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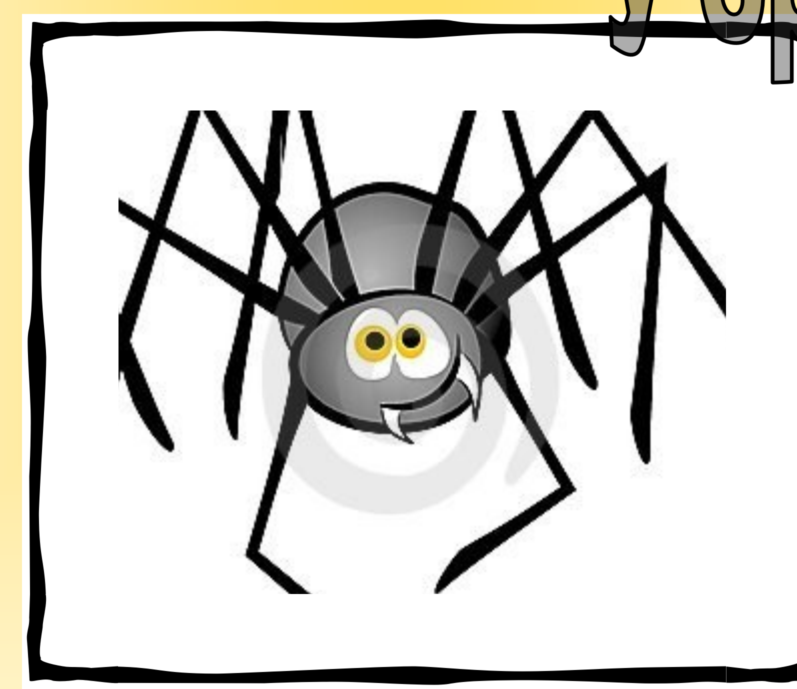
October Problem to Ponder, taken from www.NCTM.org
President's Corner

You are staying at a rural cabin, and the only method to get water is to draw it from a well. A 4-gallon bucket and a 9-gallon bucket are the only containers for carrying water to the cabin. In one trip to the well, what whole number amount of water in gallons could you bring back to the cabin in the buckets? No other markings are on the pails, and you can't do any estimating—you need to supply exact whole number amounts only!

What if you had a 4-gallon bucket and a 10-gallon bucket?
What if you had an *n*-gallon bucket and an *m*-gallon bucket?

Utah Mathematics Teacher
Fall/Winter 2011-2012
Volume 4, Issue 1

Sam the Slippery Spider



http://utahctm.org

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Call for Articles

The ***Utah Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Some of the features are: UCTM Leader Spotlight; Letter from the NCTM President; Letter from the UCTM President; Voices from the Classroom; Mathematics for English Language Learners; Puzzle Corner; Recommended Readings and Resources; the Common Core State Standards and Implementation; College and University Research; and others.

Teachers are especially encouraged to submit articles for ***Voices From the Classroom***, including inspirational stories, exemplary lessons, beginning teacher ideas; or managements tools. Sample ideas are (but not limited to) focused on teachers or districts who have successfully implemented the Common Core State Standards, Inquiry based calculus, and new math programs K-12.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Christine Walker (Christine.Walker@uvu.edu). A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included. Items for ***Beehive Math News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

UCTM Awards '11-12

Don Clark Award

Jacqueline Taylor Voyles started her teaching in Pocatello, Idaho in the year 1966. The following year she was part of the founding faculty for Springville Middle School. Jackie later finished a master's degree in Mathematics Education and assisted in the development of the Math Lab at BYU. As a faculty member at BYU Jackie has touched the lives of many future teachers. She has also had the opportunity to serve as an assistant editor for the Journal for Research in Mathematics Education. Jackie summarized much of her career in the following way, "it seems that during each piece of the journey of a career in Mathematics Education, I was in a place where there was freedom to experiment with curriculum, teaching methods, and to observe many students in their learning of mathematics. I feel very blessed to have been continually placed in situations where development and research could easily take place."

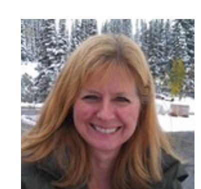
Karl Jones Award

During his second year of teaching 5th grade at Sharon Elementary School in Orem, Joey (Joseph Rino) started a math endorsement program through BYU. Though he didn't know it at the time, that decision would have huge implications on his career. The math endorsement had an immediate positive impact on his classroom teaching, but it also opened doors to teach professional development courses for Alpine School District. Joseph has demonstrated excellence in teaching and promoting student thinking in his classroom. Joseph currently is working on a Ph.D. in mathematics education at BYU.

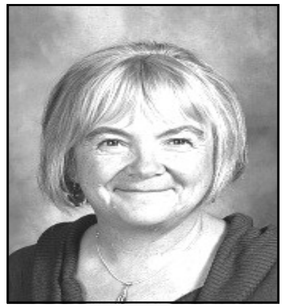
George Shell Award

Cami Perkes has loved mathematics since she was a child but didn't decide to be a math teacher until she was almost finished with college. She is now in her fifth year of teaching 7th through 12th graders at West High. Cami was in a small group of teachers, parents, and University professors to set up the Math Teachers' Circle at the University of Utah, where math teachers can collaborate and experience new mathematics. She has also participated as a mentor teacher in the Math for America SMART program for the past two years. As a participant in the common core academy and district professional development, Cami has been actively involved in the implementation of the common core in Salt Lake City school district. She currently teaches 7th grade common core math, Secondary 1, Algebra 2, and Higher Level Mathematics in the IB program.

Muffet Reeves Award



Kathy Lambert is a mathematics coach in the Salt Lake City School District and has worked with hundreds of teachers, focusing on improved curriculum, instruction, and assessment. Kathy's ongoing work with teachers includes implementing coaching cycles, organizing and leading Professional Learning Communities, facilitating elementary mathematics endorsement courses, speaking at conferences, and much more. Kathy's positive attitude, strong belief in student achievement, and her willingness to push teachers to the next level all contribute to her success as a teacher of teachers.



Presidential Award Elementary Finalist

Linda L'Ai is currently a fourth grade teacher at the Edith Bowen Laboratory School. She graduated with a B.S. in Biological Sciences from the University of California, Davis. This is also where she received her Elementary and Secondary Teaching Credentials. She received her Master's Degree and Bilingual Certification (Spanish) from the California State University, Sacramento. Her major interest is in the integration of math and science in all areas of the curriculum and providing students with learning experiences in a meaningful context. This past summer she worked with math teachers in the Mayan highlands of Guatemala.

Presidential Award Secondary State Finalists

Sharon Christensen currently teaches at Mountain Ridge Jr. High in Highland UT, and has taught Jr. high for 22 years, courses from math 7 to algebra2. She has a bachelors and masters degrees in math education from BYU, holds a gifted and talented endorsement and is a National Board Certified teacher. Sharon really enjoys working with students and helping them understand mathematics; mentoring interns, new teachers and student teachers. She says, "I enjoy working with teachers and helping them become better teachers. I also enjoy collaborating with other teachers and learning new things from them."

Renae Seegmiller has taught secondary mathematics courses including middle school mathematics through AP Calculus and Statistics for a total of 23 years in the Utah districts of Granite, Piute and Sevier as well as in Clark County District in Laughlin, Nevada. She has served as math department chair for most of those years, and as a district math specialist for Piute School District earning her doctor of education degree from University of Montana. Renae is very interested in how students think about mathematics and how they make sense of problems with a goal to help every student make sense of and become confident with mathematics. Renae has a very patient husband (Gary) who supports her in her personal and educational goals and is also the mother of two brilliant sons who are both software programmers.

Vivian Shell has a Bachelor's degree in Geology with a minor in Math from the University of Utah and has taught middle and high school math for 15 years. Vivian has collaborated with a team of teachers from across the Salt Lake City School District to create curriculum which enhances student success in Algebra. She enjoys sharing this curriculum with others and discussing approaches to teaching that are rigorous and accessible to all students and has presented units of study, both in professional development sessions and as a guest teacher in other classrooms. In recent years, Vivian has been working with the integration of Geometry and Algebra as well as the Practice Standards of the new Common Core State Standards. She is excited about the Common Core and looks forward to working with teachers to improve access to math and create a culture of mathematicians in our students.

Utah Mathematics Teacher

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Presidents Message

Logan T. Toone, UCTM President

Math Education Colleagues,

What an amazing time to be a math educator! We are in the middle of the transition to a common core that promises unprecedented rigor and coherence for the standards we teach. Despite the promise of a high-quality set of curriculum standards, we still need to remember that the mere adoption of standards is not enough. We need to take critical steps toward better instruction and increased collaboration.

I have a close friend who is a master dutch-oven chef. The dishes he creates are incredible! He makes everything from pulled pork, to peach pie. It's amazing what he can do with just a few quality ingredients, a big piece of cast iron and a bunch of hot coals. I have tried to replicate his work in my own dutch-oven, and even if I use the same ingredients and equipment as he does, my dutch-oven work never turns out as good as his. I have discovered that there is an art to dutch-oven cooking. My friend understands this, and he is a master of the art.

Teaching the common core is a lot like dutch-oven cooking. The standards are the high-quality ingredients and equipment. But high-quality ingredients and equipment are not enough. Teaching the common core in such a way as to truly foster increased student appreciation and mastery of mathematics is an art, and the art of teaching mathematics will continue to play a critical role in our classrooms.

Our challenge is to become the best we can possibly be. May we all put aside outdated and ineffective practices in our classrooms. May we seek the professional learning and skill required to teach the common core the right way. May we become better artists. I wish you the best in all your efforts!

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CCSSM and Curriculum and Assessment: NOT Business as Usual

by NCTM President J. Michael Shaughnessy
NCTM Summing Up, May 2011

Recently, I had the opportunity to participate in a working conference on issues related to mathematics curriculum and assessment under the new Common Core State Standards for Mathematics (CCSSM). Forty-four states have now adopted CCSSM, launched by the Council of Chief State School Officers and the National Governors Association Center for Best Practices in 2010. The Department of Education has funded two consortia to develop assessment instruments for CCSSM. Those assessments will go into effect in all states that have adopted CCSSM in the 2014–15 school year. As we all learn more about the Common Core Standards and the accompanying assessments, it is becoming increasingly clear to me that they provide an opportunity for us to make deep changes in the way that we teach and assess K–12 mathematics in our nation. However, as I travel to meetings and conferences throughout the country, I sometimes hear statements like, "Oh, our state already does the Common Core; it won't be much of a change for us." Or, "We're going to just wait and see what happens." Let me suggest that both of these extremes—one, an assumption that the changes called for by CCSSM have already happened, and the other, an assumption that they will come about on their own, or that CCSSM will quietly go away—are quite naïve.

Systemic changes in mathematics education will necessarily accompany the Common Core State Standards for Mathematics *because these standards are national in character*. The individual states themselves, as is their right under our Constitution, have, one by one, decided to accept the Common Core State Standards. This is *new* in the United States—never before have so many states agreed to base mathematics instruction on a common set of standards. Furthermore, these standards include both Standards for Mathematical Content and Standards for Mathematical Practice, and students' mastery of both the content and the practices will be assessed in the designs being created by the two assessment consortia. By the way, the eight Standards for Mathematical Practice are not teaching practices—they are *student practices*—processes that students need to engage in and develop facility with as they learn mathematics and solve mathematical problems. The fact that students will be accountable to the Standards for Mathematical Practice is another systemic change for mathematics education in our country. Although in the past individual states have included performance tasks and extended constructed response items calling for student work and reasoning, such assessments have been inconsistent, and sometimes not persistent, in the United States. This will change.

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The two components of a place-value numeration system:

1. Place-value —
2. Face-value —

Place-values in base 10:

Write the number __________ in base 10 in expanded form:

Place-values in base 5:

Write the number ______ in base 5 expanded form:

Name ________________________ Date__________ Period_____

Bases Worksheet
Base 5

Use the space below to draw and name the blocks:

Name:	Name:	Name:	Name:
Number of units: 1000	# of units:	# of units:	# of units:

Using the fewest number of the manipulatives pictured above, how could the following numbers be represented?

