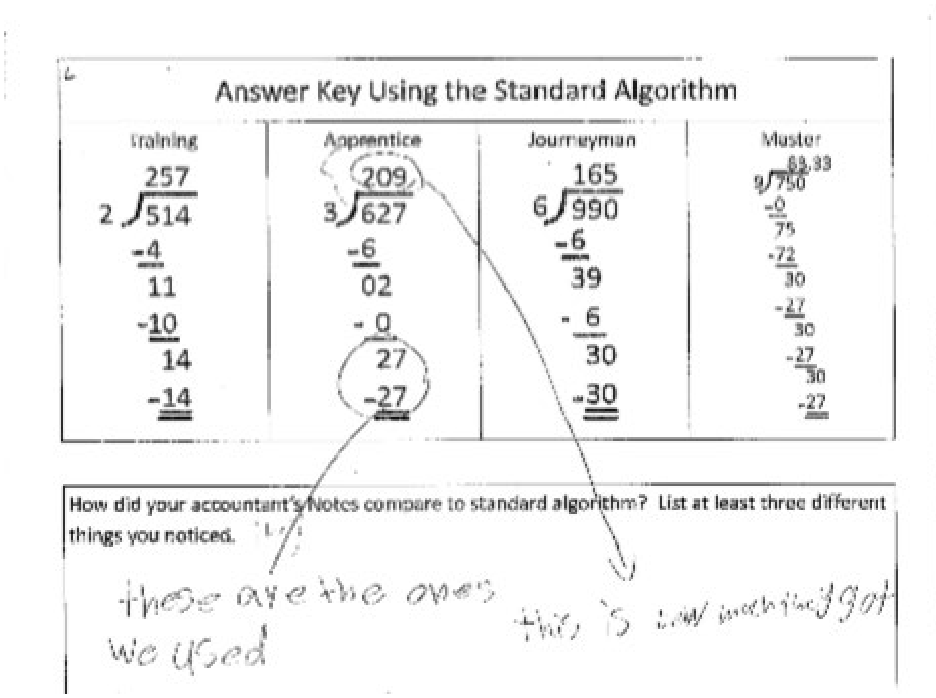


Figure 5. Student place value observation.

This connection prompted another student to ask if the “39” in 990 ÷ 6 = 165 was 39 tens or 39 hundreds. Several students voiced opinions for either answer, eliciting reasoning such as the “3” was in the hundreds place or moving from right to left the place value goes ones tens, hundreds. The original group used their notes to explain that they had 3 hundreds left after sharing 6 hundreds, so the 3 represented hundreds, but the 39 represented the number of tens they had after they exchanged the bills with the bank. The students’ purposeful question helped the class develop a shared understanding of place value within the long division algorithm.

Pose Purposeful Questions

Teachers can also pose purposeful questions to assess student understanding and facilitate reasoning and sense making (NCTM, 2014). During the group discussion, the teachers walked around and asked purposeful questions such as "what do you think this number represents?" to help students start recognizing patterns or "how do you know those numbers represented that action?" to prompt students to defend their observations. During the whole class discussion, the teachers asked purposeful questions to assess student understanding relating to the common errors such as "when and why might a "0" be placed in the quotient?"

Conclusions

The Money Makes Sense activity was designed to engage students in tasks that promoted reasoning to make connections between mathematical representations. Small-group and whole-class discussions, combined with purposeful questions facilitated opportunities for students to engage in further reasoning and connections between concrete situations, place value and the numbers represented in each stage of the standard algorithm for long division. These conceptual connections may help students as they build their fluency with the standard algorithm in sixth-grade.

As an added bonus, the Money Makes Sense activity had a positive effect on student attitude towards math and division. When one boy opened the top secret envelope, he exclaimed "oh yea, this is math!" He had forgotten that he was in math, learning about division, even though the teacher had mentioned this fact at the beginning of class. One girl explained that "it seemed like a game, but it was learning." Hands on manipulatives and the realistic application of the math helped actively engage students in the learning process. In parting, two comments made the effort put into planning and preparing this lesson worth it – "you helped us" and "can we do math again?"

References

Litster, K. (2019). *The relationship between small-group discourse and student-enacted levels of cognitive demand when engaging with mathematics tasks at different depth of knowledge levels* (Doctoral dissertation). Utah State University, Logan, UT. Retrieved from digitalcommons.usu.edu/etd/7626/

National Council of Teachers of Mathematics (NCTM). (2014). Principles to Action: Ensuring Mathematical Success for All. Reston, VA: NCTM.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., & Wray, J. (2015). *Elementary and Middle School Mathematics: Teaching Developmentally, Enhanced Pearson eText with Loose-Leaf Version -- Access Card Package (9th Edition)* Boston, MA: Pearson Education.

Do the Common Core State Standards for Mathematics Provide a Framework to Access Mathematical Knowledge through Opportunities of Conflict and Development of Emancipatory Knowledge?

Carrie Bala, Utah State University

Abstract

This study investigates whether or not current curriculum reinforces student perception of the inaccessibility of mathematics. A continuum of cognitive conflict opportunity and a continuum of knowledge type, based on descriptions by Habermas, were created as frames for analysis of the Common Core State Standards for Mathematics (CCSM). The results demonstrate an emphasis on technical knowledge and passive behavior. Conversely, the standards for mathematical practice provide opportunity for cognitive conflict and development of emancipatory knowledge.

Keywords: cognitive conflict, emancipatory knowledge, null curriculum, CCSM, mathematics

Do the Common Core State Standards for Mathematics provide a framework to access mathematical knowledge through opportunities of conflict and development of emancipatory knowledge?

Student perception of mathematics is that of a static collection of facts, with no opportunity to engage and create. “They believe that doing mathematics means applying memorized rules to problems in order to get correct answers and recalling the rules at appropriate times in response to certain types of questions” (Kamin, 2016, p.57). This perception develops from an overemphasis on technical knowledge standards, often delivered through a transmission mode of teaching. Due to pressures of high-stakes testing, “teachers have increasingly turned to teacher-centered lecture formats to deliver tested content” (Au, 2012, p.44). As such, “the consensus theory of science” dominates, with students having no opportunity to engage and conflict with the accepted knowledge. Specifically, this rigid structure “requires that institutions, commonsense rules, and knowledge be seen as relatively pregiven, neutral, and basically unchanging because they all continue to exist by ‘consensus’ “(Apple, 2004, p.78). The broad and creative field of mathematics, thus, is reduced in students’ minds to a narrow and pre-defined set of procedures.

Apple labels this narrow set of procedures as “science as truth”, to be held in opposition to “science as truth-until-further-notice” (2004, p. 93). In the first category, mathematics is a neutral and crystallized field of knowledge; in the second category, mathematical knowledge is in a process of continual change (Apple, 2004). This contrast also contains a set of oppositional student experiences, passivity and critical engagement. Au highlights that knowledge is intimately linked with “constant activity, constant interchange, and constant reflection” (Au, 2012, p.19). Additionally, Apple notes “how critical interpersonal intergroup … argumentation and conflict have been for the progress of science" (2004, p.86). Without opportunities to critically engage with ideas, progress is slowed both in individual and societal knowledge development. Thus, the contrast of student experiences in the classroom signifies a disparity in learning opportunities with significant implication.

The present study approaches one set of standards, the Common Core State Standards of Mathematics, to assess any emphasis on a specific type of knowledge and specific student experience. As standards serve to highlight the accepted knowledge for a community, a critical view will ask the following questions:

1. Is the highlighted knowledge itself transformative?
2. Does the emphasized knowledge demand an engaged, critical participation from students?

Theoretical Frameworks

In order to focus this assessment of standards, two continua were developed, a Conflict continuum and a map of Mathematical Knowledge. The Conflict continuum relies on the contrasts between Functional Reasoning and Cognitive Conflict Theory and essentially maps expected student behavior. The Functional tradition "presents a static or entropic model of society and as a consequence cannot account for change, and it overemphasizes integration" (Chilcott, 1998, p.104). The Functional tradition encourages the transmission of accepted mathematical knowledge and students' passive reception.

Conversely, Cognitive Conflict Strategy has a common pattern which includes exposing an alternative conceptual framework, creating conceptual conflict, and encouraging cognitive accommodation (Zetriuslita, Wahyudin, & Jarnawi, 2017, p. 67). This pattern presents opportunity for cognitive skepticism and students' engaged criticism. The Conflict continuum will contain these models of valued student experience as dipoles with a neutral position at its center.

The continuum of Mathematical Knowledge is structured on Habermas's categorization of knowledge. "According to Habermas, human knowledge is organized by virtue of three spheres of human interests, which he labels the technical, the practical, and the emancipatory" (Tinning, 1992, p.3). Technical knowledge, also known as instrumental knowledge, is "knowledge that will facilitate technical control over natural objects" (Tinning, 1992, p.3). This type of knowledge contains procedures, techniques, and terminology. In contrast, Emancipatory knowledge, representing criticism, transformation, and liberation, is often used in critical theory research methods. Emancipatory knowledge includes strategic awareness, conditional understanding, and self-knowledge. Hermeneutic or practical interest "generates knowledge in the form of interpretive understandings that can inform and guide practical judgment" (Tinning, 1992, p.3). Hermeneutic knowledge can be summarized as interpretation and connection and will serve as the central point of the Mathematical Knowledge continuum.

