Utah Mathematics Teacher
Fall/Winter 2015-2016
Volume 8

http://utahctm.org

Call for Articles

The ***Utah Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Some of the features are: UCTM Leader Spotlight; Letter from the NCTM President; Letter from the UCTM President; Professional Development, Mathematics for English Language Learners; Puzzle Corner; Recommended Readings and Resources; Utah Core Standards and Implementation; College and University Research; and others.

Teachers are especially encouraged to submit articles including inspirational stories, exemplary lessons, beginning teacher ideas; or managements tools. Sample ideas are (but not limited to) focused on teachers or districts who have successfully implemented the Utah Core, and new math programs K-12. Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Christine Walker (Christine.Walker@uvu.edu). A cover letter containing author's name and salutation, address, affiliations, phone, e-mail address and the article's intended audience should be included. Items include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

Presidential Awardee 2013

For 13 years, Nathan Auck has had a transformative effect on his students' educational experiences. He is a secondary mathematics teacher and specialist at Horizonte Instruction and Training Center in the Salt Lake School District, an alternative school whose population is overwhelmingly low income, underprivileged and/or minority. Here, Nathan has led the school-wide implementation of student work-sharing protocols, behavior self-reporting, and enhanced cognitive rigor. He currently oversees instructional improvement, curriculum building, and program design for the middle school, secondary, and adult mathematics programs.

In addition to serving on the Utah Senate-appointed Standards Review Committee and the Board of the Utah Conference for Teachers of Mathematics, Nathan was selected by the Utah State Office of Education to write and facilitate professional development for mathematics teachers throughout Utah.

Formerly, Nathan taught 9th-12th grade mathematics and science at a private high school for gifted and talented students, the Realms of Inquiry School.

Throughout his career, Nathan's interdisciplinary collaborations with other teachers have provided students with applicative, real-world educational experiences.

Nathan holds a B.S. in environmental science from The Ohio State University and is a certified secondary mathematics teacher.

Presidential Award Finalists

Vicki Lyons absolutely loves teaching math to her amazing and wonderful students! This year she also enjoys being a mathematics instructional coach. Vicki has taught high school mathematics, including Algebra I through AP Calculus AB & BC and AP Statistics, at Lone Peak High School in Highland, Utah ever since the school opened in 1997. Prior to that she taught one year at American Fork High School and three years at Ricks Junior College, now Brigham Young University – Idaho. Vicki also enjoyed a 2-year assignment teaching at Brigham Young University as a Clinical Faculty Associate. She is National Board Certified in Adolescence and Young Adulthood Mathematics. Vicki has designed and facilitated several workshops for the Utah State Office of Education, worked on the Utah SAGE assessments, and presented at many conferences including instruction for the Common Core State Standards of Mathematics and recently at NCTM's Interactive Institutes. In the summer she has the delightful opportunity to work on staff for the Park City Mathematics Institute Teacher Leadership Program and as a Reader for the AP Statistics exam. Vicki has a MA in Mathematics Education with a minor in Statistics and a BS magna cum laude in mathematics from Brigham Young University. Currently she is a second year doctoral candidate in Mathematics Education and Leadership at Utah State University specializing in Curriculum and Instruction. In her extra time, she loves nature and being outdoors, and daily enjoys the many trails near her home. She is a mother and a grandmother and her happiest times are spent with her family that she adores.

Mike Spencer has loved mathematics since grade school. He has always loved the challenge and knew that teaching mathematics was a field that he could be passionate about from a young age. He has been teaching at Juab High School since 2008 and taught at American Leadership High School his first year of teaching. He is currently serving as the math department head at his high school and also serves as an unofficial secondary math specialist in the Juab District. He has taught from Algebra 1 and Secondary I through Calculus and AP Statistics. Mike has facilitated for state workshops and also is a facilitator for the Mathematics Vision Project. He has been a presenter at UCTM conferences and is a district technology instructor for Juab school district. Mike loves to learn and engage in mathematics. He looks for opportunities to improve his content and teaching knowledge every chance he can. He graduated from Utah State University with a composite Mathematics and Statistics teaching degree. He also received his Masters in Secondary Education with a concentration in Mathematics from Utah State University.

Karen Feld: When I was young I always knew that I wanted to be a teacher. I would watch the teachers I had in junior high and high school and think about what I would or wouldn't do when I became a teacher. The only struggle I had what that I didn't know what I wanted to teach. I began as a music education major at Utah Valley University, but quickly determined that I wouldn't be happy teaching music. I then decided to transfer up to Utah State University and become a mathematics teacher. That choice has forever changed my life. I graduated in 2005 and got a job at Pleasant Grove Junior High, where I currently teach 7th grade students. I was also an adjunct professor at UVU teaching developmental math courses. In 2010 I decided to earn a Master's Degree in Math Education, which I obtained in 2011 from Western Governor's University. As I look at the roads that have led me to where I am today, I can't imagine anything else that would make me happier or give me more satisfaction than teaching math. It is an occupation that has shaped me and my life in ways I can't describe. I am so blessed and honored to do what I do.