4 = _____blocks _____flats _____rods _____units
17 = _____blocks _____flats _____rods _____units
89 = _____blocks _____flats _____rods _____units
100 = _____blocks _____flats _____rods _____units
220 = _____blocks _____flats _____rods _____units

Can you represent numbers larger than those above using these manipulatives? What are some larger numbers that you can represent?

_____blocks _____flats _____rods _____ units = _____
_____blocks _____flats _____rods _____units = _____
_____blocks _____flats _____rods _____units = _____
_____blocks _____flats _____rods _____units = _____

What is the largest number that you can represent with the manipulatives pictured above?

_____blocks _____flats _____rods _____units = _____

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In light of the national character of CCSSM and the new types of accompanying assessments, we can start doing certain things now as we think about curriculum, instruction, and assessment in the era of the Common Core.

With respect to curriculum, we can begin to examine the materials that we use to see how well they support the content and practice standards in CCSSM. A project is now under way to develop tools that teacher leaders, districts, and states can to use to review curricular materials to determine how well they align with CCSSM—both with the content standards and the practice standards. The tools from this project will be available to the public in the next few months. One caveat in regard to curriculum: *this is not the time for states to run off and write their own curricula based on the Common Core State Standards.* Quite a number of very thoughtful, carefully tested and piloted curriculum projects and materials have appeared over the past twenty years—many of them, in fact, have undergone several updates and revisions. We actually *do*have good curricula in this country—lots of them! Although the existing materials will need some adjustments to improve their fit with CCSSM, building on existing materials will be a much more efficient and effective process for states and districts than inventing totally new materials on their own. A move by individual states to reinvent the curricular wheel in a rush to implement CCSSM is ill advised. States would do much better to work together with other states on adapting existing curriculum materials.

With respect to instruction, implementing CCSSM's Standards for Mathematical Practice will call for engaging students much more in such processes as—

- problem solving;
- communication of mathematical ideas in meaningful classroom discourse;
- making connections across topics and to contexts;
- reasoning about and justifying solutions;
- developing a positive disposition toward mathematics;
- creating and sharing multiple representations of mathematical concepts and procedures; and modeling mathematical processes.

Excellent resources are available to teachers for instilling and developing fluency in the mathematical practices in our students. At the high school level, particularly useful resources include NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (2009) and the publications in the accompanying Focus in High School Mathematics series. Valuable resources for teachers of mathematics in prekindergarten–grade 8 include the grade-level books in NCTM's Teaching with Curriculum Focal Points series. In addition, grade-band books in NCTM's rapidly expanding Essential Understanding Series provide deep, cross-grade discussions of the most important mathematical content and offer excellent resources for teachers to use as they consider instruction based on the CCSSM.

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Finally, with respect to assessment, we need to get a head start to prepare both ourselves and our students for the new types of assessment that will be used with CCSSM. The two assessment consortia funded by the Department of Education, SMARTER Balanced Assessment Consortium (SBAC) and the Partnership for the Assessment of Readiness for College and Career (PARCC), have been sharing their respective plans for assessment with the public, and we should all become familiar with them. Both consortia plan to include extended mathematical performance tasks in their instruments. You can download presentations on both assessment plans so that you and your students can get involved in deeper assessment tasks right away, gaining experience with such tasks well before 2014. Performance tasks are available to the public from the MAP project on the MARS website and can be downloaded for noncommercial use. The Silicon Valley Mathematics Initiative also has made a collection of assessment tasks that can be downloaded. In addition, NCTM has an excellent portfolio of assessment resource materials, along with accompanying support materials for teachers, including the Assessment Sampler series.

The bottom line for all of us is that under CCSSM, it is not, nor will it be, business as usual. With the inclusion of the Standards for Mathematical Practice, we have been handed an opportunity to make some significant changes in our mathematics instruction. And for the first time we have an opportunity to have cross-state, common mathematics assessment of our students' progress in both content and mathematical practices. Together, let's make the best decisions we can as we proceed to implement these new standards.

Assessment and the Common Core State Standards: Let's Stay on Top of It!

by NCTM President J. Michael Shaughnessy
NCTM Summing Up, June 2011

In the May President's message, I mentioned that with the implementation of the Common Core State Standards, some changes will occur over the next three years in the teaching and assessing of our K–12 mathematics students. Among the changes will be new common mathematics assessments that students in most states (44 and counting) will take. Two assessment consortia, Partnership for the Assessment of Readiness for College and Career (PARCC) and Smarter-Balanced Assessment Consortium (SBAC), have obtained federal grants to develop assessment tools—both formative and summative assessment instruments—to assess students' proficiency with the content and practices specified in the Common Core State Standards for Mathematics (CCSSM) by the start of the 2014 academic year. Each state participating in the Common Core Standards will use the assessment instruments created by one of the two consortia. Some states have chosen which of the consortia they will work with, while others are still considering their plans. The assessment consortia are beginning to share their plans for assessing the Common Core Standards, and we all

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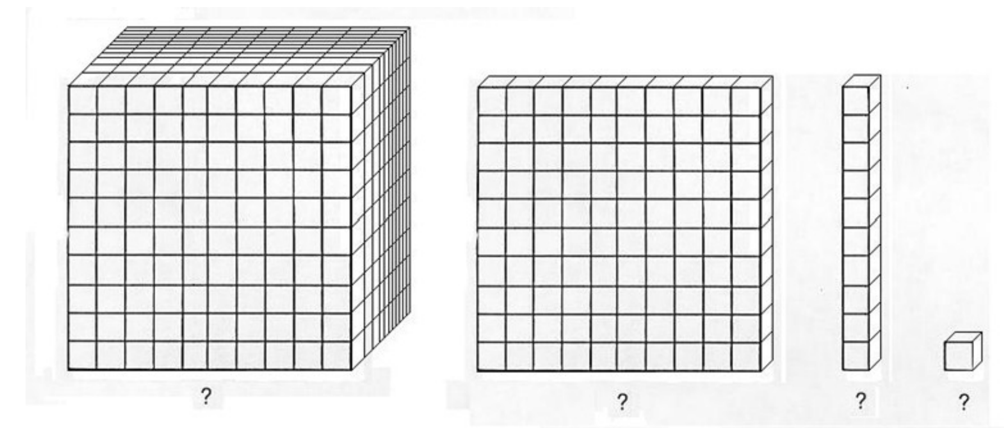
Name ________________________ Date__________ Period_____

Bases Worksheet
Base 10



Name: Block Number of units: 1000	Name: # of units:	Name: # of units:	Name: # of units:

Write 3,592 in expanded form:

How would it be represented using base 10 blocks?
_____blocks _____flats _____rods _____units

What were your thoughts and insights about this numeration system?

If students are able to grasp this concept easily and want more of a challenge, encourage them to experiment with finding the sum of ***24 and 32*** and the difference of ***i32 and 24*** in the new system. Have a few students share some of their results from this activity with the class. Ask them what numbers they converted and how the number was shown with the manipulatives.

Segment 3: Tying it Together

Explain to the students that the number system that we commonly use is a base 10 numeration system, which is the reason we group by tens with the base 10 manipulatives. The second system is a base 5 system, so the manipulatives are grouped by fives. Numeration systems have two components: place-value and face-value. The different *manipulatives used*, in both systems, represent the different *place-values*, and the *number* of blocks used is the *face-value*. The students should write a description of each component on their worksheet. With the students, identify the place-values for base10, writing these in the space provided on the worksheet. Refer to the example where 3,592 was written in words and have them write this in expanded form. Have the students write another number, this time with five or more place values, in expanded form. Next, have the students write the place values for base 5 where indicated. Choose one of the largest numbers represented with the base 5 manipulatives and write this number in expanded form. Ask the students, *"Can we represent numbers with more than four digits in the base 5 system? What would be the fifth place-value in base five? The sixth? The seventh?"* The students should write these on their worksheet.

Finish the lesson by giving the students some challenge problems to work through on their own, including the challenge to discover a quicker or simpler way to convert numbers into base 5 besides using the manipulatives. Some example homework questions would include converting numbers into a different base (base 3, base 6, etc.), or adding numbers in base 5. Probably one of the best homework assignments would be for the students to research certain aspects of the Maya, Babylonians, Egyptians and Roman ancient numeration systems. The lesson could also segue into class projects or give the students opportunities to present their research findings. The options are limitless!

Cannon, L., Dorward, J., Heal, R., & Edwards, L. (2007). National library of virtual manipulatives (version 3.0) [Software]. Available from http://nlvm.uvu.edu/

Heeren, C., Magliery, T., and Pitt, L. (1998). *Lesson 1: Binary numbers.* Retrieved from http://www.mathmaniacs.org/lessons/01-binary/

Music is the pleasure the human mind experiences from counting without being aware that it is counting. ~Gottfried Leibniz

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must keep informed of their draft assessment plans as the implementation process goes forward.

At a recent conference, "Moving Forward Together: Curriculum & Assessment and the Common Core State Standards for Mathematics," leaders of the two consortia presented drafts of their current assessment plans to a group of nationally recognized leaders in mathematics education. The PARCC and SBAC presentations are both available on the NCTM website. Both consortia included their implementation timelines, pointed to their plans for assessment support tools, and outlined some of the details of their assessment plans. For example, PARCC currently plans to include "through course" assessments at intervals throughout the school year. SBAC is planning to create adaptive tests—tests that generate subsequent items for a student depending on how he or she responded to previous items. PARCC plans to provide content frameworks, model instructional units, sample assessment tasks, and professional development modules along with their assessment instruments. SBAC plans to offer online assessment literacy guides, provide training for local development of assessment tasks, and create virtual professional learning communities around their assessment instruments.

The conference was a priority of the Joint Task Force on the Common Core Standards (NCTM, AMTE, NCSM, ASSM) and funded by a National Science Foundation grant. Presentations by the two consortia, along with panel presentations at the conference, generated much discussion and debate. The conference report includes a summary and recommendations for how the mathematics education community should continue to work to influence the CCSSM assessment and implementation processes.

The report's recommendations are extensive and detailed. They focus on concerns raised about the assessment plans and the processes for revising them, the need for ongoing professional development throughout the implementation of CCSSM and the accompanying assessments, and the importance of including expertise from the mathematics education community in the plans of the consortia for assessment design and review. The recommendations include the following:

- Ensure that the Standards for Mathematical Practice are embedded in the assessments.
- Focus attention on content changes at the middle grades.
- Design the PARCC "through-course" assessments to support teaching and learning by facilitating multiple modes of content delivery.
- Assist SBAC in the creation of scoring categories, sub-scores on constructs, and tagging systems to ensure that valid information is reported to teachers, parents, and students.
- Support long-term sustainability of assessments and an evidence-based approach to revisions to ensure that they are appropriate.
- Enlist the help of curriculum developers in resources.

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- Request and lobby for policy-level changes to lengthen the timeline and process of implementing the assessments, given the complexity of the task.

- Urge the Council of Chief State School Officers (CCSSO) to create and communicate a governing structure for current and future work on the CCSS.

Let me share a few reflections on some of the conference recommendations: Participants were in agreement that assessment of the CCSSM Standards for Mathematical Practice is a necessary condition for the success of any assessment plan related to the Common Core State Standards. The challenge and opportunity provided by CCSSM is to assess students' proficiency in selecting and using mathematical processes in addition to their content knowledge. To do so nationally is unprecedented, and to fail to do so when the opportunity presents itself would be a travesty. Even states that have not adopted the CCSSM should include the assessment of students' mastery of the Standards for Mathematical Practice in their state assessment plans. These standards echo the Process Standards in NCTM's *Principles and Standards for School Mathematics,* and the skills that they identify are indispensable to mathematics education of the highest quality.