The analysis in this study presupposes that the list of standards represents only one aspect of the curriculum. Using the term "null curriculum", Eisner (2002) argues that curriculum consists of both the intellectual processes and content areas that are present and those that are neglected (p.98). Both what is explicitly presented and what is omitted, signal what knowledge and behaviors are valued. Apple reports that often "the hidden curriculum in schools serves to reinforce basic rules surrounding the nature of conflict and its uses" (2004, p.81). Indeed, both a type of knowledge and specific behavior can be emphasized by a clear presence or a subtle omission. An analytic plane shown in Appendix A illustrates the view of knowledge and the student behavior most emphasized through the display scaled data points. The empty spaces of the plane represent the null curriculum, or the knowledge and behavior least valued.

Through these multiple aspects of curriculum, students may either find connection or alienation from mathematical knowledge. While some structures provide students opportunity to experience alternative concepts, cognitive conflict, and cognitive accommodation, many practices focus "the development of a restricted conception of thinking" (Eisner, 2002, p.98). As Apple writes, "when a society 'requires'… the maximization (not distribution) of the production of technical knowledge, then the science that is taught will be divorced from the concrete human practices that sustain it" (2004, p. 95). In fact, rigid structures and hegemonic knowledge types prove inaccessible to many students, creating and multiplying inequalities in school systems. Conversely, an emphasis on emancipatory knowledge and cognitive conflict serves to connect students to mathematical knowledge and to address social inequalities. These practices "challenge the hegemonic construction of mathematics as an apolitical, neutral, and value free discipline – a construction that does not validate mathematics as a tool for social change" (Au, 2012, p.83). In essence, the inclusion of conflict and controversy in curricula could help to eliminate some bias by situating students as a part of the creative development of knowledge and knowledge systems. As Apple notes, emphasizing disagreement in education instills an identify of "men and women as creators and recreators of values and institutions" (2004, p. 80). As such, students' expectations of classroom knowledge and behaviors are much more connected to their own lives.

The analysis in this study attempts to highlight both opportunities for students to connect to mathematical knowledge and barriers to these connections. Through a mapping of standards along each continuum, the emphasized knowledge type and behavior become visible and deficiencies are easily recognized.

Method

Sample Selection

The Common Core State Standards for Mathematics (CCSM) were selected as an example of a well-researched collection of concepts, processes, and practices. While not adopted by all states, the CCSSM form the central structure to the majority of state curriculum. The authors note “the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time” (Common Core State Standards for Mathematics, 2010, p. 4). Kamin states that while the CCSM do not include teaching methods or materials, “the content standards are the minimum requirements for what mathematically proficient students should know, understand, and do upon completion of each grade level” (2016, p. 53). Additionally, the CCSM include standards for mathematical practice that carry through all grade levels, detailing ways of engaging with mathematical knowledge and communicating with other learners. “The two sets of standards function as a roadmap of mathematical learning for K-12” (Kamin, 2016, p.53).

Specific to this study, the CCSM for High School were interpreted and categorized. Of note, the content standards and the eight standards of mathematical practice were grouped as one collection in this analysis.

Procedures

Each of the standards was coded along both the Conflict continuum and the Knowledge continuum. Descriptions from the Revised Bloom’s Taxonomy (Cognitive Process Dimensions) were used to categorize each standard along the Conflict continuum. The Revised Bloom's Taxonomy places specific actions verbs in a hierarchy of increasingly complex cognition, "with remember being less complex than understand, which is less complex than apply, and so on" (Krathwohl, 2002, p.215). The structure of six major categories of the Cognitive Process Dimension was divided into three subcategories of increasing opportunity for cognitive conflict. The first three categories, Remember, Understand, and Apply, were grouped together as descriptors of passive behaviors. The fourth category, Analyze, was isolated as a neutral subcategory, providing some opportunity for interaction with knowledge but not enough to be transformational. The fifth and sixth categories, Evaluate and Create, were grouped together as descriptors of critical behavior and opportunity for cognitive conflict. Connecting verbiage of the standards with the verbs of the Revised Bloom's Taxonomy, each standard was coded as containing Opportunity for Conflict (1), Expected Neutral Behavior (0), or Passive Behavior (-1).

As a way to reinforce interpretation of the standards along the Knowledge continuum, descriptions from the Revised Bloom's Taxonomy (Cognitive Process Dimensions) were connected to the descriptions of Habermas's knowledge types. The Factual Knowledge and Procedural Knowledge categories of the Taxonomy were mapped to Technical Knowledge of Habermas; the Conceptual Knowledge of the Taxonomy was mapped to the Hermeneutic Knowledge of Habermas; the Metacognitive Knowledge of the Taxonomy was mapped to the Emancipatory Knowledge of Habermas. According to the jointly-described categories, each standard was then coded as either Emancipatory (1), Hermeneutic (0), or Technical (-1) Knowledge.

Each standard thus was coded with a Conflict score and a Knowledge score and plotted on a two-dimensional analytic plane with the Conflict continuum on the horizontal axis and the Knowledge continuum on the vertical axis. Appendix A displays the analytic plane with the coded standards as points on the plane. Standards with the same coding overlap to form circles of differing size. The diameter of each circle is proportional to the frequency of occurrence in the list of standards. Larger diameter circles represent the valued knowledge and behavior. Smaller diameter circles represent knowledge and behavior with little emphasis in the curriculum. An absence of data circles represents the null curriculum.

Findings

Of the 187 standards reviewed, 39 percent are coded as passive behavior (-1) and technical knowledge (-1). This standard coding has the highest frequency by more than double any other coding in the study. The large circle in the third quadrant of the analytic plane displays the abundance of these standards that detail technical knowledge and invite little to no critical action from students. Including such verb use as calculate, represent, multiply, and understand, these standards emphasize the transmission of accepted procedures and performance of those procedures. Perceived as neutral and indeed arrived at by consensus of the authors, these procedures are emblematic of tested content. This type of knowledge, primarily rigid and procedural in nature, paired with an expected passive student behavior, will serve as the model for the "science as truth" viewpoint that Apple describes (2004, p.93).

The second most common standard coding was neutral behavior (0) and technical knowledge (-1). Of all standards, 19% contained this coding. This particular type of standard represents accepted practical knowledge and invites some student interaction through differentiation and organization of structural parts.

Combined with the three standards coded as containing opportunity for conflict (1) and technical knowledge (-1), 60 percent of all standards fall below the horizontal axis of the analytic plane. Thus, the majority of all standards emphasize technical knowledge above all other types of knowledge. Of those technical knowledge standards, 65% do not encourage any critical action from students.

Of all standards, 10% were coded as having opportunity for conflict (1) and emancipatory knowledge (1). This type of knowledge is strategic and includes metacognitive dimensions of understanding. Emancipatory knowledge demands students works through conflicting concepts, either internally or with peers, to create a new understanding. A total of 20 standards are coded as emancipatory knowledge standards, 18 of which are paired with expected critical action from students. Of the 20, none are coded with expected passive behavior. This type of standard provides evidence that emancipatory knowledge requires a cognitive conflict process and will serve as the model for the "science as truth-until-further-notice" (Apple, 2004, p. 93) viewpoint. It is of some import to note the relative size of the data circle on the analytic plane. In fact, it is less than a third the size of the passive/technical data circle.

Also, of note is what is absent from the analytic plane, or in very short supply. Quadrants two and four of the plane, contain a total of three standards. Only two percent of the standards are coded as containing opportunities for conflict (1) and technical knowledge (-1), demonstrating that less complex knowledge is viewed as commonsensical and not to be questioned. Similarly, there are no standards coded for passive behavior (-1) and emancipatory knowledge (1), bolstering the claim that more complex knowledge demands more critical interaction from students.

Discussion

The findings clearly highlight which knowledge type and which student behavior are given value. As the majority of standards are coded as technical knowledge and expect a passive or neutral response, the implication is that mathematical knowledge is already established, and its development is finished. The view of mathematics as a monolithic truth does little to engage and encourage critical student interest. At best, the hope is for students to accept and attempt to attach some meaning to these procedures. At worst, these standards isolate students and prevent connection to mathematical knowledge.

In the opposite corner of the analytic plane, the standards coded as containing emancipatory knowledge and opportunity for conflict remain apart. These standards are small in number but represent the best opportunities for students to connect with mathematical knowledge and to view themselves as part of its development. In fact, the authors of the CCSM view the connection between emancipatory knowledge and critical engagement as vital and recommend that “designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction” (2010, p. 8). With this emphasis, the view of mathematics requires a “robust, connected understanding” of an ever-changing, creative subject (Kamin, 2016, p.58).

While the small data point can be viewed as a discouraging lack of opportunity for students, the CCSM should be seen as an improvement over previous sets of standards, primarily for the inclusion of the highly engaging practice standards. As Kamin notes, “there is a great deal of alignment between the expertise college mathematicians expect of their students and expertise detailed in the CCSSM practice standards” (2016, p. 60). Of the 18 standards coded as having opportunity for conflict and emancipatory knowledge, seven are standards of mathematical practice. As the authors of the CCSM note, the connections made between deep knowledge and critical experiences "are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics" (2010, p. 8). The CCSM authors clearly intend to provide an expansion of critical student experience in the classroom. The framework utilized in this study could be employed for future analysis of how mathematics standards have changed over time, potentially highlighting these improvements in student opportunity in mathematics. As progressive sets of standards are mapped onto the continua, a quantitative comparison could be made in documentation of any change.

While analysis has focused thus far on the coding of specific standards and the implication for student engagement, the study also reveals a more general connection between knowledge type and behavior. It is of some interest to look at the extremes overlapping the two continua, specifically standards either coded both 1 and 1 or both -1 and -1. Of the 20 standards that are coded as containing emancipatory knowledge, none call for a passive student experience. This result would suggest that emancipatory knowledge requires critical engagement of the student, with evaluation of and accommodation for an alternative concept. The null curriculum solidifies this claim, displaying the lack of standards coded as requiring a passive response from students but containing emancipatory knowledge. Similarly, the majority of standards are coded as containing technical knowledge but requiring little to no critical action from students. This finding suggests that technical knowledge is seen as inherently neutral and thus, untouchable. The null curriculum on the analytic plane does more to develop the connection between knowledge type and expected behavior than to highlight values in curriculum.