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Karl Jones—Nong ZHAO

As a kindergarten Chinese immersion teacher, there's no doubt I have more challenges than other normal kindergarten teachers to let my students understand mathematical concepts in Chinese introduction and communication, even just numeral sense. How do I ensure mathematical success for all students? For 4 years Chinese immersion teaching in Utah, USA and 10 years English language teaching in China, I found nothing is more important than 100% students' involvement. If your students get lost in your math class once, there would be more difficult time for them to catch up in next class. So how do I get all my students involved and be successful in learning math becomes the most important in my daily lesson plan. Before I plan the lesson, I will think about what the mathematics goals, what kind of methods I can use to let students understand those goals, what mathematics activities I can use to arouse students' learning interests, how I can connect all the concepts with students' daily life and what is my differentiation teaching for different level students? 1. I do set up my mathematics teaching and learning goals as clear as I can. Because when teacher doesn't have a clear teaching goal, students will definitely lost in your class. I establish clear goals for the mathematics that students are learning, and I can use the goals to guide all the instructional decisions. 2. I like to engage all my students into my mathematics activities. Because my students are 5 years children, I like to use a lot of math manipulatives to demonstrate and hand them out to each student. In that way, I make the students understand abstract math concepts starting from concrete ideas. For example, when I introduce numbers to my students, I know alphanumeric numbers will not make any sense to them. So I need to introduce the numbers from concrete concepts and give them all kinds of manipulatives to match, connect and count naturally. Here are the numeric cards I usually use in my class and students can use them to touch, match, build, count by themselves, in whole group and in small groups. 3. I need to ensure every students is involved into math learning, so I use a lot of math games into my math teaching. Kindergarten students love to play games and it will be easier for them to learn math through games. For example, when we learn shapes, I will let the students to make shapes with their bodies, so they are not only learning the names of the shape, but also use the concept into their daily life through this making shape game. 4. Without whole group learning, differentiation leaning is very important in math teaching and learning. I love to use math centers to reinforce, reteach math concepts and play kinds of manipulatives in different small groups. Centers teaching is the best way to let all students learn and success in math. Make sure your center activities are designed for Tier1, 2, 3 students. All in all, if you want all of your students to be succeed in math learning, you need to use different way, kinds of games, manipulatives and tons of energy to get all your students involved into your math teaching. This is the key to ensure mathematics success for all students.

George Shell—Colleen Pierce

One quality that I feel makes me successful with students is the fact that I struggled with several math concepts during my school years. I understand the feelings students can have when they fail to understand a concept quickly. I use that knowledge when I teach and present concepts using several different explanations and examples. I have found success is in the details of the concepts, I repeat many times the important parts. I first started teaching using an overhead projector and faced the students. I could see their faces and especially their eyes. Students will express their understanding with a head nod or expressive eyes. I knew if my lesson was going well if they would look me in the eye and participate in the discussion. Less eye contact meant I needed to reteach. I now use a mobi and I can walk around the room and see their work as well as their faces. The students love using the mobi and want to show their work as we discuss concepts. Their faces light up when they get to share. I have also found that having a pleasant relationship with your students opens their minds to learning. Laughter and comradery work wonders in the classroom. This isn't achieved quickly. The first month, I am a drill sergeant, creating good classroom procedures and habits. Afterwards, students feel safe sharing answers and participating in classroom discussions. They don't care if they are not 100% correct all the time, they still love to share and be part of the process of learning. I feel I have created an environment where it is safe to explore which breeds success.my 26 years of teaching, I have receive many thank you letters from former students. The theme is generally the same. They have fond memories of my classroom and still remember concepts I taught them. They appreciated the education I provided them. I try to do my best to teach math to my students and hope for success.

Don Clark—Diana Suddreth

I taught mathematics in Logan and St. George for twenty years and was awarded the Presidential Award for Excellence in Mathematics and Science teaching in 2000. During my years in St. George, I collaborated with other educators in building a system of support for mathematics teachers in an era when there was very little available in remote and rural Utah. I began communicating with math leaders in the state, such as Muffet Reeves, for whom one of the UCTM awards is named, and worked to bring quality professional develop experiences to Southern Utah, while also supporting teachers with programs I designed with others. Before PLCs and common assessments were common, we implemented them in Dixie. In 2006 I came to USOE as a Title I mathematics specialist and soon became the mathematics specialist for all of Utah. It was my dream job! As mathematics specialist I advocated to bring programs to rural Utah and worked with Utah educators to develop professional development opportunities that would deepen both student and teacher understanding and enjoyment of mathematics. In 2007 I led the development of a Utah Mathematics Core that was rated A by the Fordham Foundation and in 2010 I oversaw the implementation of the Integrated Mathematics Core that was adopted by the Utah Board of Education. The early common core years were challenging, but by promoting professional and resource development, we helped teachers make monumental shifts in their classrooms. Through my work at USOE Utah is at the forefront of the Open Educational Resources (OER) movement with Utah's Mathematics Vision Project and MAISSE projects gaining notoriety across the country and beyond. Over the years, I've been actively involved with UCTM, have presented at several UCTM conferences, and have worked with the elected boards to enhance mathematics education in Utah. I started my career with a clear focus on bringing more women into mathematics and over the years have broadened my interest to equity for all. I have tried to make my work reflect my values and my strong belief that all students can and should learn mathematics and that as educators we must break down the barriers that separate students into groups that have access to higher mathematics or are denied that access. The integrated mathematics pathway Utah adopted in 2010 is just one example of the kind of state level program I believe can bring high quality mathematics into all classrooms for all students.

Muffet Reeves—Sue Pope

Sue has worked with hundreds of teachers and their principals in many schools across several districts to deepen their mathematical understanding and to develop their instructional capacity. She has done this with an unwavering mission to promote students' deep mathematical understanding. As a 6th grade teacher Sue began providing math professional development to her school's entire faculty. She developed curriculum, supported teams of teachers in lesson study, and coached individual teachers in their efforts to improve. These efforts paid off as students all across the school developed a love for math that was grounded in confidence. Those students consistently performed well (extremely well) on end-of-year state tests.

Over the years, Sue has gone from an excellent classroom teacher who began providing math professional development "on the side" to an incredible district elementary math specialist who provides math professional development to teachers in her district and who trains teachers from multiple districts to provide math professional development to their schools. Throughout this journey, Sue has remained unwavering in her commitment to teaching mathematics for conceptual, procedural, and representational understanding. Her vision that all students can think mathematically if given the appropriate opportunities and support has inspired many teachers to learn a better way of teaching mathematics. A few years ago, Sue was providing school-wide math professional development at a Title I school that had a history of underperformance.

The principal at the school was committed to helping his teachers improve their instructional capacity. Also at the school was a special education unit for students with severe and profound disabilities. These special educators initially resisted the principles being taught at the math professional development, claiming that the information didn't apply to their students. With the full support of the principal, Sue patiently encouraged and worked with these teachers. During a debriefing session after a lesson study observation, Sue changed these teachers' professional lives forever by asking them a simple question: "How do you think the lesson would have changed if you first asked your students what they knew before 'teaching' them?" This question stopped the conversation in its tracks. These teachers realized they didn't know what might happen because not only had they never done it before, but they had never even *considered the possibility!* They agreed to try. A few weeks later when Sue returned to the school, the special education teachers were excited to talk to her. They reported that they had indeed started asking their students questions, and they were amazed at what their students already knew and understood. They saw their students' capabilities in a new way, and it transformed their teaching and their students' learning.