Conference participants voiced a good deal of concern about the depth and breadth of some of the content changes that many states will need to make at the middle grades as they implement the Common Core. Participants also recommended particular attention to adjustments and revisions in the standards that may be needed at the middle grades. Aspects of both the PARCC and SBAC assessment plans generated many questions and concerns, as well as focused recommendations for each plan. Any proposed"through-course" assessments need to be designed so that they do not force content presentation and sequencing into lockstep. Participants also urged the consortia to design instruments that can report sub-scores on various categories of students' mathematical proficiency rather than one overall assessment score.

The report emphasizes the importance of developing assessments that will be amenable to research, review, and revision and pointed out the critical need for tapping the expertise of the mathematics education curriculum and assessment communities in the design and development of the assessments. The report also urges the consortia to continue to partner with NCTM, AMTE, NCSM, and ASSM, and to implement the recommendations of our Joint Task Report.

The main idea that I have taken away from this conference is that the work of developing assessments for CCSSM is a task that concerns *all* of us—curriculum and assessment leaders, mathematics supervisors, teacher leaders and district leaders. Ensuring that the recommendations in this conference report are enacted will take efforts on the part of us all. PARCC recently provided a survey for educators to provide feedback and review the assessment process as it proceeds.

As the assessment development processes continue to go forward over the next several years, we will all need to stay vigilant, to inform ourselves, our colleagues, and our peers about developments in the CCSSM assessment process. Let's stay on top of it!

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Segment 1: Base 10 Manipulatives

Before starting the lesson, give each student a copy of the Bases Worksheet (see page 43 and note on bottom of page) to guide their note-taking and learning during the lesson. Organize the students into groups of 3-4 and give each group a supply of the manipulatives. Begin the lesson by introducing the students to the base 10 manipulatives. Asses the students' understanding of these manipulatives and give a quick explanation about them as needed. Explain first what the unit is, then the rod (10 units), followed by the flat (10 rods), and lastly the block (10 flats). Help students make the connection that we group the manipulatives by tens, and when we have a group of ten, we exchange these for the next larger valued manipulative (example, 10 rods for one flat). In the space provided on the Worksheet, have each student draw what the block would look like, then count the number of units in each manipulative (this is critical for understanding the different base blocks).

Take the number 3,592 and have students write this number out in words: three thousand five hundred ninety-two. Have them discuss how this number would be represented using the manipulatives; they should present their answer using the manipulatives. Check the groups and ensure all students are arranging the manipulatives from left to right, highest value to lowest value. Let the students make the connection that this is the same as writing numbers with the highest value on the left. Students should catch onto this quickly and should be encouraged to experiment with representing other numbers. Ask the students to model some addition and subtraction problems with their manipulatives. Give students enough time to become comfortable working with the manipulatives, understanding that the exchange rate is 10 to 1.

Give each student a minute to write down, then discuss with their group, their thoughts and insights about the number system and base 10 manipulatives after this activity.

Segment 2: Introduction to Base 5

Draw the students' attention to the base 5 manipulatives on their Worksheet (page 44). Ask the students to draw (with teachers guidance) and identify similarities and differences between these manipulatives and the base 10 manipulatives they just worked with. Have each student name the manipulative (these should be the same name used for the base 10 manipulatives: unit, rod, flat, and block), count the number of units in each figure, and write this number in the space provided. It is essential that the students make the connection that these manipulatives are grouped by fives rather than by tens.

Pose the question, *"Using this technique of grouping by fives, how would you arrange 12 units using the fewest number of manipulatives?"* They should be able to come up with the answer of 2 rods and 2 units. Give the students a few other numbers to represent using the new manipulatives, using progressively larger numbers each time. The students are converting numbers into base 5 without necessarily becoming cognizant of that fact. Let the students work in pairs as they experiment with this activity, converting the numbers on their Worksheet to base 5. Monitor the students and challenge them to express the answer by drawing the appropriate number of manipulatives.

Note: Use base 5 manipulatives if they are available. This would also be an appropriate time to use the virtual manipulatives on the National Library of Virtual Manipulatives (NLVM) website (Cannon, Dorward, Heal & Edwards, 2007), http://nlvm.usu.edu, if desired.

Numerical Base Systems

Joylyn Loveridge
Utah Valley University, Student, Math Education Major

Teaching students how to work in different numerical bases and numeration systems can seem daunting at times. I have personally worked with college students who have struggled with such topics. However, I believe that if different numerical base systems are introduced in a manner that connects students' prior knowledge to the novel ideas presented, then the students' struggle for understanding will be lessened, and their level of engagement will be increased. This article illustrates one way this topic can be introduced. I believe the provided lesson helps students connect their existing knowledge of place-value and base systems to the novel concepts of different numerical base systems, thereby thus providing a scaffold for student learning. I would consider this appropriate material for the Honors Core Content for 7th Grade.

Consider the number 3,592. What does this number really mean? When was this meaning taught? How was it taught? The answer to these questions will be essential to knowing how best to introduce and teach different numerical base systems. Often numbers such as 3,592 are introduced with the aid of concrete manipulatives (or Base 10 Blocks). These manipulatives increase student learning and understanding of an abstract concept by creating a concrete and visual representation. This lesson uses this fundamental idea to create a conceptual understanding of different numerical base systems.

Note: Although this lesson is written specifically for base 5, it can easily be adapted for use in many different numerical base systems.
Materials needed:

- Base 10 manipulatives
- Bases Worksheet
- Internet access (optional)

Note: Bases Worksheet *is meant to guide the students' learning and help them "see" how to write numbers in base 5 and should be adapted to meet the teacher's instructional procedures. It is not meant to be the focus of the lesson.*

Constructive Struggling

MESSAGE 17 Constructive Struggling

THE VALUE OF CHALLENGING OUR STUDENTS

American teachers are soft.

That's the message I heard when I started my service in the Peace Corps. I was assigned to teach mathematics (in French) in Burkina Faso, a small West African country unknown to most Americans. Throughout our eleven weeks of training, our Burkinabè teacher trainers explained to us that American teachers tend to want all students to succeed and that we grade students too high. "Your students won't respect you if you are too soft," Sou and Salam reminded us on more than one occasion. Over time, I learned to put that message in perspective. I observed that the Burkinabè system was based on a philosophy that seemed to be aimed largely at eliminating students from the school system—a totally opposite goal from that in the United States, and a goal that carried its own challenges. But this experience caused me to take a closer look at what we expect of students in U.S. mathematics classrooms. I began to wonder whether our compassion for students and our desire for all students to succeed might in fact be disadvantaging them. It is now clear to me that in too many cases we are not expecting enough of our students. In fact, most mathematics teachers report to me that their students are not willing to try hard problems that they can't immediately see how to solve.

Over and over again, we hear that U.S. math students deal with less challenging mathematics than students in other countries. The content of international tests like TIMSS1 and PISA2, and the performance of U.S. students on those tests, reinforce the notion that our students may not be dealing with the same

1Trends in International Mathematics and Science Study, formerly the Third International Mathematics and Science Study (http://timss.bc.edu)
2Programme for International Student Assessment, administered by the Organisation for Economic Co-operation and Development (OECD; www.pisa.oecd.org)

level of complexity in mathematics as students in other countries. I don't advocate that the United States should copy the programs of other countries; there are too many cultural and societal differences and too many challenges for any program to be successful on a large scale when it is transported in its entirety to a different setting. But I strongly support our close examination of practices used elsewhere that might inform our work to improve mathematics teaching and learning in this country.

Spoon-Feeding Our Students

In *The Teaching Gap,* Jim Stigler and Jim Hiebert (1999) report the results of classroom observations that were part of the 1995 TIMSS. This particular part of the study sent observers to eighth-grade classrooms in the United States, Germany, and Japan. Observers categorized the level of mathematics evident in classrooms in these three countries. They noted that in U.S. classrooms, students typically dealt with a much lower level of mathematics content than students in other countries. Observers also noted that our students had far fewer opportunities to develop new mathematical learning; instead, they were simply being told what to do. Worse, observers reported that on the few occasions when American teachers chose mathematically complex tasks, their teaching approach tended to remove the complexity and reduce the difficulty of the tasks.

It appears that in the interest of having students succeed, we sometimes spoon-feed our students too much information and ask too little of them in return. We tell them what approach or tools they can use to solve a problem, or we guide them in a directed fashion that makes one path obvious, thus removing the challenge. In essence, we tell them how to solve a problem before they have a chance to tackle it themselves. Somewhere along the way, we seem to have decided that students shouldn't struggle with mathematics.

The Need for Complexity

One of the most important lessons we can learn from other countries is that sometimes mathematics is hard, and sometimes we have to struggle to figure things out, especially with problems that are complex. When we introduce complexity in the problems we ask students to solve and challenge them beyond what they think they can do, we give them the opportunity to struggle a bit—an opportunity that many students never experience in mathematics from elementary school through high school. A look at those American classrooms where teachers and students invite complexity shows that the kind of mathematics problems students can really sink their teeth into (and consequently might struggle with) are often more interesting and engaging than the problems we have traditionally provided in math classrooms. It turns out that offering students a chance to struggle may go hand in hand with motivating them, if we do it right.

Seeley, Cathy L. "Constructive Struggling" from Faster Isn't Smarter: Messages About Math, Teaching, and Learning in the 21st Century:
A Resource for Teachers, Leaders, Policy Makers, and Families.

Notes:

Mathematical Tasks by Grade Strand
Chocolate
Inquiry-based format for Middle/Junior High

Jenny was mixing herself a glass of chocolate milk (Which by the way she enjoys a strong "chocalatey" taste.) when her friend Kevin remarked,

"You certainly have enough chocolate syrup in the glass." Kevin of course was looking for a glass of his own, so he could make some chocolate milk.

Jenny responded to Kevin, "I only have a third of my glass filled with chocolate syrup."

By this time Kevin had found a glass, that was twice the size of Jenny's, and he said, "Well I am only going to fill mine one-forth of the way with syrup."

"But Kevin, your glass holds twice as much!"

Kevin, knowing that Jenny really liked "chocolatey" tasting milk said, "Jenny, tell you what, lets combine our drinks into a larger pitcher, and then split the whole amount."

Your Task:

Determine the amount of chocolate syrup in the combined larger mixture. Will the new mixture be more "chocalatey" or less "chocalatey" tasting then Jenny's original glass of chocolate milk?

Be sure that you have proof to support your conclusion. You might consider using pictures, numbers or words to support your solution strategy.

To better understand the viewpoints of others, upon finding a solution and justifying it you should then find another way to represent and solve the problem.

Adapted From:
Newton, K.J. (2010) The Sweetest Chocolate Milk. *Mathematics Teaching in the Middle School, 16, 149-153.*

Created by:
Travis Lemon and Scott Hendrickson

Constructive Struggling

Some teachers and parents may be concerned that students will become frustrated or fall behind if they are given mathematics problems that seem too hard. I offer a new way to think about this by advocating *constructive struggling*, not pointless frustration. Constructive struggling can happen when a skillful teacher gives students engaging yet challenging problems. Constructive struggling can take place when a teacher decides that one demanding, possibly time-consuming problem will likely provide more learning value than several shorter but more obvious problems. Constructive struggling involves presenting students with problems that call for more than a superficial application of a rote procedure. Constructive struggling occurs when an effective teacher knows how to provide guiding questions in a way that stops short of telling students everything they need to know to solve a problem. Constructive struggling can build from the elementary grades through the rest of a student's education as teachers continually balance the types of problems they give students. An effective teacher provides problems that range from straightforward applications of recently learned mathematics to more complex problems that require critical thinking and the connection of more than one mathematical concept, skill, or idea. As students engage in the constructive struggling needed for some of these problems, they learn that perseverance, in-depth analysis, and critical thinking are valued in mathematics as much as quick recall, direct skill application, and instant intuition.

What Can We Do?