Conclusion

Student perception of mathematics is that of a collection of facts and procedures to be memorized and appropriately applied to the right situation. This study attempts to discover whether or not current curriculum reinforces this viewpoint. In analyzing the Common Core State Standards for Mathematics, results demonstrate that the vast majority of standards contain technical knowledge and expected passive behavior. This finding seems to reinforce the student perception of mathematics. However, it should be noted that the standards for mathematical practice, recommended to permeate through all standards and subsequent classroom activities, reach levels of emancipatory knowledge and invite critical student action. Recommendation to teachers and curriculum directors would include a greater emphasis on these practice standards in order to improve student engagement. Recommendations for future research include applying the analytic framework to successive sets of mathematics standards as a way to highlight improvements in student opportunities.

More generally, the study reveals a connection between knowledge type and student behavior. Emancipatory knowledge requires students to engage in cognitive conflict, while technical knowledge invites passive student response. As such, student participation in critical educational activity could bring about connection to mathematical knowledge and the understanding to address social inequalities. Through opportunities to evaluate alternative conceptions, students may find that they are not isolated from mathematics and that mathematics is still evolving. The mathematics classroom could be a place "where plurality invites critical insight and creative experimentation, where disagreement and divergence bring reinvention and renewal, and where the meaning and value of experience is found in the transitional movement experience itself brings (Nary, 2012, p.161).

References

Apple, M. W. (2004). Ideology and curriculum (3rd ed.). New York: Routledge.

Au, W. (2012). Critical curriculum studies: Education, consciousness, and the politics of knowing. New York: Routledge.

Chilcott, J.H. (1998). Structural Functionalism as a Heuristic Device. Anthropology & Education Quarterly, 29(1), 112-114.

Eisner, E. W. (1985). The educational imagination: On the design and evaluation of school programs. New York: Macmillan.

Kamin, D. (2016). The Common Core State Standards for Mathematics and College Readiness. The Mathematics Educator, 25, 52-70.

Krathwohl, D.R. (2002). A Revision of Bloom's Taxonomy: An Overview. Theory Into Practice, 41(1), 212-218.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics for High School. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

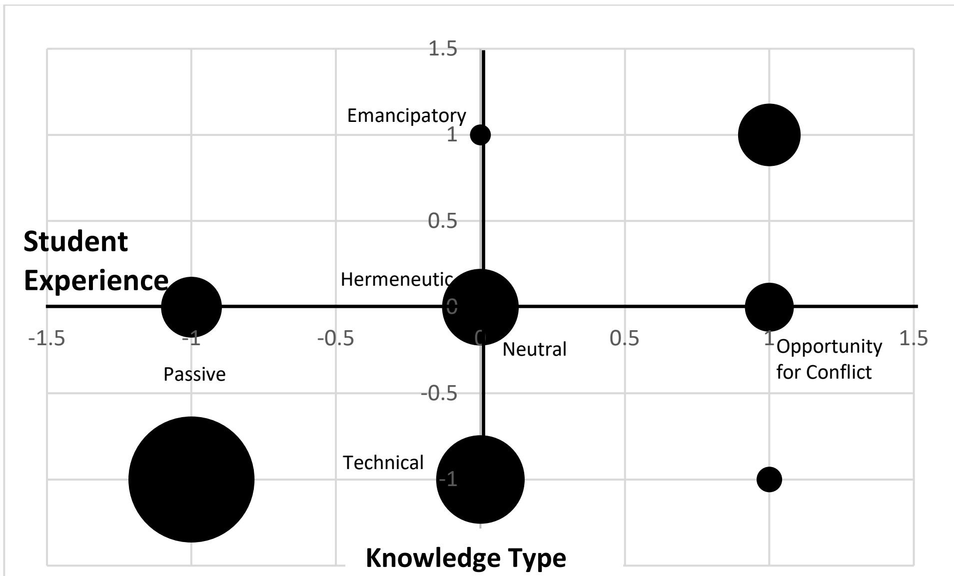
Nary, D. (2012). Democratic Visions/Pluralist Critiques: One Essential Conversation for 21st-Century Philosophy of Education. Philosophical Studies in Education, 43, 152-161.

Tinning, R. (1992). Reading Action Research: Notes on Knowledge and Human Interests. Quest, 44(1), 1-14.

Zetriuslita, Wahyudin, & Jarnawi (2017). Mathematical Critical Thinking and Curiosity Attitude in Problem Based Learning and Cognitive Conflict Strategy: A Study in Number Theory course. International Education Studies, 10(7), 65-78.

Appendix A

Frequency in Coded Standards



Interdisciplinary Mathematics and Music Instruction: A Review of the Literature

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Abstract

Because the benefits for interdisciplinary instruction are well-known, the possibility of music integration in mathematics classrooms as an impetus for deep student mathematical understanding is a compelling topic for research. A review of the literature on interdisciplinary music and mathematics reveals four common areas on which researchers tend of focus, namely the topics and strategies chosen by teachers when implementing these kinds of lessons, their beliefs of both their own abilities to integrate music with mathematics and the purpose of doing so, the influence this integration has on student achievement, and finally its impact on student beliefs. This literature review summarizes the results of empirical studies by categorizing them according to these themes. It concludes with the argument that further research is needed to understand the behavior and beliefs of secondary mathematics teachers and their students when engaging with interdisciplinary mathematics and music curriculum.

Keywords: interdisciplinary, integration, music, mathematics, interventions

Interdisciplinary Mathematics and Music Instruction: A Review of the Literature

The traditional disciplines studied in schools exist largely to maintain the organization and status quo of the school and to divide knowledge for students into more manageable chunks (Kaufman, Moss, & Osborn, 2003). However, while each discipline alone “illuminates one or more facets of the dynamic whole” (Kaufman et al., 2003, p. 7) students will learn, it is only through the interaction between multiple disciplines that the whole is truly understood. Nearly five decades ago, Piaget (1972) suggested investigation beyond the traditionally isolated school disciplines, removing the boundaries that separate them. This interdisciplinarity, he argued, is the “prerequisite of progress in [educational] research” (p. 129).

Examining the literature from more recent decades focusing on how teachers and researchers have integrated the discipline of music into mathematics reveals several exciting outcomes. The majority of articles devoted to the topic of music integration in mathematics are practitioner pieces with suggestions for distinctive music-integrated mathematics lessons that are enjoyable for students (e.g., Barger & Haehl, 2007; Blackburn & White, 1985; Edelson & Johnson, 2003; Geist & Geist, 2008; Johnson & Edelson, 2003; Moore, 1952; Rothenberg, 1996). However, results from the few existing empirical studies on this research topic can be categorized into four major themes: (a) how pre-service and in-service teachers use music-integrated mathematics curriculum, specifically their choice of topics and pedagogical strategies; (b) their beliefs about using music integration in mathematics; (c) the potential for music integration into mathematics to foster increased student mathematical ability; and (d) the potential for this integration to create positive beliefs about mathematics. This synthesize of the literature is organized according to these common themes. Following the synthesis is an examination of how the research on music-integration mathematics is lacking and what that implies for research needs moving forward.

Pre-service and In-service Mathematics Teachers' Music Integration

The literature on the integration of music into mathematics instruction in relation to teachers focuses largely on two separate areas. First, research has explored teachers' experiences with music integration, namely the specific music and mathematics topics they utilize and the pedagogical strategies they employ. Second, teacher beliefs have also been investigated, particularly beliefs about their own abilities to integrate music into mathematics and the benefits and difficulties of doing so.

The Topics and Strategies Teachers Utilized During Integration

Music and mathematical topics utilized. The research suggests that certain mathematics and music topics will commonly surface when pre-service and in-service teachers are asked to implement a music-integrated mathematics lesson. For example, An et al. (2016) coded the lesson plans and online reflections of 21 pre-service teachers who were asked to develop a lesson plan for teaching a mathematics topic primarily through the use of music-themed activities. The results revealed four main types of music activities were used: (a) music singing and listening activities; (b) music composing and performing activities; (c) musical notation learning activities; and (d) musical instrument designing and making activities. These activities spanned across all five of the NCTM (2000) content areas of numbers and operations, geometry, algebra, data analysis and probability, and measurement.

In a study of 152 pre-service teachers, An, Tillman, and Paez (2015) similarly found that following a music and mathematics intervention, student teachers could suggest music-integrated mathematics lesson possibilities in each of the five NCTM (2000) content areas. Specifically, when taking a post-test following the intervention, all 152 (100%) student teachers proposed at least one way to teach a concept that falls within number and operations, 106 (69.73%) for algebra, 126 (82.89%) for geometry, 119 (78.29%) for measurement, and finally 117 (76.97%) for data analysis and probability. It is interesting to note that the results across many studies show the most common mathematical content area mentioned by pre-service and in-service teachers following a music-integrated mathematics intervention is number and operations or number sense (An, Capraro, & Tillman, 2013; An et al., 2015; An, Tillman, Shaheen, & Boren, 2014). More specifically, An, Tillman, Shaheen, and Boren (2014) found that in their reflections, teachers wrote about how they could use music to teach the number and operations topics of counting, number relationships, the concept of fraction, real numbers, whole number computation, basic facts, and fraction computation.

Together these studies suggest the potential for music-integrated mathematics interventions as a way for pre-service teachers to learn how to integrate music into their mathematics lessons in all of the NCTM (2000) content areas.