UCTM Presidents Message

Joleigh Honey, UCTM President

What a great time to be in mathematics education! Thank you for attending this year's Utah Council of Teachers of Mathematics (UCTM) "Principles to Actions" conference. The UCTM Board has worked hard to create a conference that is informative, engaging, and interactive. We are honored to have the lead author of NCTM's *Principles to Actions: Ensuring Success for All*, Steve Leinwand, as this year's keynote and we are also excited for our second annual Ignite! session.

This year's conference theme, as you may have noticed, comes directly from the National Council of Teachers of Mathematics (NCTM) *Principles to Actions* (2014). This publication has over thirty years of research and includes six Guiding Principles. NCTM President Diane Briars has said that *Principle's to Actions* is as important today as the *Standards* publication from 1989 or the 2000 *Principles and Standards for School Mathematics (PSSM).* Linda Gojak, NCTM's Past-President, explains that this publication goes beyond standards:

> Over the past twenty-five years, we have learned that standards alone—no matter their origins, authorship, or the process by which they are developed—will not realize the goal of high levels of mathematical understanding by all students. More is needed than standards. For that reason, NCTM has developed Principles to Actions: Ensuring Mathematical Success for All, the next in its line of landmark publications guiding mathematics education into the future. (pg vii, *Principles to Actions*).

What a great time to be in mathematics education in Utah! I am honored to be part of this great community. As mathematics educators in Utah, we are discussing what it means to implement high quality tasks- and that implementing quality tasks requires us to pose purposeful questions

Presidents Message Continued

that promote reasoning and problem solving. We know that discourse and listening to our students making sense of mathematics is essential to their understanding and learning. As professionals, we are working to improve our craft of using evidence of student thinking to assess progress toward understanding mathematical goals. When the Teaching Practices from *Principle's to Actions* were introduced, I heard many teachers make connections to the *Five Practices for Orchestrating Productive Discussions* (Smith, 2011). For several years, mathematics teachers in Utah have been working together as a community of learners to improve our understanding and implementation of this research. We have been working on balancing conceptual understanding and procedural fluency. We also have open dialogue about long held beliefs, some of which may be unproductive.

What a great time to have open discussions about our beliefs and how our beliefs impact access and equity. While the Teaching Practices provide explicit guidelines and examples of good instruction, perhaps our greatest discussions from *Principles to Actions* is the Essential Element of Access and Equity. I encourage every school and every district (Local Education Agency) to read this section and openly discuss beliefs about access and equity. According to the authors, the obstacles created that prohibit access and equity are "seldom, if ever, erected purposely to limit participation or achievement… Rather, they emerge in part from a set of beliefs" (pg 62). It is important to note that the authors stress that "these beliefs should not be viewed as good or bad, but rather as productive when they lead to change or unproductive when they limit student access to important mathematics content and practices" (pg. 62-63).

What a great time to make a difference! Thank you for attending this year's Utah Council of Teacher's of Mathematics (UCTM) "Principles to Actions" conference, and thank you also for your dedication toward making mathematics achievement accessible for each and every one of our students.

NCTM Featured Publication

In the call for manuscripts for the 2015 Focus Issue: Creating Classroom Communities, the Editorial Panel requested manuscripts that would support *Mathematics Teacher* readers in exploring the notion of classroom as community in mathematics. The call highlighted two themes.

Classroom communities embrace individuals and foster communication. But how do we, as teachers, make that happen? Six feature articles in this Focus Issue, and five more in the upcoming months, highlight ways to capitalize on the diversity in our schools. Equitable discourse, student participation, opportunities to listen, supportive environments, variety, and collaboration all play a role in establishing strategies and norms that we can adopt or adapt for our own classrooms.

Foundations of Geometry

Alexis Gagon, Student, Utah Valley University

You know those classes that you are reluctant to take? The ones that are "so hard" and "impossible." Well, my peers made Foundations of Geometry sound like *that class*. It was my junior year in college, and I had finally decided that I wanted to become a math teacher and Foundations of Geometry was one of the required courses.

In addition, it was my first upper division course that was proof-based. I had some experience with "proof" in high school, but on a limited basis. Such as prove the Pythagorean Theorem or the "sum of the interior angles of a triangle results in 180 degrees". However, in high school I was never required to formally prove any Theorem. Given those circumstances, the Foundations of Geometry was my first proofs course.

I was beyond overwhelmed. The content and proofs felt like a foreign language, and seemed impossible to learn. I had to spend hours and hours doing homework every night, yet my dilemma was that I never felt like I could fully grasp the concepts. I did not have much success on the first exam. Yet, I continued to persevere in the homework and continued to struggle. However slowly, I began to understand and grasp the concepts that originally seemed foreign to me just weeks before.

I did pass the course with a fairly decent grade. I recognized though that my struggle wasn't about the grade, it was about the struggle, and what I truly learned from the course. I completed the Foundations of Geometry course with a deeper knowledge about the origins of geometry, beyond what I had known before.

More importantly, I recognized the value of struggle, and that through struggle is the retention of mathematical knowledge. As a future teacher, my goal will be to ensure that constructive struggle is a key component in my classroom.

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Moving Ahead: Opportunities & Priorities

by NCTM President Diane J. Briars
Test Your Test Sense, February, 2015

As we move into the second half of the school year, many of us are feeling increasing pressure to engage in special "test prep" activities intended to help our students do well on end-of-year high-stakes tests. How knowledgeable are you about effective actions to prepare yourself and your students for upcoming high-stakes tests? Take this short test and find out!

1. It is valuable to analyze released assessment items from the PARCC and SBAC assessment consortia, in addition to your district and state standards and curriculum documents, even if your students will not be taking these assessments in the spring. True or False?