Of course we want students to succeed, and we don't want students to dislike math class. Perhaps the way to help them most, both in terms of success and attitude, lies in the counterintuitive notion of finding the right level of struggle or challenge—a level that is both constructive and instructive. The business community tells us that the ability and willingness to tackle a problem that is not easily solved is one of the most important traits of a well-educated adult in the twenty-first century. If we do our job well and make students think just a little harder, we can prepare them to take on some of the most difficult problems we face today as well as the unknown problems we are likely to face tomorrow.

Seeley, Cathy L. "Constructive Struggling" from Faster Isn't Smarter: Messages About Math, Teaching, and Learning in the 21st Century:
A Resource for Teachers, Leaders, Policy Makers, and Families.

Reflection and Discussion

FOR TEACHERS

• What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
• What teaching actions or strategies do you think support or inhibit students' willingness to accept the struggle that goes with solving a challenging problem?
• How long do you allow your students to wrestle with a complex problem before you offer increasingly guided assistance? How frequently do you provide such an opportunity?
• How can you determine the right amount of frustration and struggle for any given student on any given task?
• How can you help your students develop the confidence and persistence necessary to persevere through a challenge?

FOR FAMILIES

• What questions or issues does this message raise for you to discuss with your son or daughter, the teacher, or school leaders?
• How can you help your daughter or son understand that it's OK to struggle with a math problem sometimes?
• In what ways can you help your son or daughter with math homework without spoon-feeding all of the steps needed in order to solve a challenging problem?
• How can you help your daughter or son develop confidence and persistence in tackling hard mathematics problems? How can you support the teacher's efforts in developing student confidence and persistence?

FOR LEADERS AND POLICY MAKERS

• How does this message reinforce or challenge policies and decisions you have made or are considering?
• How can your mathematics program support students in learning the value of working through a hard or complex problem?
• What kinds of professional learning opportunities can you offer teachers to help them learn how to determine and incorporate appropriate levels of struggling in their mathematics teaching?

Mathematical Tasks by Grade Strand
Sam the Slippery Spider
Inquiry-based format for grades 3-6

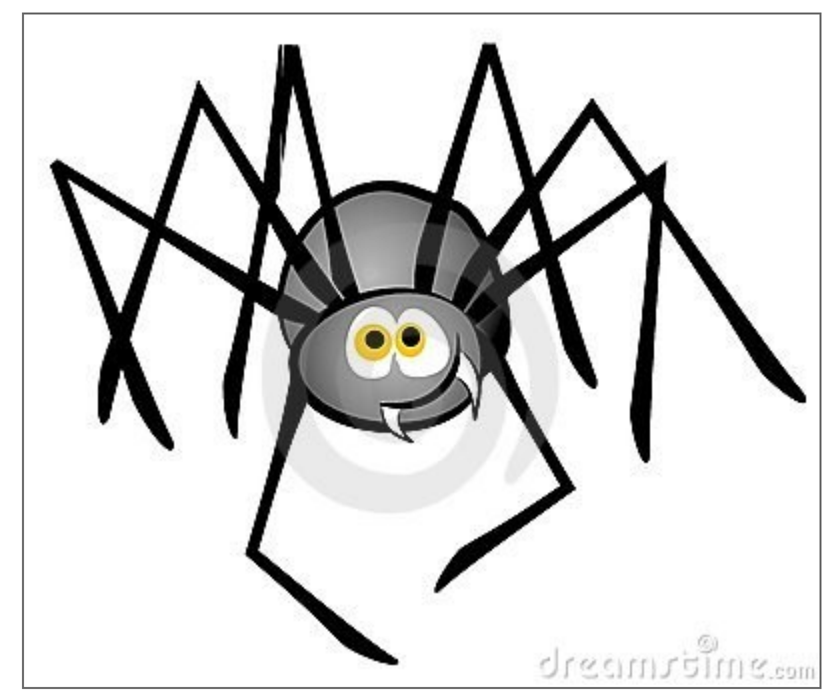
Sam the Slippery Spider

Sam is a garden spider. Everyday he climbs 10 centimeters up his dragline to make a web. Unfortunately, it has been very windy, and every night he gets pushed back down 4 centimeters.

Today Sam will start climbing back up his dragline from the ground level (he headed for the ground yesterday to escape a wasp). He wants to build an orb web 24 centimeters above the ground where he finds the best bugs.

How long will it take before Sam can start making his orb web? Write an equation to represent Sam's journey.

The teacher will need to see all of your work. Be sure to include the following:
- Pictures, charts, graphs, or t-tables that support your explanation
- A written explanation with detailed sentences
- The equation or number sentence
- The answer (Ask yourself: Is my answer reasonable? Why or why not?)
- The solution in more than one way or related to other situations.



APS Mathematics Standard: *Number Sense and Operations; APS/RDA;/CHF: Performance -Based Mathematics Assessment 2001-02.*

Mathematical Tasks by Grade Strand
Bunk Beds

Inquiry-based format for grades K-2

Lesson Objective:
Compose and decompose numbers and record each composition or decomposition by a drawing or writing an equation.

Launch:
Annie got a new bunk bed for her birthday. She was so excited; she asked her mom if she could invite some of her friends over to see it. Her mom told her she could invite 4 friends. Annie made the arrangements so her friends could come over Wednesday after school.

On Wednesday, Annie's friends couldn't wait to see her bed. Once they got to her house, they all rushed into her bedroom. Annie and her friends wanted to try out the bed so they all found a spot to sit. When Annie's mom came to check on them, she saw 3 girls on the top bunk and 2 girls on the bottom bunk. What are some other ways the girls could have been sitting on the bunk beds?

Take suggestions from the class and ask the child who shared how they know everyone would have a seat. Represent the number sentence suggested on the board.
For example, "There could be one person on the top and four on the bottom because I know that one plus four is five."
Teacher writes: 1 + 4 = 5
"There could be 5 on the bottom and none on the top. Five and zero makes five."
Teacher writes: 5 + 0 = 5

Explore:
Tell the children that they are going to pretend they are the ones who got the new bunk bed and were able to invite friends over to see it. Allow students to work with a partner. Assign each partnership a number of friends they could have between 5 and 9 (numbers may be adapted). *Remember students will need to add one more to the number of friends in order to include themselves.*

Have students record their work on a piece of paper. Walk around the room in order to see what strategies the children are using and answer any clarifying questions. If a pair thinks they have found all the ways to arrange their friends on the bed, ask "How do you know?"

For the Debrief,
Look for a pair who organized their number sentences in a way that makes it easy to see a pattern.
Look for a pair that recognized how the commutative property works in this situation. For example, 2 + 3 equals 3 + 2, but how they are arranged on the bed is different.

Debrief:
Selected students share their strategies. Ask questions such as:
Are there any other ways to arrange ___ friends on the bunk beds?
How do you know you found all the ways?"
How is __________'s strategy the same as ___________'s strategy? How is it different?
Did you try any other strategies before this one?
Which strategy is the most efficient?
Will these strategies work for bigger numbers?

Assessment:
Collect student work. Sort into three groups: those who've got it and are ready to move on, those who need a little more practice, and those that need extra help. How will you support each learner with tomorrow's lesson?

Cameron, A. , & Fosnot, C. (2007) *The Double-Decker Bus*. (pp. 13-19). Portsmouth, NH: Heinemann.

RELATED MESSAGES

• Message 16, "Hard Arithmetic Isn't Deep Mathematics," makes the argument that mathematics must include more than computation.

• Message 32, "Yes, but . . . ," examines some of the reasons we think students don't learn challenging mathematics.

• Message 2, "Untapped Potential," reminds us that many students can do much more challenging work than we currently expect of them.

• Message 31, "Do They Really Need It?," discusses the value of letting students tackle challenging mathematics, even if we aren't sure whether or when students will use it.

FURTHER READING

• *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* (Stigler and Hiebert 1999) considers differences between American mathematics teaching compared to mathematics teaching in other countries, including differences in how students may be encouraged to struggle with hard problems.

• *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics 1991) remains one of the richest descriptions of the nature of worthwhile tasks and classroom discourse that pushes students' thinking and develops mathematical understanding.

• NCTM's Illuminations website (http://illuminations.nctm.org) provides a rich online source of student activities to develop mathematical thinking and understanding.

• *Exemplars: Standards-Based Assessment and Instruction* (www.exemplars.com) provides a variety of tasks for several subject areas, including mathematics. The tasks cover a range of difficulty levels and are designed to challenge students' thinking.

• My website on Burkina Faso (http://csinburkinafaso.com) includes photos and stories about life and teaching during my Peace Corps assignment from 1999 through 2001.

Mission Impossible...Accomplished

Sandy Coxson
Spanish Fork Junior High, Nebo School District

"Good morning, Mrs. Smith," says the principal, "I have your teaching assignment for next year. We would like you to teach a new math class – a combination of four other math classes. You will have students coming from two different previous classes but will be expected to teach the new curriculum regardless of the insufficient background knowledge of some students. Oh, and by the way, there are no books, no scope and sequence for the course, nor are there any resource materials for either teachers or students." Does this sound like something out of a nightmare? Yes…but this is a reality for hundreds of teachers teaching the new common core classes this year.

Teachers, schools and districts have been scrambling to put together materials that not only satisfy the core skills but reflect the deeper intent of the writers of the new common core standards. However, at best, it is a very difficult task that has been given to us and many problems have surfaced. It isn't realistic to take assignments entirely from current textbooks because budgets can't cover the cost of giving each student two to four different books. Copyright laws prevent xeroxing pages from those texts. More importantly, many of these new lessons must draw from concepts taught in several different traditional classes. Existing materials are aligned to the old core and most need to be adapted to fit the new core. Creating new materials is a very time-consuming task especially when using just a word processor. These are very real problems that exist in classrooms, schools and districts across the state of Utah.

Nebo school district is no different than any other as we are in the process of creating materials for the Secondary I Math course. We have adapted existing materials, activities and resources as well as created new ones that will align with the new core. One of the resources that we have found that helps us more than any other is the Kuta test and assignment generators, called Infinite Pre-Algebra, Infinite Algebra 1, Infinite Geometry and Infinite Algebra 2 (all found at *Kutasoftware.com*). In literally a minute or two, even a technological novice can generate an entire worksheet or test, especially if a single concept is to be assessed. The inclusion of several concepts can take a bit longer but still requires only a few clicks of the mouse. In addition, with the ability to import questions from any of the four courses into any of the others, it makes the possibilities almost endless. Although not everything in the core is included in this software, we have found enough to greatly reduce the time it takes to create materials.

7. Look for and make use of structure.

Mathematically proficient students…

- discern a pattern to identify rules or properties.
- separate complicated ideas into their individual parts.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students…

- generalize from patterns noticed.
- look for more efficient strategies.
- identify general methods or a general formula.
- evaluate reasonableness while working the problem.

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students…
- demonstrate number sense.
- translate concrete and pictorial representations into symbols.
- provide real-life context for a number expression or equation (e.g. create a story problem for 3 x 5).
- recognize the meaning of the answer.

3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students…
- formulate an argument (e.g. How did I get my answer? Will this strategy always work?).
- communicate their strategy using pictures, numbers, or words.
- justify their conclusion (i.e. How do I know my answer is correct?).
- make conjectures based on what appears to be correct and has not yet been disproven.
- ask questions of others to clarify thinking.
- compare effectiveness of strategies.

4. **Model with mathematics.**
   Mathematically proficient students…
- recognize mathematics in every day life.
- make estimations to simplify the situation.
- use diagrams, graphs, and charts to identify key ideas and draw conclusions.

5. **Use appropriate tools strategically.**
   Mathematically proficient students…
- effectively use a variety of tools.
- identify which tools are appropriate in a given situation.
- use technology to deepen understanding.