Pedagogical strategies employed. Two studies focus on the instructional practices involved when pre-service and in-service teachers develop and implement music-integrated mathematics lessons. An and Tillman (2014) coded 78 music-integrated mathematics lesson plans developed by 45 graduate and undergraduate students studying elementary mathematics education. Lesson plans were analyzed for their content in five areas: (a) "the objective of integrated music-math;" (b) "the rationale of music-math integration;" (c) "a guided sequence for students' investigation;" (d) "math pedagogy based on music activities;" and (e) "math pedagogy that transcended music activities" (p. 31). Results showed that almost all pre-service (91.9%) and in-service (95.1%) teachers developed an in-depth route for how to guide students through an interdisciplinary music and mathematics activity as well as help the students finish the activity if needed. Percentages in each of the other four categories were similarly higher with in-service than pre-service teachers. In general, the findings contribute to the knowledge of music integration in mathematics by providing evidence that following an intervention, in-service mathematics teachers are more likely to employ all five pedagogical techniques described above. This confirms a perhaps intuitive notion that in-service teachers are more comfortable with the pedagogy involved in incorporating music in mathematics than their pre-service counterparts.

An et al. (2015) took a slightly different and more detailed look at the pedagogical strategies described by 152 pre-service teachers on five open-ended questions following their participation in an intervention that involved the creation of music and mathematics lessons with the assistance of an experienced university professor. The five open-ended questions aimed to understand what kind of music activities and pedagogical strategies the pre-service teachers would use for "helping students make sense of challenging mathematical concepts, and their strategies for connecting music and mathematics for their classroom" (p. 13). The results were divided into NCTM's (2000) five content standards and showed that pre-service teachers prior to the intervention were more likely than not to suggest superficial uses of music when teaching mathematics in every content strand. For example, the teachers used musical lyrics to learn arithmetic or played music quietly in the background to improve classroom ambiance. However, on the post-survey, the teachers were able to produce activities involving deeper integration of music and mathematics, contextualizing the mathematics by "planning rhythm, investigating intervals, and transferring chords," "creating melody, organizing musical form, and arranging instrumentation," and "musical instrument making such as designing and crafting instruments" (An et al., 2015, p. 19). The intervention appears to have potential for strengthening pedagogical content knowledge and curriculum knowledge.

While An and Tillman (2014) gave particular attention to general pedagogical strategies, An et al. (2015) took a closer look at what the pedagogical strategies particularly in relation to music and mathematics integration might look like. Together, both studies create an understanding of how a music-integrated mathematics intervention might influence the pedagogical choices teachers make in their mathematics classrooms when attempting to integrate music activities.

Teacher's Beliefs about Music Integration in Mathematics

The literature spanning the topic of music integration in mathematics classes is notably focused on pre-service and in-service teachers' beliefs regarding their ability to successfully use music in their mathematics classrooms, as well as their beliefs of the possible benefits and challenges of doing so.

Beliefs about their ability to integrate music in mathematics. In the study of 152 pre-service elementary teachers mentioned above, An et al. (2015) conducted an intervention where teachers were given an opportunity to participate in over eight hours of activities involving "contextualizing mathematics education through: (a) music composition and playing processes, and (b) musical instrument design and construction processes" (p. 12). The researchers used mixed-methods analysis of a 30-question Likert-style survey aimed at understanding four components of their self-efficacy in teaching music-integrated mathematics: (a) "self-efficacy for teaching mathematics via interdisciplinary pedagogy;" (b) "self-efficacy for motivating students to participate in mathematics tasks;" (c) "self-efficacy for teaching mathematics within music contextualized pedagogy;" and (d) "providing a positive mathematics classroom environment" (p. 13). A paired samples t-test was used to find any statistically different scores on a pre and post-survey given before and after the intervention to examine any significant differences in self-reported self-efficacy.

Results showed that all four aspects of self-efficacy measured increased between the pre-survey and post-survey with effect sizes of medium large or large. Overall evidence was found supporting the notion that interdisciplinary music and mathematics pedagogy has a capability to enhance pre-service teachers' self-efficacy for "meaningfully contextualized instruction that generates student engagement with difficult academic content" (An et al., 2015, p. 18). This finding supports previous research showing how contextualized mathematics and inquiry can lessen teachers' anxiety towards both the teaching of mathematics and towards mathematics itself (Gresham, 2007; Furner & Berman, 2005; Gresham, 2008).

An, Ma, and Capraro (2011) present a notably similar study of 30 pre-service teachers who participated in a brief 90-minute interactive intervention led by a university professor that involved applying mathematical principles to music composition activities. These teachers were given a pre and post survey to examine their beliefs about music-integrated mathematics. The survey focused on their beliefs towards music-integrated mathematics activities in four areas: (a) the engagement of the activities; (b) the relationship between music and mathematics; (c) mathematics itself; and (d) confidence in mathematics learning. Results confirm statistically significant improvement across all four of these areas following the intervention.

Together, these investigations demonstrate the potential of music-integrated mathematics interventions and lesson design to foster positive teacher beliefs towards the relationship between music and mathematics and their ability to carry out quality interdisciplinary music and mathematics instruction.

Beliefs of the benefits and challenges of music integration in mathematics. It is also important to consider teachers' beliefs towards lessons with integrated music and mathematics, as these beliefs are significantly correlated to pedagogical practice (Zdzinski et al., 2007). An et al. (2016) investigated 21 pre-service elementary teachers across 391 reflective essays, lesson plans, online discussion entries, and transcribed interview responses produced during and after a music-integrated mathematics intervention. Coding revealed student motivation as the most commonly mentioned benefit to incorporating music into their mathematics lessons. In fact, increasing student motivation in mathematics was mentioned by every single pre-service teacher. These results are consistent with An, Tillman, Shaheen, and Boren (2014), who studied the online reflective essays of 53 elementary education pre-service teachers following a music-integrated mathematics intervention. Results showed 94.59% of responses mentioned using music in mathematics to make mathematics an enjoyable experience. More specifically, teachers mentioned how the lessons enhanced engagement, made lessons more entertaining and fun, and fostered stronger motivation and interest.

On the other hand, the teachers studied by An et al. (2016) most often mentioned classroom management and control as a challenge they faced during their music and mathematics lessons. They complained of the disorderly pace and the tendency for students to talk out of turn or not follow instructions.

In summary, teachers who participate in music-integrated mathematics interventions believe these interdisciplinary lessons are promising for promoting positive student beliefs towards mathematics. This belief is consistent with empirical findings in the literature, as will be summarized shortly hereafter.

Student Outcomes with Music Integration in Mathematics

In addition to the investigations seeking to understand how pre-service and in-service teachers engage with these interdisciplinary music and mathematics lessons, attention has also been given to how these lessons affect students' mathematical abilities and beliefs.

The Potential of Music Integration for Improving Student Mathematical Achievement

Despite what has become, perhaps through word of mouth, a common assumption that musical participation can increase overall academic ability, the empirical evidence on this relationship does not necessarily support that claim (Cox & Stephens, 2006). However, when narrowing investigations down to the influence of certain musical activities during mathematics classes on academic achievement within that mathematics class, the results are more promising.

An et al. (2013) confirmed that a mathematics teacher's participation in a music integration intervention can positively affect the mathematical achievement of his/her students. Two classes with a total of 46 students in either first or third grade saw statistically significant improvement between a pre and post-test designed to assess whether students could apply their mathematical knowledge to the real world. Between the pre and post-tests, students participated in a series of 10 music-integrated mathematics lessons that their teacher created through regular collaboration with the authors. An et al. (2013) attributed their mathematical success to specific qualities of these lessons.

> Students can communicate mathematics ideas with their peers, represent mathematics concepts with multiple forms, connect mathematics content with different real life situations, think mathematics meanings from reasonable and logical perspectives and solve mathematics problems by using various problem solving strategies. (p. 15)

Courey, Balogh, Siker, and Paik (2012) sought to understand how music and mathematics lessons can specifically strengthen students' understanding of fractions. They found that after receiving 12 40-minute mathematics lessons that incorporated the ideas of fractions in music composition, a classroom of third-grade students performed significantly better on a fractions worksheet than the control group who received traditional instruction. In fact, the instruction seemed to particularly help struggling students who entered the intervention at a low level (as evidenced by a pre-test).

An and Tillman (2015) similarly studied changes in scores on a series of mathematics tests given before, during, and after an intervention of music-integrated mathematics lessons with a classroom of 28 students. Findings again confirmed statistically significant improvement between each test.

Teachers believe that one of the benefits of music integration in mathematics is its potential for improving students' mathematical ability (An, Tillman, Shaheen, & Boren, 2014), which appears to be reflected in empirical research. Together these studies suggest music-integrated mathematics lessons cultivate a greater mathematical understanding than traditional instruction.

The Potential of Music Integration for Improving Student Mathematical Beliefs

Several studies have focused on the influence of music-integrated mathematics instruction on student beliefs and dispositions. An, Tillman, Boren, and Wang (2014) used a series of independent and paired samples t-tests to investigate how the beliefs of students from two third-grade classes would differ when one teacher participated in an interdisciplinary music and mathematics intervention and the other did not. Throughout a nine-week period, the teacher leading the control classroom taught lessons using a traditional method from a textbook provided by the district, while the teacher leading the experimental classroom created and implemented 14 music-integrated mathematics lessons with the help of the authors. Following these lessons, both classes of students (n=56) were given a 36-question survey designed to assess their "(1) mathematics success, (2) mathematics attitude, (3) mathematics confidence, (4) mathematics motivation, (5) mathematics usefulness, and (6) mathematics beliefs" (p. 6). Results showed the treatment group saw significant improvement in their dispositions between pre and post-test in each of the six areas. In addition, while the pre-test showed no significant differences between treatment and control group, the difference in post-tests between the treatment and control group was significantly different in five of the six areas in favor of the treatment group and with medium to large effect sizes. Brock and Lambeth (2013) saw similar improvements in students' attitudes towards and confidence in mathematics following their teacher's participation in a music-integrated mathematics intervention.