1. True.

Analyzing the standards and related curriculum documents, such as the Common Core State Standards for Mathematics (CCSSM) progression documents, is an essential foundation for understanding what students are expected to know and be able to do. However, examining tasks can further clarify those expectations in a number of ways, including illuminating the types of problems that students are expected to solve, the reasoning they should be able to demonstrate, and the quality of explanations they should provide. Tasks released by PARCC and SBACare particularly valuable to examine because these two consortia are developing assessments to measure all aspects of mathematical proficiency: conceptual understanding, procedural fluency, applications and problem solving, reasoning, and important habits of mind for using mathematics (that is, the Standards for Mathematical Practice).

For example, consider the following CCSSM standard for seventh graders in the domain Ratios and Proportional Relationships: "Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error"* (7.RP.A3). What content does this standard encompass? And what tasks would you expect students to solve to demonstrate proficiency as expected by this standard?

Consider the released PARCC task *TV Sales* (2013) below:

Grade 7: TV Sales

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

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Part A

A 32-inch television is regularly priced at $295.00. What is the total price of the television, including sales tax, if it was purchased on sale? Fill in the blank to complete the sentence. Round your answer to the nearest cent. The total cost of the television is $___________.

Part B

Adam and Brandi are customers discussing how the discount and tax will be calculated.

Here is Adam's process for finding the total cost for any item in the store.

- Take 10% off the original price.
- Then, add the sales tax to the discounted price.

Adam represents his process as:

$$T = \underbrace{0.9p}_{\text{sale price}} + \underbrace{0.05(0.9p)}_{\text{sales tax}}$$

Here is Brandi's process for finding the total cost for any item in the store.

- Determine the original price of the item, including sales tax.
- Then, take 10% off.

Brandi represents her process as:

$$T = \underbrace{1.05p}_{\text{T.V. price plus tax}} - \underbrace{0.10(1.05p)}_{\text{10\% off discount}}$$

In both equations, T represents the total cost of the television and p represents the regular price. Are they both correct? Use the properties of operations to justify your answer.

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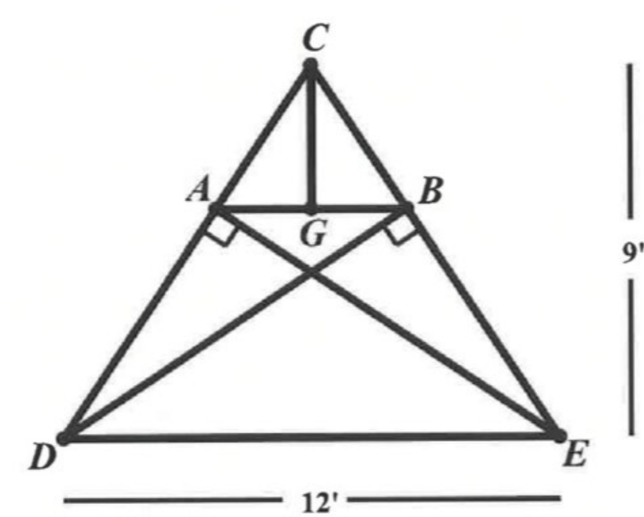
all the resources, and then later has students search and share their own resources as well. Finally, having students create their own resources to share with one other allows them to fully construct and own the material to have it become part of their personal knowledge. This doesn't work for all age groups as there is a certain level of responsibility that the learner accepts as part of the process. However, if you want to help your students discover how to learn rather than just what to learn it will help make them better lifelong learners.

References

Keller, F. S. (1968). "Good-bye, teacher..." *Journal of Applied Behavior Analysis*, *1*(1), 79–89. http://doi.org/10.1901/jaba.1968.1-79

Mathematical Question of the Day!

Mario is designing an A-frame for the lodge of a ski resort. Below is a scale drawing of his design.
Given: C lies over the center of the building
$\overline{AB} \parallel \overline{DE}$
$\angle DAE$ and $\angle EBD$ are right angles.



What are the lengths of all segments in the diagram?

First five correct submissions will win a prize. Please submit your solutions to Christine.walker@uvu.edu.

It is amazing to see how the little things can make the biggest difference."

I have seen the same thing in my instruction. Whether I create the video myself, or give them another resource or have them find it themselves, they are taking responsibility for their own development. But flipping the instruction by itself doesn't help tackle the problem with guiding students through a deeper conceptual understanding.

Personalized Instruction

The concepts of Personalized Instruction have been looked at for decades beginning with B.F. Skinner's teaching machine and later looked at by other Behaviorists such as Keller (1968) in his article titled "Goodbye teacher . . ." where the idea of self-paced instruction was analyzed. Even today the aim at finding self-paced software that can eliminate the need for a teacher is trying to gain traction in some circles. However, a self-paced course removes elements of student-to-student and student-to-teacher interactions which is vital to education (depending of course upon your personal learning theory belief). Even computerized adaptive systems making predetermined assumptions based upon "statistically common errors" can miscalculate where personal struggles might actually be in understanding. However, while this approach towards a student-centered curriculum, rather than teacher-centered, is a valuable effort it lacks some of the basic constructs of learning in a social constructivist theory in working collaboratively with others.

Personalized Flipped Instruction

The focus upon our education is on how we learn, and not only upon what we learn. The process of Personalized Flipped Instruction is that the instructor can give students a variety of resources or have the students research the topics and share the resources with one other. The process would be to analyze the information and help determine the value of the different resources and learn from the differences between the procedures which will help them make connections. Many times when I would share a video created by someone else with students they would tell me that the instructor in the video did the task differently than how I showed them. This is a great opportunity to discuss the differences and try and understand how those differences relate to each other. These discussions help enrich the overall conceptual knowledge by helping the students make big picture connections.

This process is best done through scaffolding. For the first topic the instructor provides

Would your interpretation of the standard have encompassed a task that calls for applying both a discount and tax to the same price, as in Part A of the PARCC task? Did it encompass evaluating and explaining two different methods for calculating price, as in Part B? And did your interpretation encompass representing the sale price and price including tax as a product, that is, as 0.9p and 1.05p respectively?