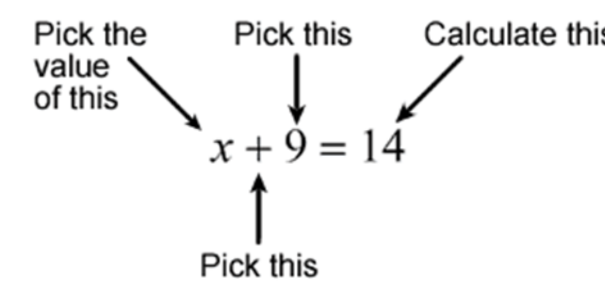
6. **Attend to precision.**
   Mathematically proficient students…
- communicate precisely using symbols or words.
- use vocabulary and commonly agreed upon definitions in discussions.
- calculate accurately and efficiently.

The following are just some of the features and capabilities of this software. For a complete list, you can visit their website.
- Choose a wide range of topics in each course
- Make multiple versions
- Vary spacing
- Create your own questions
- Edit existing questions
- Edit or create your own instructions
- Write multiple choice or short answer documents
- Convert to a PDF using an external program

The whole idea behind this software is that it does not utilize a test bank of questions. Instead, it generates the questions from randomized components. The box below is an excerpt from their website, under Frequently Asked Questions, explaining this:

What do you mean it can create an unlimited number of questions?
Our software generates the questions based on the options you have selected. It does not choose from a list of prewritten questions -- that would probably be slower, take up a lot more memory, and not result in nearly as many possible questions. Our software randomly chooses the variables and numbers in the question so that the question conforms to the options you picked.

Here's a simple example: You have selected one-step equations with numbers up to 10. For the first problem, the program would begin by picking the answer (21 possibilities: -10 to +10), the operation (4 possibilities: +, –, ×, ÷), and the other number in the operation (21 possibilities: -10 to +10). The program would then calculate the value on the other side of the equal sign and write the problem, formatting it nicely:

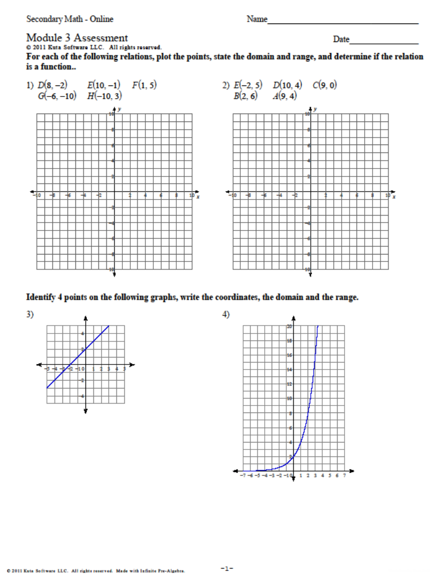


For this setup, there are 1764 theoretically possible questions. It's not infinite, but it's a lot. In reality, the number of questions it would make would be slightly lower because the program would weed out bad questions such as "0x = 10" or "x ÷ 1 = 4." Also, for this type of question, there are other options such as one for excluding problems like "x – (–4) = 10" so it may also exclude these.

You can see that our software can't create an *infinite* number of questions in the strict sense (that's impossible), but it can create all theoretically possible questions for the given options (minus the bad questions). In most cases, that's a huge number of questions; a number that means you don't have to worry about running out of questions or stumbling over the same ones.

Again, some examples of front pages of documents created for the Secondary I Math core. (Note: The program that enabled me to embed the images in this document would not allow me to capture the other pages.) *Document 1* uses PreAlgebra, Algebra 1 and Algebra 2 programs merged into one document and took about 10 minutes to generate. *Document 2* uses the Geometry program and took about 2 minutes to complete. *Document 3* was created in the Algebra 1 program and took about 4 minutes.

Document 1


Secondary Math - Online Name____________________

Module 3 Assessment Date__________

For each of the following relations, plot the points, state the domain and range, and determine if the relation is a function..

1) $D(8, -2)$ $E(10, -1)$ $F(1, 5)$ $G(-6, -10)$ $H(-10, 3)$

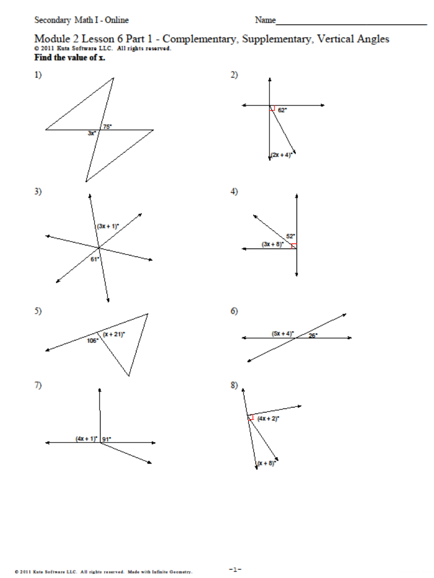
2) $E(-2, 5)$ $D(10, 4)$ $C(9, 0)$ $B(2, 6)$ $A(9, 4)$

Identify 4 points on the following graphs, write the coordinates, the domain and the range.

3)

4)

Document 2

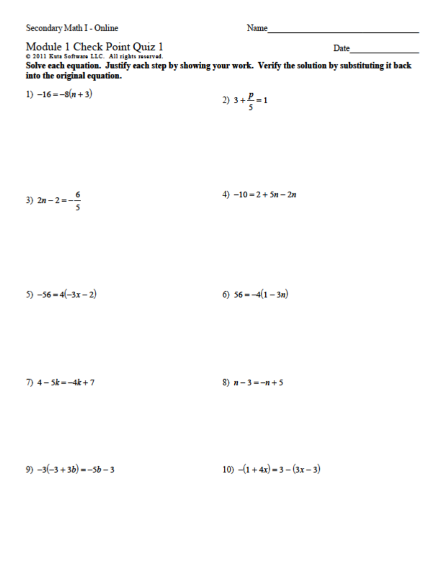

Secondary Math I - Online Name____________________

Module 2 Lesson 6 Part 1 - Complementary, Supplementary, Vertical Angles

Find the value of x.

1) 2) 3) 4) 5) 6) 7) 8)

Document 3


Secondary Math I - Online Name____________________

Module 1 Check Point Quiz 1 Date__________

Solve each equation. Justify each step by showing your work. Verify the solution by substituting it back into the original equation.

1) $-16 = -8(n + 3)$

2) $3 + \frac{p}{5} = 1$

3) $2n - 2 = -\frac{6}{5}$

4) $-10 = 2 + 5n - 2n$

5) $-56 = 4(-3x - 2)$

6) $56 = -4(1 - 3n)$

7) $4 - 5k = -4k + 7$

8) $n - 3 = -n + 5$

9) $-3(-3 + 3b) = -5b - 3$

10) $-(1 + 4x) = 3 - (3x - 3)$

Although Kuta software may not solve all of the problems arising from teaching the new common core standards, it at least makes our lives a little easier. We can use the program for what it was intended – creating worksheets, quizzes and tests. With the time this saves us, we can spend our efforts in creating other materials and activities that will deepen the understanding for students of the core concepts in ways that worksheets cannot.

Standards for Mathematical Practice

The Common Core State Standards for Mathematics (p. 6) state that:

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). "

A summary of the Mathematical Practice Standards from the Alpine School District Website, May 2011:

What Good Mathematicians Do

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students…

- understand the context of the problem.
- plan a solution strategy.
- use manipulatives or pictures to represent the problem.
- monitor their thinking as they solve the problem.
- are flexible in approaches to solving problems (when one doesn't work, try another).
- make connections to similar problems.
- determine reasonableness of an answer.
- understand the strategies of other students.
- make connections between strategies.

continued to talk about how the lesson was summarized. Several students had shared their work with the class and we both agreed it looked a lot like "show and tell"; a parade of students with no particular connection between what each person shared. My mentor's advice was to utilize the time students are working to look for distinct approaches students take, and then consider the sequence of how students will present each of their unique approaches. As a result, students in the class are exposed to many ideas that build on one another and serve to provide clarity and sense making. When classroom discussion, based on student's ideas, is orchestrated in such a way that connections and patterns are clear there is almost visible enlightenment of student understanding. These are thrilling class periods for me.

Life comes at you fast during the first year of teaching. I had worked 20 hours a week while taking 18 credit hours per semester in college but I have never been as busy as I was while handling the responsibilities of a teacher for the first time. I am grateful for lessons I learned about preparation and classroom discussion because they have made me more effective in helping students understand mathematics. I anticipate continuing to become aware of, and deal with instructional issues such as those I have learned about so far, and look forward to improving my ability to promote student understanding.

"There are two versions of math in the lives of many Americans: the strange and boring subject that they encountered in classrooms and an interesting set of ideas that is the math of the world, and is curiously different and surprisingly engaging.

Our task is to introduce this second version to today's students, get them excited about math, and prepare them for the future."

Jo Boaler (*What's Math Got to Do with It?*, Penguin 2008)

Secondary I is Not Algebra I

Diana Suddreth
STEM Coordinator, Teaching and Learning,
Secondary Education
Utah State Office of Education

It's the first day of school and Roberto is excited about his new mathematics class, Secondary I. Last year he took Algebra and was very successful in developing an abstract way of looking at and using mathematics. Roberto watches his teacher draw a number line on the board and label the numbers -1, -2, -3, -4. The teacher tells the class that these numbers are called "integers" and shows the students how to add integers using the number line. Roberto is bored. He did this last year. As the weeks progress Roberto's teacher completes a unit on numbers. Roberto is excited that finally he and his classmates will have an opportunity to do something new. The teacher writes on the board 5x+3=8, x=?. Roberto prepares himself for more boredom. He did this last year too.

A few miles down the road Angelique is nervous about her first day in Secondary I. She took Pre-algebra last year and did well enough that she was recommended for the Honors section of Secondary I. On the first day of school Angelique's teacher presents the following problem.

> Suppose you were the owner of a chain of eight fast-food establishments, each making a profit (or loss) on burgers, fries, shakes, and sodas. You are wondering which establishments are the most profitable overall and which products are most profitable individually. You are also looking for differences in profitability among the various establishments. How might you organize and analyze the information to begin to answer your questions?

Angelique's teacher then arranges the class into groups and gives each group data representing the sales from each establishment. The group task is to find a way to organize the information in a way that can be analyzed. After the groups share their strategies in a facilitated discussion, Angelique's teacher identifies one that lends itself to the development of a matrix as a way of organizing and analyzing information. Over the next week, the class uses matrix operations to review addition and subtraction of rational numbers in new contexts. Angelique loves math and is confident that she can succeed at the honors level.

In many ways, the content of the two classes is the same. Both teachers are helping their students review and improve their ability to operate with rational numbers. Roberto's teacher has chosen a traditional method, perhaps taken from the first chapter of the Algebra I book. Angelique's teacher recognized that computational fluency with and understanding of rational numbers is a fundamental skill but it is also a Grade 7 standard and therefore must not take valuable time away from continued learning in mathematics. The teacher chose to teach something new from the Honors Common Core curriculum, matrices, while also renewing some practical skills from previous grades in computation to spark student interest while attending to critical needs.

Roberto's teacher continues to misunderstand the content of Secondary I, by beginning his second unit introducing Grade 8 material as if it were something new. He fails to recognize that he can use student experiences with solving equations in Pre-Algebra and Algebra to "Create equations that describe numbers or relationships" or "represent and solve equations and inequalities graphically" as outlined in the Secondary I Core (CCSS, 2010). This change in focus would capture student interest through new learning and application. In addition, Roberto's teacher has missed his chance to ask students to "analyze and explain the process of solving an equation" by reducing the process to an algorithmic skill, most likely mastered in Pre-algebra or Algebra. Rather than capturing the essence of the Common Core and using that to interest students in the usefulness and beauty of mathematics, Roberto's teacher has reinforced the notion that mathematics is something you do to get a right answer.

So what are the differences?