This research illustrates how a teacher's participation in a music-integrated mathematics intervention can positively influence his/her students' mathematical dispositions.

Discussion and Implications

Though not a widely ventured area of study in mathematics education, the literature on interdisciplinary music and mathematics offers promising results for both teachers and students. Mathematics lessons that incorporate musical activities seem to have potential for engaging teachers in a wide variety of pedagogical strategies that span across all five of the NCTM (2000) content strands. These lessons also have the potential for improving both teacher and student beliefs towards mathematics, and student mathematical achievement.

Examination of empirical research on this topic reveals that the literature is lacking in several ways. First, Song An and his various teams at the University of Texas El Paso seem to be responsible for the majority of knowledge in this area. With his traditional intervention design and similarities in methodologies across many studies, knowledge is perhaps limited in how interdisciplinary music and mathematics might play out in different scenarios and contexts.

An seems to be primarily interested in pre-service elementary teachers (e.g., An et al., 2011; An & Tillman, 2014; An et al., 2015; An et al., 2016; An, Tillman, Shaheen, & Boren, 2014), and occasionally practicing in-service elementary teachers (An et al., 2013). Consequently, the field lacks understanding of how pre-service and in-service *secondary* mathematics teachers interact with music-integrated lesson planning and instruction. No research has given attention to how these lessons affect the mathematical beliefs and achievement of students at the middle or high school level, or how the content areas and pedagogical strategies used by secondary teachers might differ from elementary teachers.

The research on music integration in mathematics classrooms is both interesting and promising. However, questions left unanswered about how different groups of students and teachers participate in and benefit from this interdisciplinarity should motivate more researchers in the field to give their attention to this area of study.

References

An, S. A., Ma, T., & Capraro, M. M. (2011). Preservice teachers' beliefs and attitude about teaching and learning mathematics through music: An intervention study. *School Science and Mathematics*, *111*(5), 236-248.

An, S. A., Capraro, M. M., & Tillman, D. A. (2013). Elementary teachers integrate music activities into regular mathematics lessons: Effects on students' mathematical abilities. *Journal for Learning through the Arts*, *9*(1), 1-19.

An, S. A., & Tillman, D. (2014). Elementary teachers’ design of arts-based teaching: Investigating the possibility of developing mathematics-music integrated curriculum. *Journal of Curriculum Theorizing*, *30*(2), 20-38.

An, S. A., & Tillman, D. A. (2015). Music activities as a meaningful context for teaching elementary students’ mathematics: A quasi-experiment time series design with random assigned control group. *European Journal of Science and Mathematics Education*, *3*(1), 45-60.

An, S. A., Tillman, D. A., Boren, R., & Wang, J. (2014). Fostering elementary students' mathematics disposition through music-mathematics integrated lessons. *International Journal for Mathematics Teaching & Learning*, *15*(3), 1-19.

An, S. A., Tillman, D. A., & Paez, C. (2015). Music-themed mathematics education as a strategy for improving elementary preservice teachers’ mathematics pedagogy and teaching self-efficacy. *Journal of Mathematics Education at Teachers College*, *6*(1), 9-24.

An, S. A., Tillman, D. A., Shaheen, A., & Boren, R. (2014). Preservice teachers' perceptions about teaching mathematics through music. *Interdisciplinary Journal of Teaching and Learning*, *4*(3), 150-171.

An, S. A., Zhang, M., Tillman, D. A., Lesser, L. M., Siemssen, A., & Tinajero, J. V. (2016). Learning to teach music-themed mathematics: An examination of preservice teachers' beliefs about developing and implementing interdisciplinary mathematics pedagogy. *Mathematics Teacher Education and Development*, *18*(1), 20-36.

Barger, R., & Haehl, M. (2007). Guitars, violins, and geometric sequences. *Mathematics Teaching in the Middle School*, *12*(8), 462-466.

Blackburn, K. T., & White, D. L. (1985). Measurement, mathematics, and music. *School Science and Mathematics*, *85*(6), 499-504.

Brock, D., & Lambeth, D. (2013). The effects of music on basic mathematics fact fluency for third grade students. *Online Submission*, *2*(2), 43-60.

Cox, H. A., & Stephens, L. J. (2006). The effect of music participation on mathematical achievement and overall academic achievement of high school students. *International Journal of Mathematical Education in Science and Technology*, *37*(7), 757-763.

Courey, S. J., Balogh, E., Siker, J. R., & Paik, J. (2012). Academic music: Music instruction to engage third-grade students in learning basic fraction concepts. *Educational Studies in Mathematics*, *81*(2), 251-278.

Edelson, R. J., & Johnson, G. (2003). Music makes math meaningful. *Childhood Education*, *80*(2), 65-70.

Furner, J. & Berman, B. (2005). Confidence in their ability to do mathematics: The need to eradicate math anxiety so our future students can successfully compete in a high tech globally competitive world. *Dimensions in Mathematics*, *18*(1), 28 – 31.

Geist, K., & Geist, E. A. (2008). Do re mi, 1-2-3, that's how easy math can be: Using music to support emergent mathematics. *YC Young Children*, *63*(2), 20.

Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. *Early Childhood Education Journal*, *35*(2), 181-188.

Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre-service teachers. *Teaching Education*, *19*(3), 171-184.

Johnson, G. L., & Edelson, R. J. (2003). Integrating music and mathematics in the elementary classroom. *Teaching Children Mathematics*, *9*(8), 474.

Kaufman, D., Moss, D. M., & Osborn, T. A. (Eds.). (2003). *Beyond the boundaries: A transdisciplinary approach to learning and teaching.* Westport, CT: Greenwood Publishing Group.

Moore, A. I. (1952). Mathematics of music. *Journal of Education*, *135*(3), 85–86.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Piaget, J. (1972). The epistemology of interdisciplinary relationships. In L. Apostel (Ed.), *Interdisciplinarity: Problems of teaching and research in universities* (pp. 127-139). Washington, DC: OECD Publications Center.

Rothenberg, B. S. (1996). The measure of music. *Teaching Children Mathematics*, *2*(7), 408-410.

Zdzinski, S. F., Ogawa, M., Dell, C., Ching Yap, C., Adderley, C., & Dingle, R. (2007). Attitudes and practices of Japanese and American music teachers towards integrating music with other subjects. *International Journal of Music Education*, *25*(1), 55-70.

Using Noticing of Mathematical Strengths as a Vehicle for Empowerment

Amy Kinder, President of UCTM

There is a long history of inequitable practices in the United States which include tracking, focusing on gaps between groups, and limiting access to worthwhile mathematics tasks. The impact of these actions has had devastating effects on students (Boaler, William, & Brown, 2000; Esmonde, 2009; Gutiérrez, 2002, 2008, 2012; Hand, 2012). More and more research has focused on the great need to shift to more equitable practices (Gutiérrez, 2012; Hand, 2012; Secada, 1992). One line of research that has garnered tremendous positive attention is the use of videos to increase teachers' abilities to notice students' mathematical strengths. Noticing mathematical strengths is define as collaboratively viewing classroom videos with a focus on making sense of students' thinking by looking for mathematical strengths that students exhibit. Then, as a group asking questions, pressing each other's thinking and critically analyzing the mathematical moments (Hand, 2012; Jilk, 2016; Mason, 2011; Sherin, 2007).

Noticing students' strengths allows decreasing discriminatory practices and assisting students in building identities as mathematicians (Borko, Koellner, Jacobs, & Seago, 2011; Gutiérrez, 2012; Jilk, 2016). Video clubs were as a platform for reflecting and refining practices with teachers (Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Sherin, 2007; Sherin & van Es, 2008; van Es & Sherin, 2007). Research on video clubs have shown compelling evidence that when teachers have time to slow down and reflect it has the power to shift mindsets and beliefs about students' abilities, as well as strengthen the teacher's in-the-moment decision-making skills (Castro, Clark, Jacobs, & Givvin, 2005; Hand, 2012; Sherin, 2007; van Es

& Sherin, 2009). The research reveals that there are still hurdles to overcome with teacher beliefs. It remains to a challenging task to act on student thinking at the moment and how to train and implement effective ways to utilizing equitable noticing with videos at the school level (Hand, 2012; Sherin, 2007; van Es & Sherin, 2009). The noticing research so far is quite compelling because of the effects that research on teachers' instruction, beliefs, and empowerment of students. Three themes that emerged as factors contributing to inequality in mathematics education: (a) tracking, (b) focusing on gaps between groups, and (c) limiting access to worthwhile mathematics tasks. This paper seeks to reify these current equality issues and to offer two teacher practices that have been suggested to enhance some of these endemic problems. The two practices are known as "noticing of mathematical strengths" and "video clubs."

Historical Inequities and Beliefs that Effect Noticing

Early in the 1900s, schools were established and began increasing in size from one room to multiple room schoolhouses. Many teachers started requesting that individual students receive higher-level mathematics instruction. This line of thinking led to the development of advanced-level mathematics classes, and these classes were considered unnecessary for most students who were destined to work in factories (Kilpatrick & Stanic, 1995). This move created a necessity for a challenging, rigorous mathematics experience for a small percentage of privileged students, while other students were left to focus on computation and rote memorization (Boaler et al., 2000; Gutiérrez, 2012). Ellis (2005) suggested that this practice of denying students access to higher mathematics content continued largely unhindered until the 1970s. With this shift in education practice, the mindset that rigorous mathematics is not meant for everyone became firmly entrenched in the United States of America education belief system (Gutiérrez, 2009).