As the TV Sales item illustrates, tasks provide valuable information about how to interpret the standards, and about content and reasoning that students should have the opportunity to learn. They are also valuable examples that you might incorporate into your own assessments or instruction, even if your students will not be taking the PARCC or SBAC assessments or if you teach in a state not implementing CCSSM.

Of course, PARCC and SBAC released tasks are not the only sources of high-quality assessment tasks that illustrate expectations related to standards. NCTM has many publications that include such tasks, including the growing Putting Essential Understandings into Practice Series. Also NCTM will soon be releasing the *Discovering Lessons for the Common Core State Standards* e-book series, which explicitly links CCSSM to articles and resources in the NCTM practitioner journals. Other sources include the Balanced Assessment in Mathematics Project,Inside Mathematics, the Implementing the Mathematical Standards Project, theMathematics Assessment Project, the National Assessment of Educational Progress (NAEP) Questions Tool, and the Illustrative Mathematics Project.

2. It is important to cover all the content standards that will be on the test, even if doing so means having time for only superficial instruction of some topics. True or False?

2. False.

Ideally, teachers give students the opportunity to learn *with understanding* all the content on the end-of-year test. But sometimes, despite our best efforts, we can't make that happen. We fall behind in our intended pacing and have too little time to teach everything. In this situation, it is more productive to teach fewer of the remaining topics with understanding than to try to cover all of them superficially. The value of depth over breadth is illustrated by the Trends in International Mathematics and Science Study (TIMSS) study (Ginsburg and Leinwand 2005). The Singapore curriculum contains a smaller percentage of the TIMSS test topics than does the typical U.S. curriculum; yet Singapore outperforms the United States. Why? A partial answer is that Singapore teachers teach the topics that are in their curriculum in more depth.

Monitoring your pacing and adjusting it periodically are also helpful, so that students have adequate time to learn the most important content before the test.

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3. Which of the following practices are likely to improve your students' test performance? (Select all that apply.)

a. Stop teaching to review and give practice tests during the weeks prior to the test.
b. Provide opportunities for ongoing review and distributed practice within effective instruction.
c. Provide opportunities for students to develop clear expectations about the test—for example, becoming familiar with question types, characteristics of high-quality responses, and technology that the tests will use.
d. Teach students how to become effective self-assessors who take an active role in monitoring their own learning.

3. (b), (c), and (d) are likely to improve your students' test performance; (a) is not.

Although (a)—stop teaching new content and spend the weeks before the test reviewing and giving practice tests—may appear to be an effective way to increase test scores, research indicates just the opposite—test scores are actually lower in schools where teachers spend large amounts of time on this type of test preparation (see, for example, *From High School to the Future: ACT Preparation—Too Much, Too Late*).

Instead, providing opportunities for ongoing review and distributed practice, along with feedback, as suggested in (b), helps students solidify their knowledge and promotes retention, reflection, generalization, and transfer of knowledge and skill (Rohrer 2009). Review problems can be incorporated into daily warm-ups, nightly homework, quizzes and tests, or classroom questions. Regardless of how you build in the practice, be sure to provide feedback, both to ensure that students are practicing correct content and to help them take responsibility for their own learning. The sooner you start providing distributed practice, the more your students will benefit.

Test scores are also higher when students know what to expect—the types of questions that will be on the test and how their responses will be evaluated, as described in (c). For example, White and Frederiksen (1998) found that teaching seventh-grade science students the characteristics of high-quality work, and showing them how their work was likely to be evaluated, reduced the achievement gap between the highest- and lowest-achieving students by half. In addition, on average, the weakest students in the "clear expectations" classes were outperforming all but the very strongest students in the control classes who had not had such instruction.

Teaching students to take ownership of their learning and assess the quality of their work, as outlined in (d), is another effective strategy for increasing test performance. Student ownership needs to be an explicit expectation, supported by regular opportunities for students to analyze the quality of their work and reflect on their progress toward stated learning goals. Clear expectations and ownership of learning are two of Dylan Wiliam's key strategies for effective formative assessment (Benefits of Formative Assessment, NCTM, 2007).

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students can find. In fact, I challenge you to find one procedural task that we ask our K-12 students to perform that could not be found anywhere online. Therefore, if I was a student today, why would I wish to spend my time sitting in a classroom watching an instructor perform the same procedure on a whiteboard? Or, is there more offered to me from the classroom instruction and experience? But probably an even more important question is if we have all this free procedural tutorials and information available online through YouTube, Khan Academy, MyOpenMath, MOOCs (Massively Open Online Coures), etc., what benefits do instructors and classrooms offer? The key to successful mathematical instruction comes from something outside of content knowledge: personalized human interpretation. I find that by having students find and share resources there is so much more power to then constructing the knowledge rather than being passive learners of the material.

Have you ever finished a task and never quite understood what you just did? Or how about when you tried to perform a task you didn't have personal confidence you could do? For example, what if I asked you to cook a Thai meal. Have you cooked before? What is your comfort level in the kitchen? What is Asian Fish Sauce and where can you even find it? We want the transfer of knowledge to come quickly, even if you have cooked before, just not Thai. But for someone who focuses completely upon a procedure, changing an ingredient is similar to changing an entire process of cooking.

Flipped Instruction

There are many interpretations of flipped instruction with the traditional definition of "flip" being that the lecture and homework from the traditional classroom. However, more than just flipping the lecture and homework comes the justification that you are using classroom time in a more valuable way for what does matter and that more informative information can be done elsewhere. The other day I was discussing this topic with a colleague and she mentioned about an experience she recently had with her online instruction:

> "I had learned a new trick on how to multiply the higher times tables (12-15) and so I made a video that I posted for my class (one student had mentioned to me that he was hoping to learn some new tricks). I didn't realize how much my video would help out my students, but I had 2 students comment on it in the Lesson Questions Discussion Board where I posted the video. The student I e-mailed it to sent a thank-you back as well.