The Standards for Mathematical Practice

The primary difference between the 2007 Mathematics Core and every course in the Mathematics Common Core is captured in the Standards for Mathematical Practice which "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students." (CCSS, 2010, p. 6) These statements of what mathematically proficient students do must be attended to in very explicit ways. Students can develop reasoning abilities and construct arguments with *careful lesson design*. There are times in Secondary I where the mathematical content may overlap that of the 2007 Algebra course. Teachers should capitalize on these overlaps to turn student attention to making mathematical arguments and logically explaining why certain algorithms work. They can also use these previously learned skills while working with more complex problem solving or modeling problems that use familiar skills to solve what appear to be unfamiliar problems.

> While all teachers agree that they want their students to "Make sense of problems and persevere in solving them" (CCSS, 2010, p. 6) it is more difficult to design lessons that explicitly develop these practices while at the same time building mathematical understanding and skill. Teachers must intentionally provide learning experiences that give opportunities to connect mathematics to new problems in ways that, while challenging, are also accessible for students of all abilities.

Learning Curve of a First Year Teacher

Kolby Gadd
American Fork Junior High, Alpine School District

As I entered my first year of teaching I was largely unaware of what an exciting time this is in math education. With efforts to increase student understanding underway and gaining momentum, it truly is fascinating to be involved. One manifestation of these efforts is the implementation of the Common Core State Standards for Mathematics. According to these standards students are expected to learn the Standards for Mathematical Practice as well as the Standards for Mathematical Content. The core states, "Expectations that begin with the word 'understand' are often especially good opportunities to connect the practices to the content." Surely these sets of standards must be taught concurrently and it is suggested that student understanding is the impetus for this type of learning. Further, it is apparent that "mathematical tasks of sufficient richness" are requisite to fully developing understanding and skills students will need for college and career readiness. Throughout my first year, learning to teach with such tasks and in a way that will increase student understanding motivated, and continues to motivate, my efforts to improve. Lessons learned include how to effectively prepare, and how to orchestrate classroom discussion.

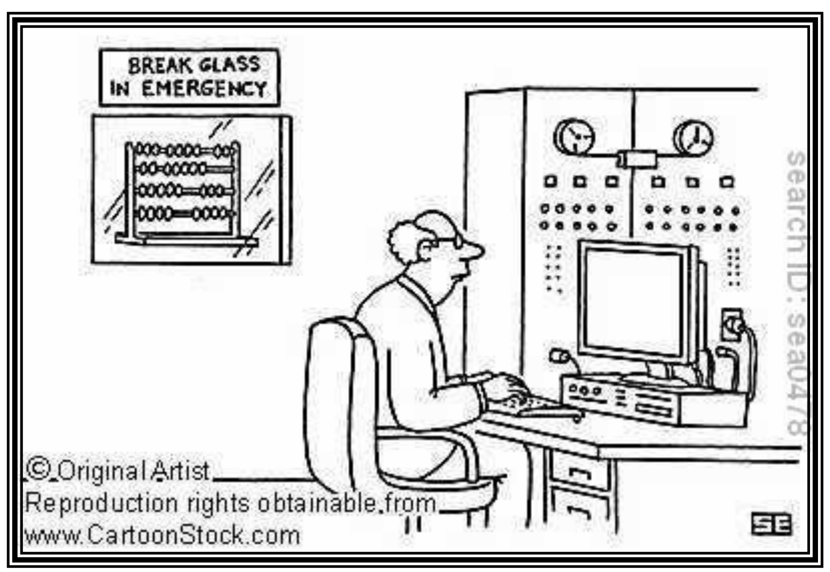
In college I was told preparation makes the difference in lessons. As I began teaching, my preparation centered on what I would do; I focused on things I would say, examples I would give, et cetera. I found a plethora of student thoughts and schemas overwhelming. I wanted to embrace student intuition and natural reasoning but struggled to sort through the diversity. As my first year progressed, my preparation tended to lean more towards what students would do. I thought about ways students would make sense of a problem or different methods they might use to find a solution. I considered questions to ask and suggestions to offer that could move students toward understanding. For me, the nexus of understanding is prior knowledge: students must do what makes sense to them if they are to develop understanding. As I have anticipated student thinking I have been better prepared to engage students in a process leading to understanding.

Becoming aware of student thinking has helped me build understanding for individual students. Learning to use student thinking as a base for classroom discussion has helped build understanding for the entire class. My mentor observed a lesson early in the year and afterward asked what I thought my role was as the students were working. I could not give an adequate response to his question. We

It is worth mentioning that parental support, while required at Maeser, is still something that we have to encourage and request. When family and friends ask me about what it is like to teach at Maeser, I refer to parental support as "the deal-breaker". I feel this one aspect is a hallmark of any successful educational program. Although not every parent at a typical school will be able to dedicate time for tutoring and/or grading, teachers would do well to identify as many possible ways to involve the parents, according to respective schedules.

These ideas, while unique to charter schools, are certainly not exclusive. While it may be easier to collaborate with colleagues and petition parental support in a small school, those teachers, departments, and schools who are committed to incorporating these helpful strategies can and will find a way.

To conclude, may I generalize the notion of collaboration while I address the implementation of the common core in Utah. I recently attended a SMECC meeting and was struck by the generosity and outreach from my fellow educators, all of whom came from other schools and districts. I have experienced collaboration on a small level, but they showed me that it is possible, even ideal, on a macro level. Because of their counsel and guidance I feel more confident moving forward with implementing the core in my own department. May I urge each of us, whether on a department or state-wide level, to continue to build up the math education community through the free exchange of ideas and resources.



"The hardest thing being with a mathematician is that they always have problems."
Tendai Chitewere

Teachers must pose problems that allow all students entry while at the same time allowing high ability students to explore mathematics beyond what is required for basic proficiency. In Secondary I this means the teacher might present some data that fall in a pattern and ask students to identify the pattern. Some students might describe a pattern by relating one term to the next, others might graph it, and others might find an equation. Teachers may ask students to speculate on the source of the data and justify their hypotheses. These low threshold/ high ceiling problems give all students an opportunity to enter a mathematical experience while simultaneously opening up opportunities for additional exploration for high ability students.

By attending to instruction that includes mathematics tasks with high cognitive demand such as those that engage students in doing mathematics (Stein, Smith, Henningsen, Silver, 2009)) teachers prepare their students for the depth of knowledge that will be required on accountability assessments at the same time that they equip them with disposition towards and expertise in the practices and proficiencies of the Standards for Mathematical Practice. Teachers typically concentrate mathematics instruction on the lowest levels of cognitive demand, memorization and procedures without connections. While it is no-doubt important that students memorize math facts and formulas and have the ability to reproduce certain algorithmic skills, Secondary I requires much more.

Teachers can help students understand concepts such as function, by presenting tasks that while procedural, make connections within mathematics or to other fields. For example, when students are learning about function, understanding is enhanced through multiple representations such as mappings, equations, graphs, and stories. Teachers can provide opportunities that reveal levels of understanding by posing tasks such as "Compare the function $f(x)=3x + 5$ to its graph and show how the 3 and the 5 are revealed in the graph". While identifying slopes and intercepts is very procedural, presenting tasks that make connections, such as those between equations and graphs, build deeper levels of understanding.

The Practice Standards are also supported through "Doing Mathematics" tasks. Constructing viable arguments and critiquing the reasoning of others (CCSS Practice 3) is required when students are asked to explain processes or relationships. In a typical Algebra I class students are often asked to solve linear equations, but in Common Core Standard A.REL.1 students must "explain each step…following from the equality of numbers" and "Construct a viable argument to justify a solution method" (CCSS, 2010). This can be done in many ways from holding a classroom discussion about knowing which step comes next to requiring algebraic proofs in a more formal way such as two-column format. Teachers may choose from a variety of ways to reinforce logical thinking through reasoning, but they must explicitly provide these experiences for students or those practices will not be developed.

Teachers can further develop the Practices by designing learning cycles that allow students to develop, solidify, and practice understanding (Hendrickson, Hilton, Bahr, 2008). By providing opportunities that allow students to explore and discuss mathematics, instead of expecting them to watch and absorb mathematics, teachers lead students to look for and make use of structure, express regularity in repeated reasoning, and critique the reasoning of others. Students become active and engaged learners through explicit attention to the Standards for Mathematical Practice.

Designing learning experiences that explicitly develop the Mathematical Standards for Practice is new work for teachers. Refocusing from solely developing skill to supporting the growth of understanding takes constant vigilance. The first step is to begin analyzing existing lessons using the lenses of cognitive levels and depth of knowledge. In time teachers will be able to select and design appropriate tasks and experiences for students that will rightfully develop deep understanding that supports procedural skill.

It's not too late for Roberto's teacher to make the kinds of changes that will result in Roberto's engagement with and success in Secondary I. The teacher can focus on the fourth Standard for Mathematical Practice, "model with mathematics", and the second Standard for Mathematical Practice "reason abstractly and quantitatively" by extending the students' work on linear equations to compare linear function with exponential function. Asking questions that compare rates of change in linear and exponential models that result from real world scenarios such as population growth, spark students' interest and expose them to new mathematics.

The Standards for Mathematical Content

Secondary I has some very specific content differences from Algebra I making it both more accessible to all students and more rigorous at the same time. (See Table 1) Because the content standards focus on building understanding, they provide opportunities for students of all abilities to engage with the mathematics in legitimate and deep ways. When students are first encouraged, and then required to provide justification for their processes and answers, they build meaning that is more difficult to forget than learning step-by-step procedures. This becomes the basis for learning all content in the Common Core.

At the outset Secondary I begins at a higher level than Algebra I. Skills such as solving simple linear equations and knowledge of rational numbers is foundational to Secondary I, but should not be the focus of new learning in the course. Teachers lose valuable time and student interest when they begin the year by reviewing previously studied topics without adding a new element such as problem solving or modeling. During the transition years, there may be topics with which students are less familiar and teachers will need to fill in knowledge gaps; however, through effective use of formative assessment, teachers will be aware when they might be reinforcing previously learned skills as opposed to developing new understanding.

More specifically, Secondary I has a significant focus on function, which was formerly studied in a more surface way in Algebra 2. Teachers must recognize that focusing on function goes far beyond being able to recite the definition or describe a graph as "function or not a function". A strong focus will build understanding of the origin of functions and explore common functions that model mathematical relationships in life, such as the exponential functions used in finance. Constructing and comparing linear and exponential models (Unit 2) is far more than learning to graph linear or exponential functions independently of one another in sequential lessons. Furthermore, these comparisons lead to rich classroom experiences that engage students due to clear real-world applications that also expose students to previously unfamiliar areas of mathematics.

Another clear difference in the Secondary I core is the focus on geometry, formerly relegated to its own course. The geometry of the Common Core is grounded in transformations in the plane, where students build an understanding of congruence through rigid motions and use these transformations to develop and use geometric constructions based on geometric properties. Where students were previously "given" definitions, postulates, and theorems, they now use inductive reasoning to "develop definitions" (Standard G.CO.4) that can be used to predict the effects of rigid motions and justify whether the results produce congruent figures. (Standard G.CO.6) While familiar geometric ideas such as ASA, SAS, and SSS are still essential, understanding how those criteria result from geometric realities is equally valued.

Lessons of a Charter School

Ted Gilbert
Karl G. Maeser Preparatory Academy

The vision and mission of Karl G. Maeser Preparatory Academy is likely similar to most high schools in Utah. We empower and inspire students to think critically, learn continually, and communicate effectively. Just as each individual teacher brings his or her natural strengths to the classroom, various school environments also have inherent benefits. This article addresses two areas of strength which promote student learning that are typically present in charter schools. These two categories, size and parental support, are strong factors in helping Maeser math students succeed.