Tracking Perpetuates Deficit Noticing on Student Abilities

The idea that individual students deserved access to rich mathematics, while others are relegated to lower-level classes became an institutional norm (es Van, Hand, & Mercado, 2017; Gutiérrez, 2002, 2008). As school sizes grew, the need to maintain this separation led to the development of in-school tracking systems. Teachers' observations of perceived ability grouped students into "lower" and "higher" performing groups (Gutiérrez, 2002). By the 1980s, researchers pointed out how the education system perpetuated numerous negative aspects of students' perceptions of mathematics (Boaler et al., 2000; Gutiérrez, 2012). Once students were "tracked" in middle school, rarely were opportunities offered to move to higher levels of mathematics classes. Instead, lower-performing students focused on remedial fact practice or drills (Gutierrez, 2008). Boaler interviewed students from the classes considered low achieving by teachers and found "students were particularly concerned about the low level of their work and talked at length about teachers ignoring their pleas for more difficult work" (Boaler et al., 2000, p. 635).

Today, tracking remains a part of the education system, and that is primarily due to schools continuing to utilize this practice as a means to justify the support of struggling learners (Boaler et al., 2000; Esmonde, 2009; Gutiérrez, 2009). While evidence shows that tracking has minimal positive effects on higher tracks of students, the lower tracks have a significant negative impact (Boaler, 2008; Gutiérrez, 2002).

Achievement Gap Perceptions Add to Deficit Noticing on Student Abilities

Another practice that has shown detrimental effects in building students' identities as mathematicians is the continued focus on the achievement gap between races and socioeconomic classes (Esmonde, 2009; Gutiérrez, 2002). Gutierrez (2008) contends that concentrating on the achievement gap encourages "deficit thinking and negative narratives about students of color and working-class students" (p. 4). A disproportionate focus positioned on these gaps highlights students' deficits, rather than identifying the source of inequities. Some school systems use the achievement gap as a means to justify the lowering of expectations for students (Boaler et al., 2000; Esmonde, 2009; Gutiérrez, 2002, 2008, 2012; Hand, 2012).

Power of Noticing Using Video in Reforming Teaching

Video clubs that focus on noticing mathematical strengths have shown promising results in combating educators' historically inequitable practices (Erickson, 2011; Hand, 2012; Jilk, 2016). Watching videos is not a new practice; for the past twenty years, videos have been used in a variety of ways to refining teacher skills. Frequently, videos have been used to model teaching for teachers to emulate or short scenarios to assist teachers with decision-making or problem-solving. This work focuses on analyzing research that uses video differently. The focus on watching videos is to make sense of students' thinking by collaborating, look for mathematical strengths, ask questions, press each other's thinking and critically analyze the mathematical moment on the video (Hand, 2012; Jilk, 2016; Mason, 2011; Sherin, 2007). This paper seeks to rectify these current inequality issues and to explore two teacher practices that have been suggested to improve some of these problems. The two methods are known as "noticing of mathematical strengths" and "video clubs." The four critical points of viewing videos are: (a) focus on strengths, (b) build mathematical identities, (c) the need for time to collaboratively reflect, and (d) the struggle to move away from deficit thinking.

The research also reveals that there are still hurdles to overcome when using video with teachers, especially dealing with teachers' deficit beliefs. Added to those challenges is that learning to seize and act on student thinking at the moment is a difficult task. The final challenge is how to provide meaningful professional development and training (Hand, 2012; Sherin, 2007; van Es & Sherin, 2009). Even with some of these obstacles, the noticing research with video, so far, is quite compelling because of the effects that research on teachers' instruction, beliefs, and empowerment of students as mathematicians.

Utilize Video to Learn to Focus on Noticing Mathematical Strengths

Researchers developed video clubs as a way to have teachers collaboratively work together to analyze video. Studies found that video clubs for noticing for mathematical strengths can radically change teachers' viewpoints from focusing on the deficits to moving to the strengths (Castro et al., 2005; Hand, 2012; Sherin, 2007; van Es & Sherin, 2009). Cohen (1994) found that when a teacher notices what positives student bring to the class, it changes viewpoints and allow students to be seen as mathematically capable. Teachers reported that they found themselves interacting differently with students in their courses due to viewing them as more capable mathematicians (Mason, 2011; Sherin, 2007; Sherin & van Es, 2008; van Es & Sherin, 2009). Seago explored how video can engage educators in "actual practice of teaching" and to learn to in real-time "interpreting the mathematical logic of student thinking, analyzing the mathematical territory of a problem . . . [and] designing probes to elicit student mathematical understandings" (2004, p. 276). Video clubs provide a collaborative avenue to learn to focus on noticing mathematical strengths.

Building Math Identity Through Utilizing Video

Noticing mathematical strengths during a video club creates a forum where teachers begin to reframe their thinking and when focusing on examining students' thinking. With this reframing, teachers can become more apt to see strengths, instead of deficits, which in turn will strengthen students' mathematical identities (Boaler & Staples, 2008; Esmonde, 2009; Gutiérrez, 2011). As teachers view students as capable mathematicians, it opens the door to utilizing more meaningful mathematics, instead of focusing on using skill-based instruction (Gutiérrez, 2002; Hand 2012). Students build their mathematical identity through daily interactions with teachers and peers. As positive interactions increase, it provides opportunities for the student to begin to view themselves as mathematicians (Esmonde, 2009, Jilk, 2016).

Many video club studies found that as teachers attended more club meetings teachers' noticing of strengths increased (Hand, 2012; Mason, 2011; Sherin, 2007; Sherin & van Es, 2008; van Es & Sherin, 2009). After participated in video club meetings; teachers reported seeing more strengths on a day-to-day basis in the classroom (Jilk, 2016; Sherin & van Es, 2008; van Es & Sherin, 2009). The discussions during video clubs strengthened teachers' abilities to see strengths in their students that they may have overlooked due to previous dispositions on students. There is research that contends if teachers do not actively focus on strengths, it will be challenging to notice occurring in the classroom. (Mason, 2011; Rosaen et al., 2008). When teachers focused on strengths, their instruction transformed into more rigorous mathematics tasks that support the development of richer mathematical understandings by students Jilk, 2016; Mason, 2011; Secada, 1992).

Noticing to Assign Competence to Student's During Instruction

An important outcome for some of noticing research is that teachers increase assigning competence to students. Assigning competence is defined as "publicly naming an intellectual strength that is being used by the student(s) in a moment to move the group work forward or further the team's mathematical understanding" (Esmonde, 2009, p. 1012). Assigning competence is beyond just telling students that they are doing great, complementing their work or telling them they are trying hard. Assigning competence is connected to the student feeling empowered. Assigning competence comments might sound like this, "Raul, really added to our understanding of finding the area of the triangle with his model and justification of his reasoning. Let's use his thinking and Lucy's and compare approaches. Great use of models and explanation of your strategies."

While watching the video, the teacher can see how comments and conversations with students can leave students empowered. Using videos allows a real-time opportunity to notice if students are being empowered during mathematical discussions (Jilk, 2016, Sherin & van Es, 2008). Competence can change students' ideas and perceptions about the true meaning of what it means to be a powerful mathematician (Esmonde, 2009; Hand 2012; Jilk, 2016; Mason, 2011). It is important in breaking down inequities for students to see themselves as powerful mathematicians.

Videos Clubs Provide a Refuge to Slow Down and Reflect

A common theme that came up repeatedly in the research with noticing with videos is that it "affords the luxury of time" (Sherin, 2007, p. 293). When teachers can take time to examine a lesson, it allows noticing specific actions and discussions that students are having (van Es & Sherin, 2002). Many times in video club's teachers would view the videos repeatedly with each time bringing new insight or focus (Borko et al., 2008; van Es, Hand, & Mercado, 2017; van Es & Sherin, 2009). Researchers found that teachers start to "imagine themselves in the future acting (responding) more appropriately than before" when they return to their classrooms to work with students (Mason, 2011, p. 38). With time always seeming to be in short supply in education, studies have shown that the time spent in video clubs has a positive impact on student achievement, building mathematical identity, and changes the way the teacher plans mathematical tasks for the students (Jilk, 2016; Sherin & van Es, 2008; van Es & Sherin, 2009).

Struggles Remain with How to Move from Highlighting Deficits to Strengths

One of the hardest teaching skills is to learn to notice mathematical strengths and when to take advantage of mathematical moments (Mason, 2011; Rosaean et al., 2008; Sherin, 2007; Sherin & van Es, 2008). Teachers are immersed in a culture that focuses on examining students that are falling behind, needing Tier 2 or Tier 3 interventions, and students who are not yet proficient with grade-level standards. The culture of many schools is to use professional development time to focus on highlighting and discussing these deficits. When trying to develop a strengths-based model, it is running contradictory to years of focusing on filling holes in students' knowledge and closing achievement gaps. Sherin & van Es (2008) contends, "we are unknowingly trained to identify learners' mistakes and misunderstandings. We analyze what students do not know or cannot do, and then we try to close the gap with what they need to understand", (p. 28). In "Pedagogy of the Oppressed," Freire, (1970) argues that educators are not products of the past actions in mathematics classes. He contends that teachers need to transform their teaching beyond how they learned in school.

> Many of these leaders, however (perhaps due to the natural and understandable biases against pedagogy) have ended up using the 'educational' methods employed by the oppressor. They deny pedagogical action in the liberation process, but they use propaganda to convince (p. 68).