Value of Personalized Flipped Instruction

Sam Gedeborg, MET, Instructional Designer, Utah Valley University

First off, there is no silver bullet, or one-size fits every task in education. The other day I was doing some binge watching of Dr. Quinn: Medicine Woman and the topic of education came up. One actress, in discussing the topic with her friends, said, "I thought the purpose of an education was to teach you how to learn, not what to learn." While procedural knowledge and skills are important and valuable, there has been a focus and an increase in trying to help our students understand the conceptual knowledge, as evident by the Common Core Math Standards. While there are many different ideas and theories when it comes to learning, sometimes mixing a few ideas can create something even better, as in the case of Personalized Flipped Instruction.

Future Knowledge

We can see the demand and need from society to have more of a critical mindset of knowledge and an overall conceptual understanding from our students. It used to be that you would pull a book off the shelf when you wanted to perform a DIY project which would teach you the procedure (or algorithm if you will). However, with the increase of the Internet and YouTube we can find many different videos on any given procedural topic, and just-in-time (JIT) for that matter. In fact, I remember the other day needing to change out my alternator in my car and spending about half an hour looking over a variety of videos to understand how to best perform the task. John Dewey once said, "If we teach today's students as we taught yesterday's, we rob them of tomorrow." We are seeing a glimpse of how students will access knowledge in the future, and the technology that gives them this information at their fingertips.

Granted, there are certain tasks that require quick recall – I don't want my doctor needing to look up a certain surgery on YouTube right before she operates on me. This knowledge should be quickly accessible to her and preferably it will be something she has performed dozens of times. Yet, there are some tasks which technology has made our lives easier and more accurate.

Mathematical Knowledge

There are many videos on the internet made on how to solve math problems that

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4. Good instruction is the best test preparation strategy. True or False?

4. True.

Consistently providing your students with high-quality instruction and assessment that incorporates the effective mathematical teaching practices identified in *Principles to Actions: Ensuring Mathematical Success for All* is the most effective way to prepare your students to do well on high-stakes assessments. Such instruction takes students beyond rote learning of facts and procedures to learning with understanding, which enables them to apply and transfer their knowledge and skills to new problems, as well as apply them to familiar situations—exactly what is needed to perform well on high-stakes tests.

Developing your test sense in the ways suggested by this short test can have tangible benefits for you and your students alike. Implementing these ideas can lower everyone's stress and increase confidence and test performance—and, most important, improve teaching and learning every day in the classroom.

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Using Writing in the Mathematics Classroom

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Numerous studies have shown that incorporating writing into the learning process has a significant benefit in deepening understanding (Biancarosa & Snow, 2004; Graham & Perin, 2007; Baxter, Woodward & Olson, 2005). The majority of these studies have used written composition and described its effects on improving reading comprehension. Although there has been far less research conducted on the connection between writing and conceptual understanding in mathematics, it is likely that incorporating writing into mathematics instruction would have similar benefits by deepening student conceptual understanding of mathematical concepts. When students demonstrate conceptual understanding they are more able to use this knowledge to solve problems, use it flexibly, and avoid common misconceptions.

Additionally, it is clear that the Standards for Mathematical Practice call for students to engage in reading, writing, and speaking about mathematics. Mathematics instruction that aligns with the Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010) is demanding that students be able to communicate their thinking and ideas while building conceptual understanding of the concepts and ideas being learned. Students who articulate and justify their mathematical thinking and reason through their own and their peers' explanations will develop deep understanding that is essential to continued success in mathematics (National Council for Teachers of Mathematics, 2014).

This article describes two instructional strategies that can be employed to engage students in speaking and writing about mathematics as an avenue to deepening their understanding of various mathematics concepts. Although this particular lesson focuses on multi-digit subtraction with regrouping, the strategies presented can easily be modified to coordinate with any mathematics concept.

It is a difficult balance to maintain. Students must encounter new ideas in such a way as to make new connections in their brains that lead to lasting learning. The Comprehensive Mathematics Instruction (CMI) framework recommends that students use tasks to develop, solidify, and practice understanding (Hendrickson 2012). The teacher consistently launches tasks, discusses findings, and debriefs to package understanding for long-term memory. In practice, tasks are an effective way to deepen understanding. Tasking can be ineffective if used as an activity with little learning value. A task should always tie into the desired learning outcome. Also, taking a long time on a simple concept to deepen the understanding can pay dividends later. Sometimes you must make easy things hard to make hard things easy.

Direct instruction still has a place. Are we using direct instruction to help students understand or are we spoon-feeding? It can be good or bad.

Conclusion

Well-designed instruction and learning tasks combined with well-designed assessments are like safety inspections for learning and understanding. They allow us to find and fix issues before serious problems arise and place our students in a precarious situation with the potential of an irreversible tragedy. Honest grading practices communicate vital information to all stakeholders concerning students' progress within and mastery of the State Core. This allows for more effective decisions at all levels of the education system regarding educational components such as planning, instruction, intervention, remediation, policies, mandates, and teacher training. We believe that sound assessment design and grading for learning is fundamental for improving student achievement.

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Using the butterfly method, students can perform DOK 1 tasks of only one variety:

Find the sum: $\frac{3}{5} + \frac{1}{7}$

Students have no understanding of the required math concept, which is to replace fractions with equivalent fractions; therefore, they cannot reason through higher-level questions. Consider the following higher-level query:

Circle the numbers in the list that could be used as a common denominator to find the sum $\frac{3}{2} + \frac{4}{5}$.

7 10 12 15 20 25 30

The first question gives no information about student mastery of the math standard, because students can solve it with a "trick." The second question reveals specific information about student understanding.

The butterfly effect is often mentioned in dynamic systems as an illustration that little things can have big effects, such as a butterfly flapping it's wings in one part of the world having an effect on the weather patterns in another part of the world. The reference made here is far less arbitrary as we refer to the effect on students' learning when understanding is ignored in favor of a trick that is quickly learned and just as quickly forgotten. Many students learn to hate fractions because they only learned how and never saw the 'why'. Such students see a set of conflicting rules of arithmetic reminiscent of spelling rules in English that seem to change at a whim.

Teachers often come up with cutesy little ways of doing things that can be quickly learned and just as quickly forgotten—making mathematics a collection of things forgotten and rarely understood except for the few privileged students who learn anyway. It is important to assess DOK 2 and 3, but it is of greater importance that the students are taught at DOK 2, 3, and 4 as well.