First, charter schools are relatively small, with close-knit student bodies and teachers. The smallness allows students and teachers to work very effectively together. Specifically, teachers in cohesive departments are able to provide an extra support system for each other. This leads to increased collaboration on planning lessons and general curriculum implementation. For example, at Maeser, we have three separate teachers who each instruct at least one period of geometry. These three teachers collaborate together to create captivating lesson plans centered on mathematically important ideas. This pooling of resources not only helps the students receive better instruction, the teachers also have more time in which to mentor and guide students rather than herd them through classes. For example, we have a math lab each day after school where any student can come to receive free tutoring. Without a collaborative department, our teachers would not have the time to dedicate to after-school tutoring. Rather than the individual approach, we have chosen to work together and "tag-team" math lab and consistently be available for advice and brainstorming regarding our lesson plans. The partnership in our Maeser community extends to students as well. Our AP math students volunteer their time to come tutor their peers in the math lab each day. While not all schools have the benefit of a "small-town" community feel, it may be worth looking into how this can be achieved through other, or additional, measures.

The second built-in strength at Maeser is parental support. This year, our teachers filled out a survey regarding specific classroom/department needs and the parents of our students received the results. One effect of this survey has been two parents coming in weekly to help with tutoring and grading in my own classroom. This has freed up my time so I can spend more time mentoring students and working with them one on one. Another aspect of a strong parental showing is student accountability. When students know that their teachers have talked, and will continue to talk with their parents, they are much more likely to align their behavior in class and at home. Parent-teacher conferences have been golden opportunities to establish such relationships.

So, what is next? A team made up of last summer's facilitators met in October to begin designing the 2012 Common Core Academy for K-5 teachers in mathematics. We have made some decisions that will enhance their experience. For example, we will be sure to connect each task to the classroom by asking teachers to reflect on how they can use what they have learned in their own classrooms. We will also change the focus from individual standards to the grade level CCSSM critical areas, found in the introduction to that grade level. We are seeking teachers to help facilitate the CCA next summer. K-5 teachers who meet the qualifications are invited to apply. Contact David Smith at david.smith@schools.utah.gov for an application. You can then determine if you meet the qualifications and send in the application.

Ministry of Math Comics Education
Department for Learning
Exam Sheet for:

A-Levels

Time allowed: 1 Minute

1. Continue this sequence in a logical way:

M T W T __ __ __

2. Correct this formula with a single stroke:

5 + 5 + 5 = 550

3. Please write anything on the line:

____

4. Draw a rectangle with 3 lines:

What about honors?

One of the most difficult challenges for teachers is ensuring that honors courses do not become stagnant for students nor appear to be a repeat of previous courses. High ability students thrive on challenge and can quickly become bored, and possibly disruptive, if their needs are not met. Teachers must provide new learning experiences, deeper learning, and legitimate applications if they are to honor both the core and their students. Furthermore, parents must be assured that the Common Core is leading to the deeper understanding that will lead to success in higher levels of mathematics, such as calculus. This is a mathematical content issue as well as a public relations issue. If students are doing the same homework they did in previous courses or using the same books, parents have every right to be not only skeptical, but angry. It is the responsibility of the individual teachers to ensure that this does not happen at any level. No student should be studying previously learned material due to the transition to the Common Core. The depth of the regular Secondary I course combined with the additional topics for honors students provide sufficient materials to challenge high ability students, deepen their content knowledge, and prepare them for continued study.

Conclusion

The transition from teaching Pre-algebra and Algebra is as difficult for many teachers as the transition students are making from their 8th grade course to Secondary I. Nevertheless, teachers must fight against the tendency to use familiar lessons that may have been effective in meeting the demands of the 2007 Core when working with students to achieve the vision for student learning outlined in the Common Core. Where algorithmic skill was often the focus of previous mathematics courses, building understanding along with procedural skill is increasingly necessary for students to do mathematics in ways described in the Common Core. Where solving linear equations was once the focus, building an understanding of function and comparing linear and exponential equations now take center stage. Where teacher lesson-planning and preparation were the primary subjects in professional development opportunities, now student learning becomes paramount. Teachers may certainly capitalize on similarities between the 2007 Core and the Common Core, but they must be daily vigilant in providing rich and differentiated learning experiences for their students that lead to increased mathematical proficiencies at every level.

"I have not failed. I've just found 10,000 ways that won't work."

Thomas Alva Edison (1847-1931)

Provo City School District's Response to the Secondary Math I Course

Ron Twitchell
Math K-12 Curriculum Specialist; Provo City School District

Motivation for a new textbook

With the mandate to implement the new Common Core State Standards (CCSS) and no textbook that would meet the needs of the students in Provo City, we decided to take a path that may seem unusual to other districts, but one we had traveled before and create our own math book for Secondary Math I. Although many publishers with products that claimed to be aligned to the new CCSS approached us, we found the materials less than desirable for our needs. Even though there were concepts from to the new CCSS, the approach was less than welcoming especially for a block schedule. We had encountered this before with traditional algebra textbooks. Our experience was that the regular algebra textbooks were not developed for a block schedule and many of our at risk students were not able to keep up with a delivery approach that necessitated two sections per class period in order to cover the content of the algebra course.

Several years ago, in response to this deficit, the mathematics department at Provo High School wrote a textbook to be used in our algebra 1 courses for students that had been identified as students at risk. The book was written with our "at risk students" and the block schedule in mind. The book was designed to be more than just a textbook it was also a notebook. Students were able to write their notes in the book and work the examples provided rather than just look at examples that were completed without explanations.

A couple of years later, Provo City School District opened the Center for Advanced Studies (CAS), a school with grades 4 – 6 designed for students needing Gifted and Talented instructional opportunities with an emphasis in mathematics and science. Again, the needs of our students would not be met by purchasing a textbook so I rewrote the Provo High Algebra book with adaptations for those students in the 5th and 6th grades ready to take algebra 1. This book kept the notebook format found in the Provo High text. With these two experiences behind us, we felt that we could write a book that would better meet the needs of our students.

Fourth, participants designed their own mathematical tasks. These tasks are the beginning of a bank of mathematical tasks that will be available for K-6 educators in the state. They are available at http://cca6summer2011.wikispaces.com for now. In the future they will be available on the USOE website as well as hosted on uen.org.

A team of fourteen facilitators hired from across the state met for four days last spring to design the Common Core Academy. The facilitators were teachers, mathematics coaches, a district mathematics specialist, and one administrator with a solid background in mathematics teaching. They spent time understanding the power of rich mathematical tasks. They discussed adult learning theory. They bonded and had a great time together.

The facilitators then created and sequenced a series of mathematical tasks based on thirteen standards in the sixth grade CCSSM. These standards were judged by a committee of math coaches and district mathematics coordinators to be those that would be the most difficult for sixth grade teachers to understand and implement. Facilitators then created each of the four days of instruction.

Each day of instruction had a theme. The first day was devoted to the Standards for Mathematical Practice in the CCSSM. As participants worked through the tasks of the first day they looked for how those standards were implemented in each tasks. The second day focused on the cognitive demand of the tasks used that day. They learned about the importance of "constructive struggling" where students are encouraged to persevere in solving real world mathematical problems in order to really understand the mathematics involved. On the third day facilitators led teachers to understand the elements of a balanced mathematics classroom. They used a three-part lesson design template to help participants see how mathematical tasks lead to not only conceptual understanding, but also help students with the procedural understanding they need to solve problems. The theme of the fourth day was writing and sharing mathematical tasks. Participants were taught how to use the three-part lesson template to design a task. They then created their own tasks and shared them with each other.

Overall, participants had a very positive experience in the CCA. Though a little hesitant at first, as participants experienced task-based learning for themselves, most became excited about implementing rich mathematical tasks in their classrooms. They exhibited the Standards for Mathematical Practices in their own learning in the tasks, helping them see how those standards can work with children. The tasks they completed were designed for adult learners, treating teachers as professionals instead of pretending they were children.

Participant evaluations revealed that most participants learned a great deal from the CCA. They felt that their time was well spent. They were enthusiastic about the relationships they formed with other teachers from their regions and from other areas around the state. Some were even planning on-line means of staying in touch or planning reunions. They had fun with each other as they hiked, went to dinner together, and explored that region of the state.

Of course, the greatest outcome of the CCA is that we reached our goals; teachers understood the standards more fully and learned or remembered mathematical concepts that were fuzzy for them, and they learned another type of teaching they can use in the classroom. Perhaps even more important, however, is that teachers had fun with the mathematics. They learned that math does not have to be scary or even hard. When you learn mathematics in a real context, have to struggle with it a bit (or a lot), and can share solutions to a problem with each other, you come to understand it. Things that you understand no longer have any capacity to frighten or burden you. That is the power of the Common Core Academy.

2011 Common Core Academy

David Smith
STEM Coordinator, Teaching and Learning
Elementary Education
Utah State Office of Education

The 2011 Common Core Academy (CCA) in Sixth Grade Mathematics was held over seven weeks in June and July, 2011. Overall, about three hundred twenty-five teachers from all regions of the state participated in a very different experience than what occurred in previous Core Academy sessions. The participants did not receive ready-made lesson plans and materials for use in their classes. Instead, the course facilitators designed four days of instruction that had the potential to change the way participants view mathematical understanding and impact the way they teach mathematics in a very positive way. This academy was different by design from the ground up. How was it different?

First, the CCA was based on the Common Core State Standards in Mathematics (CCSSM) that were adopted by the Utah State Board of Education in June, 2010. These standards are different than those we have had in the past. Properly implemented, the standards have the promise of leading students to really understand mathematical concepts and to be able to use those concepts to solve real world problems.

Second, the CCA was designed to help teachers understand certain key standards. Facilitators created a learning-task based atmosphere that engaged participants as adult learners. As they allowed themselves to become immersed in the mathematics, teachers came to appreciate the concepts being taught and became excited about their implementation. Our goal was to assist teachers in truly understanding mathematical concepts they may not have understood or had perceived as inconceivable in the past.

Third, the CCA was designed to help teachers understand another way of teaching mathematics. They experienced the power of rich mathematical tasks in helping learners understand mathematical concepts. Participants experienced the integration of the CCSSM mathematical practices into everyday math lessons and saw how students can become mathematically proficient. They also began to see how mathematics instruction can be balanced between direct instruction and task-based instruction that allows learners to explore mathematical concepts.

The Process

The process was hasty by default. The secondary math teachers in our district and I waited for the curriculum guides from the state office of education to give us some more specific direction than can be found in the CCSS documents. As we waited for the curriculum guides, we did take some professional development time to become acquainted with the CCSS documents.

The task of reading and understanding the CCSS documents was difficult because of some ambiguous language and the fact that the content was dispersed throughout two different documents. When the curriculum guides were available the district provided two days of training in May of 2011 to begin the task. During these two days, we worked together to create a list of essential skills and a scope and sequence. These two documents along with the curriculum guides from the state office of education and our algebra book used at CAS became our foundational work for creating a textbook that would meet the needs of our students.

During the summer of 2011, math teachers from Provo High, Timpview High, and Independence High were given the task of becoming familiar with the content of the new CCSS Secondary Math I course. We sent one teacher from each high school and each middle school to the Common Core Academy training with the intent that insights and training would be shared with all other teachers.

We set the work dates of August 1 – 5 of 2011 as our week of intense training. Due to budget constraints, each teacher was allowed to select any three of the five days to come and work on the CCSS materials. The teachers were paid a stipend of $100 per day for the three days they worked during that week. The dedication of the teachers was demonstrated by the great participation we had from the teachers knowing that $100 was nowhere near their daily rate.

During this first week of August, teachers were given the CAS algebra book that had been aligned with the scope and sequences established in May. The teachers volunteered to work on the missing sections with the intent of getting the first half of the book completed by the end of the week in order to have it printed in time for distribution before the first day of classes on August 22. We have also set the deadline of Thanksgiving break for the completion of the second semester.

Many districts throughout the state have been given a copy of the first semester book with the understanding that this book is far from complete. In fact, we have started the revision process of the first semester book with the intent of cleaning up the content as well as the problem of too many voices in the current work. We were surprised to find that it is distracting to have so many different voices in the writing of the book. We are going to make revisions with a small committee which should help eliminate this problem.

Future Goals

We are hoping to have the second semester book complete by Thanksgiving in order to have sufficient time to do some quick editing and still have time to print the book before classes start for the second semester in January 2012. Teachers are keeping notes about the material that has been presented in the first semester making sure to communicate those items that were not as successful as desired and identifying what was used in place of those items.