Video clubs have provided an avenue to move away from focusing on deficiencies, but focusing on strengths. Video clubs gives teachers a forum to develop, practice, and refine these skills. Sherin (2004) contends that utilizes video assists in developing "analytic mindset" that is "a different kind of knowledge for teaching—knowledge not of 'what to do next,' but rather, knowledge of how to interpret and reflect on classroom practices" (pp. 13-14). Changing the culture of education to view strengths instead of deficiencies will take significant time and training.

Next Steps in Research

When teachers use video clips of themselves teaching to notice mathematical strengths, it creates opportunities for students to be viewed as capable learners of rich mathematics (Borko et al., 2011; Gutiérrez 2012; Jilk, 2016). The creation of video clubs provides a platform for collaborating reflecting and refining practices in a safe environment (Rosaen et al., 2008; Sherin, 2007; Sherin & van Es, 2007). Video clubs afford the unique opportunity to slow time down and allow for some deep reflection and analysis as well as strengthening the teacher's in the moment decision-making skills (Castro et al., 2005; Hand, 2012; Sherin, 2007; van Es & Sherin, 2009). An outcome on noticing with videos is that it has the strong possibility of creating an environment that has the power to challenge some teachers' deficit mindset beliefs on students' abilities (Mason, 2011; Sherin, 2007).

The next steps for the research are: (a) how to improve training, (b) how to deal with persistent deficit mindsets, and (c) determining which protocols/actions have the most impact day to day instruction. Professional development and training on video noticing is not widespread; it is still in its infancy. Struggles remain on how to best provide teachers time to participate in video clubs. Additionally, the issue of dealing with deficit mindsets is an overwhelming task. Shifting a teachers' thinking is not a quick, overnight fix it requires extensive time and reflection. Finally, each of the video clubs in the study focused on different nuances and asked teachers different questions, so a comparative study might be useful in narrowing down what are the key components and actions that make a difference a teacher's daily instruction.

Our current education system is fraught with problems, yet there are glimmers of hope for creating instruction and experiences with students that leave them inspired and viewing themselves as capable mathematicians able to understand rich mathematics.

References

Boaler, J., William, D., & Brown, M. (2000). Students' experiences of ability grouping: Disaffection, polarization, and the construction of failure. *British Educational Research Journal*, *26*(5), 631-648.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110*(3), 608–645.

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed ability approach. *British Educational Research Journal, 34*(2), 167–194.

Borko, H., Koellner, K., Jacobs, J., & Seago, N. (2011). Using video representations of teaching in practice-based professional development programs. *ZDM Mathematics Education, 43*(1), 175-187.

Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, R. Phillip & V. R. Jacobs (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17-34). New York: Routledge.

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. *Review of Educational Research, 79*(2), 1008-1043.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning*, *4*(2&3), 145-187.

Gutiérrez, R. (2008). Research commentary: A “gap-gazing” fetish in mathematics education: Problematizing research on the achievement gap. *Journal for Research in Mathematics Education, 39*(4), 357–364.

Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance. *Science and Math: Equity, Access, and Democracy*, *18*(3), 9–16.

Gutiérrez, R. (2011). Context Matters: How Should We Conceptualize Equity in Mathematics Education? *Equity in Discourse for Mathematics Education, 55*(2), 17-33.

Hand, V. (2012). Seeing power and culture in mathematics learning: teacher noticing for equitable mathematics instruction. *Educational Studies in Mathematics*, *80*(1), 233-247.

Jilk, L. M. (2016). Supporting teacher noticing of students' mathematical strengths. *Mathematics Teacher Educator, 4*(2), 188-199.

Kilpatrick, J. & Stanic, G. M. A. (1995). Paths to the present. In I. M. Carl (Ed.), *Prospects for school mathematics* (pp. 3-17). Reston, VA: NCTM.

Mason, J. (2011). Noticing: Roots and branches. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 35–50). New York: Routledge.

Rosaen, C. L., Lundeberg, M., Cooper, M., Fritzen, A., & Terpstra, M. (2008). Noticing noticing: How does investigation of video records change how teachers reflect on their experiences? *Journal of Teacher Education, 59*(4), 347-360.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 623-660). New York, NY, England: Macmillan Publishing Co, Inc.

Sherin, M. (2007). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, B. Barron, & S. Derry, *Video research in the learning sciences*. (pp. 383-395). New York: Routledge.

Sherin, M. G., & van Es, E. A. (2008). Effects of Video Club Participation on Teachers Professional Vision. *Journal of Teacher Education,60*(1), 20-37. doi:10.1177/0022487108328155

van, Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks,* 251-270.

van Es, E. A., & Sherin, M. G. (2009). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education,13*(2), 155-176.

Six Principles of High-Quality Instruction

(From the 2008 Utah Mathematics Teacher)

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Steven R. Covey, author and management expert, explains in his book Principle-Centered Leadership (1989) about an interesting phenomenon. In surveying 200 years of success literature he found that around 1940 the literature began to change from a character ethic to a personality ethic. In the character ethic the core push was to develop fundamental traits such as service, honesty, industry, patriotism, integrity, self-discipline, and benevolence. The personality ethic changed the focus to “human relations techniques, on influence strategies, on image building, on getting what you want, . . . on success programming and people manipulation tactics” (Covey, 1989, p xi). The continuation of this trend has yielded many quick-fix and band-aid approaches to increase success in various enterprises, both individual and institutional. However, these latter approaches, rarely succeed when separated from the principles associated with the character ethic, especially when evaluated with the test of time.

Focusing on foundational principles is a change from the current conversations around mathematics education. Debates have focused around the effectiveness of methods (lecture, group work, manipulatives, writing about mathematics, seatwork, etc) and the implementation of various educational philosophies (reform, traditional, etc.). Current research findings have found that effective instruction is largely independent of instructional method or strategy. For example, when educational researchers judge the quality of instruction by observing lessons, both instruction that is considered “reform” as well as “traditional” are rated at the highest level, as well as rated at the lower levels (Weiss, 2003). We can conclude that different "styles" or "methods" can be equally effective.

Another example can be found in international studies of middle school instructional quality. The two highest achieving countries in the latest TIMSS video studies, Japan and Hong Kong SAR, teach very differently from each other when comparing teaching styles. Japanese instruction often looks similar to what many refer to "reform" instruction with more time spent on fewer problems and lots of student-to-student interaction with presentation of student work as a central focus of discussion. Mathematics instruction in Hong Kong looks, on the surface, a lot like US instruction with teachers giving interactive lectures, having students doing some seatwork during the lesson, and largely focusing on understanding and performing mathematical procedures (Hiebert et al., 2005; NCES, 2003).

These findings raise the question, what is it about instruction, if not the style or the method, that determines effectiveness? We argue that the answer is found, at least partially, in the ideas of Stephen R. Covey mentioned earlier: foundational principles. In this case it is foundational principles of high-quality instruction. Principles are much like a compass, showing the direction you should go but not dictating how to get there. The principles are not so explicit as to tie a conception of high-quality instruction to a particular form, but, if the principles are understood well, they can be used to evaluate instructional quality of varying forms.

In the remainder of this short paper we explain six principles of effective instruction that emerged from a study of Japanese middle school mathematics teachers. These principles come from a study of mathematics teacher education in Japan. Student teachers in Japan teach fewer lessons than their US counterparts. This enables the cooperating teachers to spend a lot of time discussing lesson plans and instructional decisions with the student teacher before the lesson is ever taught. In our study the student teachers and the cooperating teachers spent about three one-hour sessions discussing the lesson plan before the lesson was taught and about an hour after it is taught. We used the conversations, 19 pre-lesson conversations and 7 post-lesson conversations, to explore what the cooperating teachers viewed as important in designing and teaching a good lesson.

Principle 1: *High-quality mathematics instruction intellectually engages students with important mathematics.*

This principle appears to be the most central feature of a high-quality mathematics lesson. This was a topic in every single one of the 19 pre-lesson conversations between cooperating teachers and student teachers. Not only was it most frequently discussed across conversations but the other five principles are all closely tied to this single central principle of high-quality mathematics instruction. Although US teachers also emphasize engaging students as important, they tend to emphasize physical engagement rather than intellectual engagement (Wilson, Cooney, & Stinson, 2005; Wang & Cai, 2007). Japanese teachers focus explicitly on intellectual engagement. Below is an excerpt of a conversation where this is illustrated. CT U is looking at ST M's lesson plan for the first time. The lesson is introducing the idea of a variable to the students. After looking at the lesson plan for a few minutes and asking some clarifying questions the following conversation takes place.

CT U: Are there any places that students use their head?

ST M: There is no such a place. Nothing at all.

CT U: Your plan is to do this and this, right? Students won't use their head at all. I don't know what you plan is in this part but they won't use their head either. This part only requires them to fill in blank. I don't know if you really want to do that yet. You didn't plan to stimulate student's "thinking process." So it will be quite a mediocre lesson. Do you think this [problem?] will make students think?

ST M: No, I don't think this will make them think.

It is clear that to the CT this lesson would be "mediocre" because there is nothing in the lesson to stimulate student's "thinking process." He asks a simple question "Are there any places where students use their head?" This one question summarizes this first principle well, because if the answer is no, then there is no hope of it being a good lesson.

Principle 2: *High-quality instruction is guided by an explicit and specific set of goals that consist of student motivation, student performance, and student understanding.*

Every Japanese lesson plan begins with a set of goals. The goal statements are very important to Japanese teachers. The goal statements help Japanese teachers balance between mathematical skills and conceptual understanding, something that is often dichotomized in the mathematics education literature. The goals also helped the teachers balance two other issues, to make the mathematics interesting and meaningful to the students while maintaining high mathematical standards. The goals help to guide teachers in developing a lesson. The cooperating teachers continually referred back to the goal of the lesson to see if the activities suggested by the student teacher were aligned with the goals. They even went beyond checking for alignment but they challenged to student teachers to come up with the best activities that they could that would accomplish all of the goals of the lesson.