We, as teachers, often scaffold our way down and out of DOK 3 and 4 tasks by giving the students too much help because we hate to see them struggle. The struggle is where the learning takes place. The one talking is the one learning.

Teaching Practices that Address Depth of Knowledge

Students need a healthy amount of frustration and confusion to open up their minds and learn.

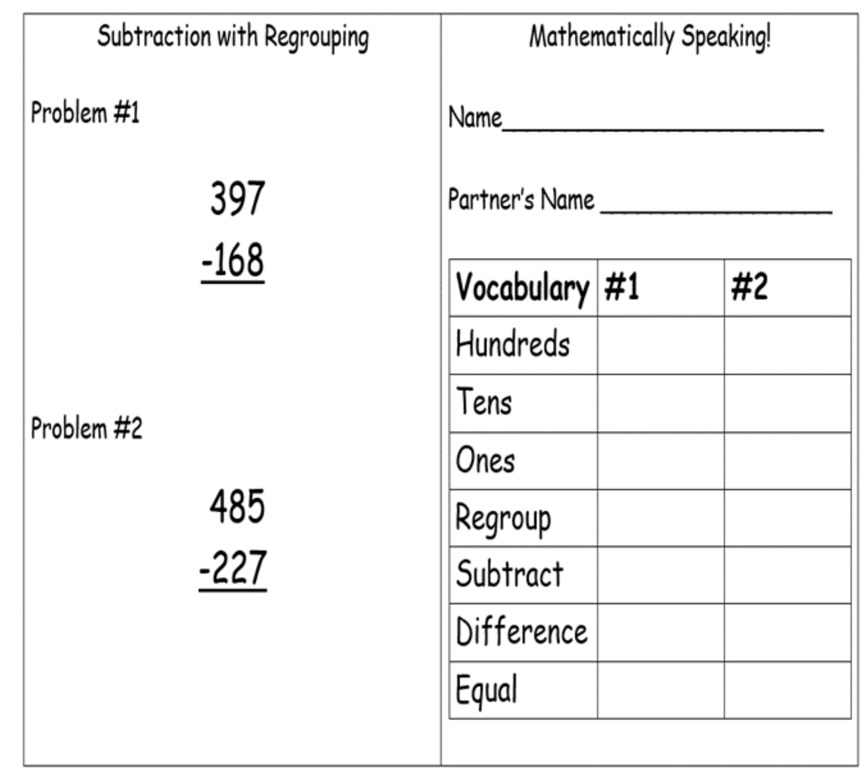
Background Information

A math lesson integrated with writing math lesson was presented during the second week of third grade as a review of students' understanding of multi-digit subtraction that required regrouping. We were interested in finding out which students understood the concept and which students would need additional instruction. We used two instructional strategies to investigate students' conceptual understanding: 1) Mathematically Speaking (Santa Cruz, 2009) and 2) a written response frame.

Strategy 1: Mathematically Speaking

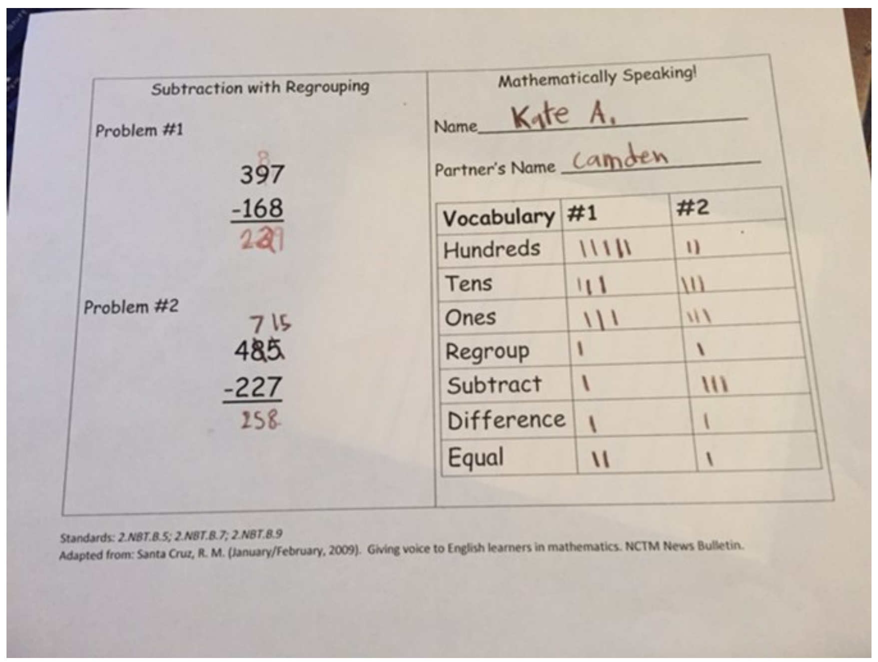
The lesson opened with the teacher introducing the Mathematically Speaking template (see Figure A). First, the teacher modeled a multi-digit subtraction problem that required regrouping, similar to those found on the template. As the teacher explained and solved the problem the students were asked to keep track of which vocabulary terms the teacher incorporated into her verbal explanation. Upon finishing the modeled problem, the teacher then explained to the

Figure A: Mathematically Speaking Template



Standards: *2.NBT.B.5; 2.NBT.B.7; 2.NBT.B.9*
Adapted from: Santa Cruz, R. M. (January/February, 2009). Giving voice to English learners in mathematics. NCTM News Bulletin.

students that they would be working in partners to solve similar multi-digit subtraction problems. Partner 1 would solve problem #1 and partner 2 would solve problem #2. The students were asked to solve the problem and explain the process they used to do so. As part of their explanation, their partner would be tally marking which of the key vocabulary terms the students used during their verbal explanation. The partners were to encourage each other to use all of the vocabulary at least once during their explanation. Through requiring students to verbally explain their reasoning, students were able to “solidify and strengthen their understandings of mathematical processes and concepts because in the process of verbally explaining something to others, students often clarify for themselves what they mean” (Fogelberg et al., 2008, p. 57).