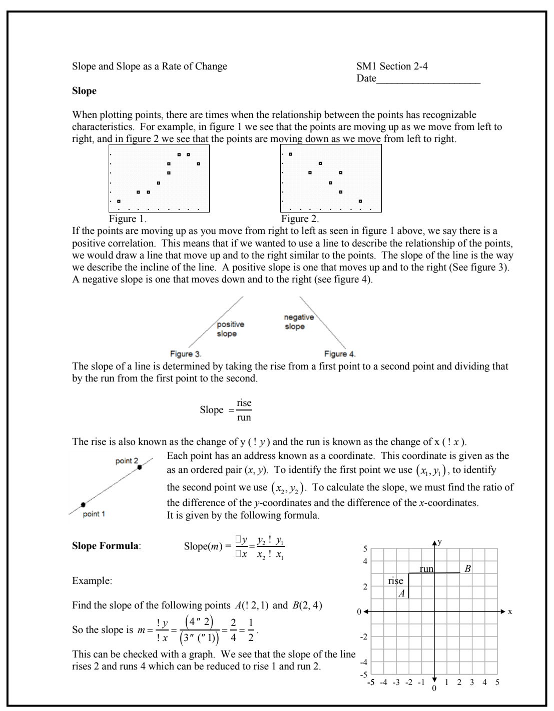
The teachers are also working on worksheets and tasks that will go along with the student book. These are kept on a wiki page that we set up for that purpose. We have found that a wiki page is beneficial because it allows teachers to not only download material, but to upload as well. They are also able to make notes to each other. The wiki page can be found at http://psdsecondarymath1.pbworks.com/w/page/40584586/FrontPage we are more than willing to allow others to access the materials that we have published to this sight, but we are restricting authorship to those teachers in our district.

The link at the bottom of the first page entitled PHS THS secondary math assignments will take you to the assignments page that has assignments for each section of the student text. Please keep in mind that these are still in draft form. We kid each other about the fact that we are living the video clip that shows mechanics building a plane while it is in flight. What we are doing is definitely "on the fly."

We are also working in preparation for creating the same kind of book for the Secondary Math II course. We feel that doing so will provide a resource that better fits the needs of our students who are taking the courses in the block schedule. It also makes each participant more aware of the content of the course. I know that the process of writing the first book has required each teacher to look at the course material in depth and especially with an eye set on sequence and concept development.

A final part of the collaborative process we are engaged in is to create common assessments. The teachers are using their Professional Learning Communities to create common assessments. The ability to meet together during the year also allows for communication between buildings rather than just within a specific department. We feel that these efforts are in the best interest of the students in our district.

Lesson content


Slope and Slope as a Rate of Change — SM1 Section 2-4 — Date____________

Slope

When plotting points, there are times when the relationship between the points has recognizable characteristics. For example, in figure 1 we see that the points are moving up as we move from left to right, and in figure 2 we see that the points are moving down as we move from left to right.

Figure 1. Figure 2.

If the points are moving up as you move from right to left as seen in figure 1 above, we say there is a positive correlation. This means that if we wanted to use a line to describe the relationship of the points, we would draw a line that move up and to the right similar to the points. The slope of the line is the way we describe the incline of the line. A positive slope is one that moves up and to the right (See figure 3). A negative slope is one that moves down and to the right (see figure 4).

Figure 3. Figure 4.

The slope of a line is determined by taking the rise from a first point to a second point and dividing that by the run from the first point to the second.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The rise is also known as the change of y (! y) and the run is known as the change of x (! x).

Each point has an address known as a coordinate. This coordinate is given as the as an ordered pair (x, y). To identify the first point we use (x_1, y_1), to identify the second point we use (x_2, y_2). To calculate the slope, we must find the ratio of the difference of the y-coordinates and the difference of the x-coordinates. It is given by the following formula.

Slope Formula: $\text{Slope}(m) = \frac{\Delta y}{\Delta x} = \frac{y_2 ! y_1}{x_2 ! x_1}$

Example:

Find the slope of the following points $A(!2,1)$ and $B(2,4)$

So the slope is $m = \frac{!y}{!x} = \frac{(4''2)}{(3''(''1))} = \frac{2}{4} = \frac{1}{2}$.

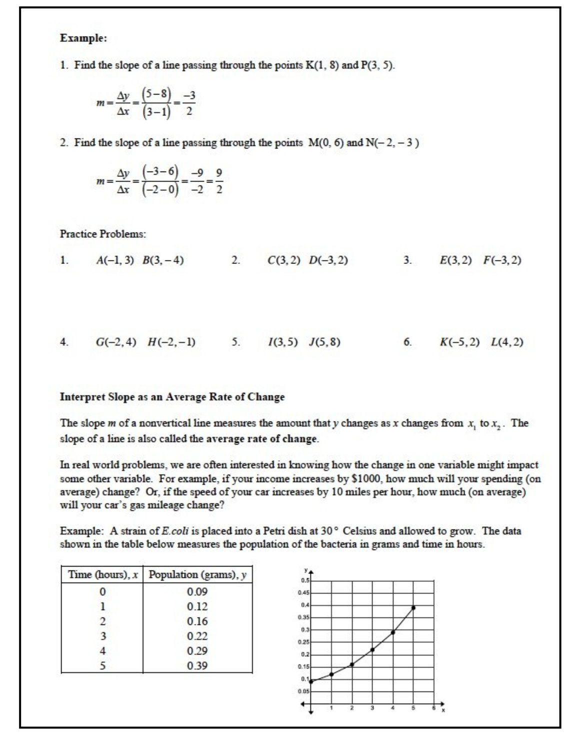
This can be checked with a graph. We see that the slope of the line rises 2 and runs 4 which can be reduced to rise 1 and run 2.

Fig. 1.

Each lesson has informational material that teachers refer to during instruction. You might notice that there is more wording that is normally found in a math textbook. Also, there is space provided for students to work the practice problems that are given during the instruction of the lesson. New vocabulary is in bold print and students are held responsible for that vocabulary. The teachers who worked on the project created all graphics. Please note that this is really a draft of what we will eventually have to work with so as we share the book with others, we ask that you not judge the instructional ability of our teachers by the text that is provided. An example of the textbook is provided in figure 1.

Notice in fig. 1 there is more information than you might typically see in a textbook. There is also plenty of open spaces for students to write in the book as the teacher gives instruction. Teachers used TI-Smart View ®, HandiGraph ®, and math type ® to create the graphs and the mathematical notation found in the text.

In fig. 2, you can see that there are practice problems in the middle of the page. These are not to be confused with the homework that will be assigned to this section. The homework is found on the wiki page. These practice problems are part of the instructional strategy and are done in class. At the bottom of fig. 2 you can see the concept development of the definition of slope to slope being a constant rate of change and then in fig. 3 this concept is moved to the exponential rate of change which is what the Secondary Math I course does. Once again, notice the areas provided for practice and note taking.


Example:

1. Find the slope of a line passing through the points K(1, 8) and P(3, 5).

$$m = \frac{\Delta y}{\Delta x} = \frac{(5-8)}{(3-1)} = \frac{-3}{2}$$

2. Find the slope of a line passing through the points M(0, 6) and N(−2, −3)

$$m = \frac{\Delta y}{\Delta x} = \frac{(-3-6)}{(-2-0)} = \frac{-9}{-2} = \frac{9}{2}$$

Practice Problems:

1. $A(-1,3)$ $B(3,-4)$ 2. $C(3,2)$ $D(-3,2)$ 3. $E(3,2)$ $F(-3,2)$

4. $G(-2,4)$ $H(-2,-1)$ 5. $I(3,5)$ $J(5,8)$ 6. $K(-5,2)$ $L(4,2)$

Interpret Slope as an Average Rate of Change

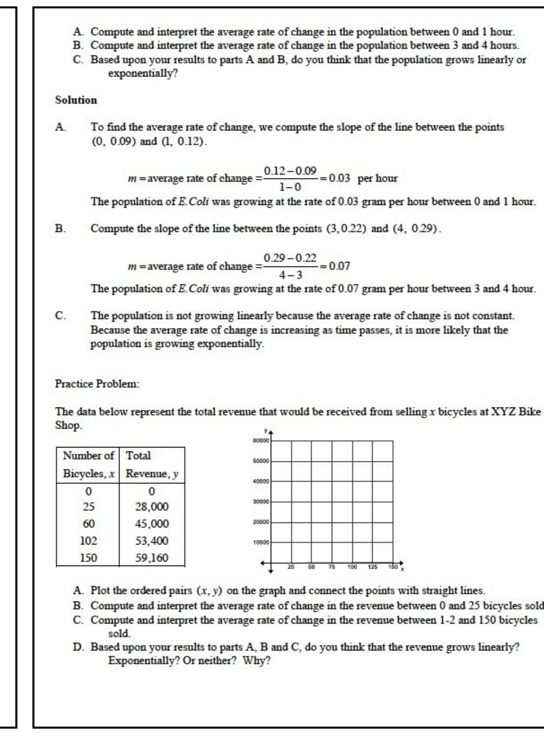
The slope m of a nonvertical line measures the amount that y changes as x changes from x_1 to x_2. The slope of a line is also called the **average rate of change**.

In real world problems, we are often interested in knowing how the change in one variable might impact some other variable. For example, if your income increases by $1000, how much will your spending (on average) change? Or, if the speed of your car increases by 10 miles per hour, how much (on average) will your car's gas mileage change?

Example: A strain of *E.coli* is placed into a Petri dish at 30° Celsius and allowed to grow. The data shown in the table below measures the population of the bacteria in grams and time in hours.

Time (hours), x	Population (grams), y
0	0.09
1	0.12
2	0.16
3	0.22
4	0.29
5	0.39

fig. 2


A. Compute and interpret the average rate of change in the population between 0 and 1 hour.
B. Compute and interpret the average rate of change in the population between 3 and 4 hours.
C. Based upon your results to parts A and B, do you think that the population grows linearly or exponentially?

Solution

A. To find the average rate of change, we compute the slope of the line between the points (0, 0.09) and (1, 0.12).

$$m = \text{average rate of change} = \frac{0.12-0.09}{1-0} = 0.03 \text{ per hour}$$

The population of *E.Coli* was growing at the rate of 0.03 gram per hour between 0 and 1 hour.

B. Compute the slope of the line between the points (3, 0.22) and (4, 0.29).

$$m = \text{average rate of change} = \frac{0.29-0.22}{4-3} = 0.07$$

The population of *E.Coli* was growing at the rate of 0.07 gram per hour between 3 and 4 hour.

C. The population is not growing linearly because the average rate of change is not constant. Because the average rate of change is increasing as time passes, it is more likely that the population is growing exponentially.

Practice Problem:

The data below represent the total revenue that would be received from selling x bicycles at XYZ Bike Shop.

Number of Bicycles, x	Total Revenue, y
0	0
25	28,000
60	45,000
102	53,400
150	59,160

A. Plot the ordered pairs (x, y) on the graph and connect the points with straight lines.
B. Compute and interpret the average rate of change in the revenue between 0 and 25 bicycles sold.
C. Compute and interpret the average rate of change in the revenue between 1-2 and 150 bicycles sold.
D. Based upon your results to parts A, B and C, do you think that the revenue grows linearly? Exponentially? Or neither? Why?

fig. 3

Conclusion

While it is not ideal to create a textbook during the school year that it is being implemented, it does provide some important benefits. The teachers who have worked on this project have a deeper understanding of the core content as well as a good understanding of the pace, order and concept development that should be addressed in the efforts to teach the core. It has unified the teachers between buildings of sister schools. Teachers from Independence High, Provo High, and Timpview High have opened channels of communication that had not previously existed. Teachers also have a sense of pride in the work. It will be nice to have more time to prepare for the Secondary Math II course but the greater benefit is the fact that we have walked this path already.