Principle 3: *The flow of high-quality instruction begins from a question or a problem that students see as problematic*.

As students intellectually engage in the problem or in answering the question they learn the lesson's hatsumon or big mathematical idea. The flow of mathematical ideas follows a natural path from what students currently understand and know to the new material of the day's lesson. In our analysis of the data we found frequent references (13 of the 19 conversations) to a concept that the Japanese call the "flow" of a lesson. Flow includes the whole logical structure of the lesson as planned (how it builds on students' ideas, how the task creates a need for the mathematics, etc) as well as how the lesson actually plays out (building on specific student comments, transitions, etc). The lesson flow answers the natural questions raised by principles one and two. In which problem, questions, or activities will the students intellectually engage (principle 1) that will best address the goals of the lesson (principle 2), and in particular, will raise the hatsumon or big idea of the lesson. Ideally, the hatsumon can be developed largely based on work the students do, but lessons vary on how well the Japanese teachers reach this ideal.

Principle 4: *High-quality instruction is created with close connections to past lessons and to build a basis for future lessons*.

The lessons have strong connections within a unit as well as connections across grades. The lessons in a unit help students progress to ways of thinking, writing, and representing mathematics evident in the discipline of mathematics. One interesting result here is that lessons within a unit changed depending on the placement within a unit: beginning, middle, or end. Japanese teachers teach lessons at the beginning of the unit in a very open-ended, exploratory fashion. However, at the end of the unit they lesson are more "focused" and are taught in a more explicit fashion. Although the analogy is not perfect, the beginning lessons look more like proposed "reform" instruction while the ending lessons look more "traditional." However, all lessons still strive to have students doing intellectual work in a way that naturally builds connections to the new material.

Principle 5: *High-quality instruction adapts so that all students are engaged in mathematical work that appropriately challenges their current understanding.*

It is clear from the pre-lesson conversations that differentiating instruction is important to these Japanese mathematics teachers. More than half of the conversations, 10 out of 19, discussed adapting instruction for different kinds of students. Adaptations of the lesson are done differently than the current US differentiated instruction literature recommends. Much of the current US literature pushes for differentiation along learning style classification, focuses on differences between individuals, and is not content specific (Gregory & Chapman, 2002).

Japanese teachers emphasize commonalities among students rather than the differences. They craft lessons based on knowledge common to all students in the class, but challenge all students. Of course the instruction is more effective for some students than others and is more challenging for some than others. Part of the lesson planning process in Japan is to consider how to adapt the lesson to students who are struggling or who are not challenged. The Japanese then adapt instruction by considering two groups of students, those that understand specific content and those that are struggling to understand. For those that understand and are not challenged, they adapt the material to make it more challenging. For the students that are struggling they provide hints or carefully adapt the task so it is still challenging these students, but at their level. Below is a quote from a student teacher who explains how she failed to do this in a lesson she had just taught. There were some that solved the problem very quickly, and there were others who couldn't do anything at all. I wasn't able to follow up on those two groups. Now I can, but at that time I wasn't sure what should have been said. Nobody was able to come up with all four methods, but there were groups that used two or three methods. To those groups, I said, “Are there any other ways?” or “How would elementary school students solve this problem?” But, there was little reaction to those questions, and I wonder if my questions weren’t appropriate.

Principle 6: *High-quality instruction requires a well thought out, detailed lesson plan that addresses the previous five principles and ties them together in a coherent lesson*.

We admit that this principle is less about instruction itself and more about what is needed for good instruction to take place. However, it was clear that this was an extremely important principle that cooperating teachers wanted student teachers to learn. It is also clear from the post-lesson conversations that both the cooperating teachers and student teachers thought that many of the disappointments in the lesson could have been solved by better preparation and more “research” on the part of the student teacher.

Conclusion

These principles seem to be a good basis for understanding what is necessary for students to learn mathematics with understanding. A couple of researchers surveyed all examples they could find of projects, programs, curriculum, or systems where students successfully learned mathematics with understanding (Hiebert & Grouws, 2007). They could only find two things that were common among all of these efforts. The first one is that learning with understanding was an explicit focus. This finding tells us that learning for understanding will not come as a by product of focusing on something other than understanding. This focus is part of principle 2. The second commonality was that students had to struggle with the mathematics, that is, they had to do some intellectual work during the lesson. This corresponds to principle 1. These two principles then are necessary for students to learn with understanding. So if we want instruction to help all students learn mathematics with understanding we need to ensure that these two principles are present in each lesson. The other four principles we found the Japanese teachers focused on mainly support the implementation of these first two principles.

References

Covey, S. (1989). Principle entered leadership. Executive Excellence: Provo, UT.

Gregory, G., & Chapman, C. (2002). *Differentiated instructional strategies: One size doesn't fit all*. Corwin Press: Thousand Oaks, CA.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404).

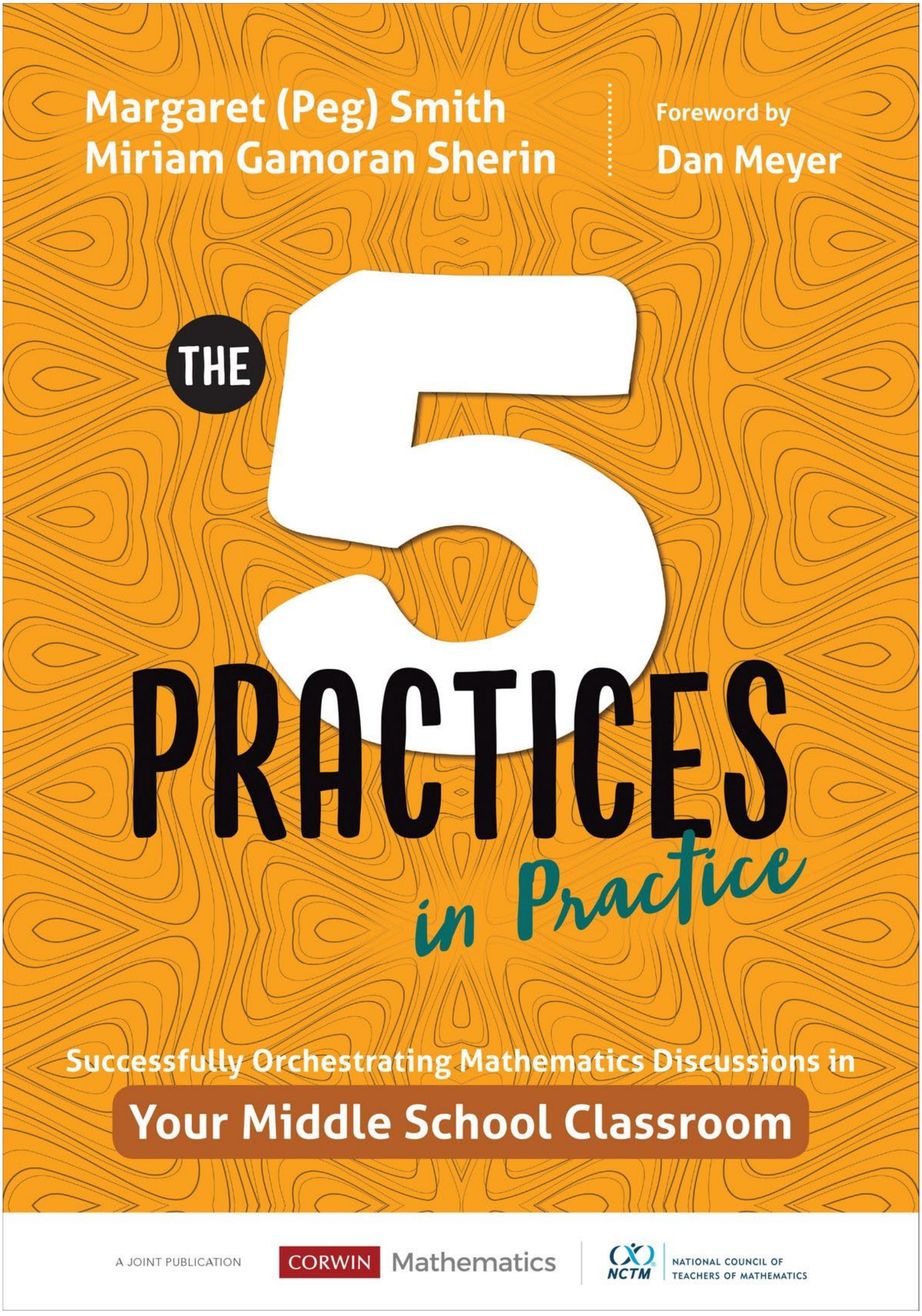
Greenwich, CT: Information Age. Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., et al. (2005) Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 Video Study. *Educational Evaluation and Policy Analysis*, *27*(2), 111-132.

National Center for Educational Statistics (2003). Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study Washington, DC: U.S. Department of Education, National Center for Educational Statistics.

Wang, T., & Cai, J. (2007) United States teachers' views of effective mathematics teaching and learning. *ZDM Mathematics Education*, *39*(4), 315-327.

Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education*, *8*(2), 83-111.

Weiss, I. R., Pasley, J. D., Smith, P.S., Banilower, E. R., & Heck, D. J. (2003). Looking inside the classroom: A study of K-12 mathematics and science education in the United States. Retrieved January 5, 2006, from http://horizon-research.com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