Strategy 2: Written Response Frame

After students completed explaining their thinking to their partner, students were then asked to write how they solved the problem. The teacher modeled how to complete the written response frame using the problem used at the beginning of class. Students were given two options for their written response: 1) open-ended response in which they were given a blank piece of paper to provide their response, or 2) the provided written response frame (see Figure B).

high levels and develop depth of knowledge in the content.

Troubling Teaching Practices

There are some troubling teaching practices that teachers often use. As an example, we have, many times in our careers as mathematics teachers, heard students refer to the butterfly thing when adding fractions or wanting to cross-multiply when adding fractions. For a very long time, we did not understand what these students meant. Phil Daro, the keynote speaker for the 2014 Utah Council of Teachers of Mathematics Conference, first introduced us to this concept, called the butterfly method. As shown in *Figure 6*, you are to multiply in an x pattern (the butterflies wings) to get the 4 and 9; you then add across the top and multiply across the bottom, and through what must seem to students an arithmetic sleight-of-hand, the answer magically pops out. The gaps in student understanding arising from the butterfly method will not be found with drill-type recall questions. The same teachers teaching low level processes and other similar methods often employ low level questions on assessments. This offers the teacher a pat-on-the-back feeling for the fleeting successes of a student being able to do it for the test. Very few students will gain a deeper understanding of adding fractions through the Butterfly Method, as deeper understanding is not planned into the teaching and learning process (just nice when it happens). Also, retention of the concept tends to not last since the mathematical meaning of adding fractions is missing.

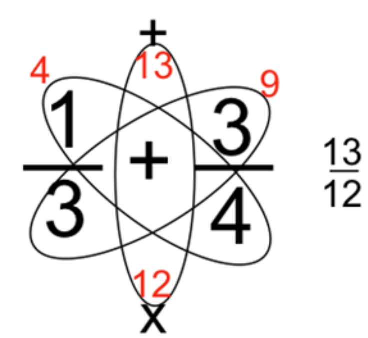
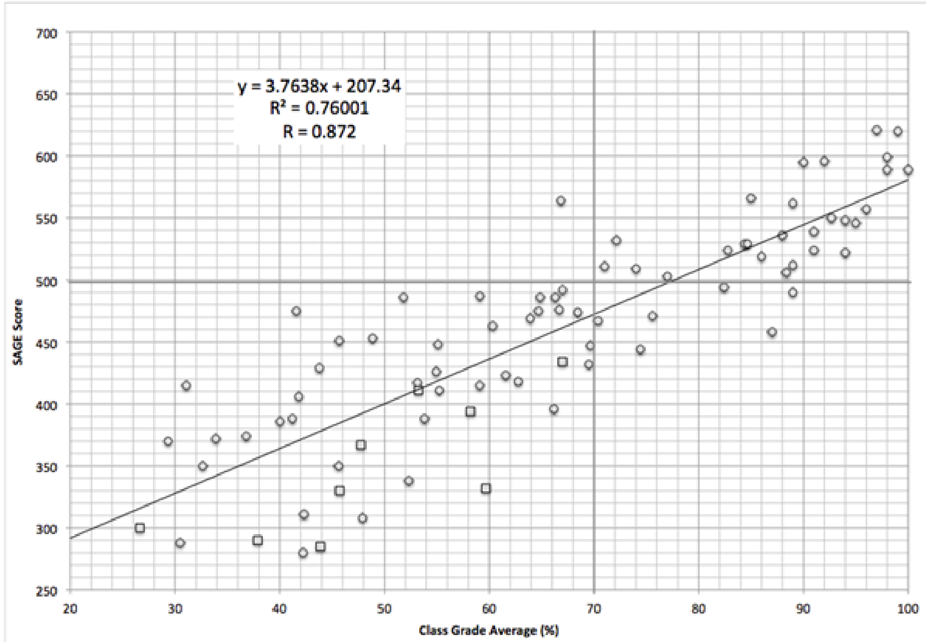


Figure 6

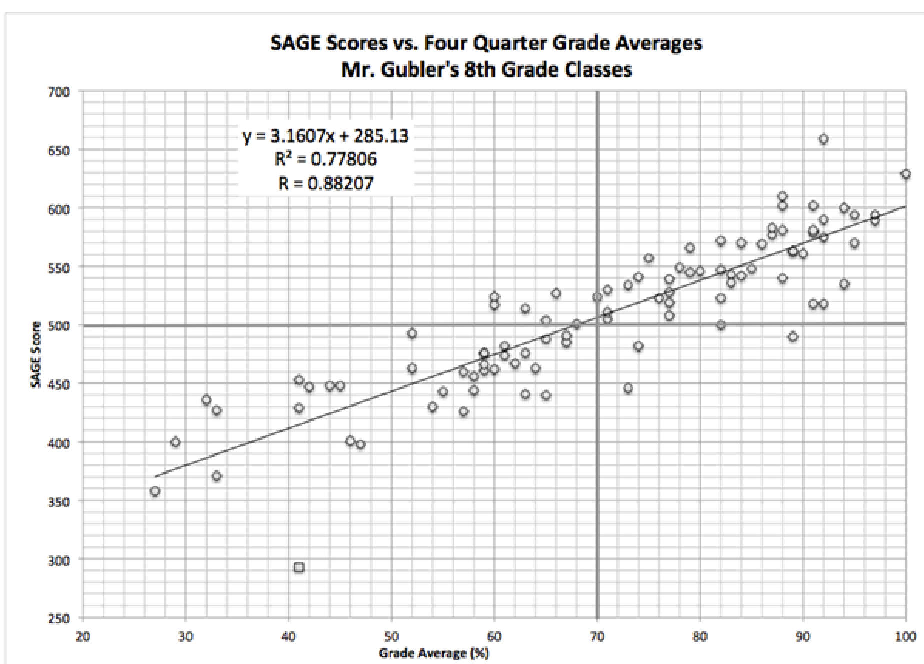
Consider the standard from grade 5:

> *Add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. (5.NF.1)*

Graph 4. *Mr. Goodrich: 8th Grade Math Classes SAGE 2015 (n=84)*



Graph 5. *Mr. Gubler: 8th Grade Math Classes SAGE 2015 (n=95)*



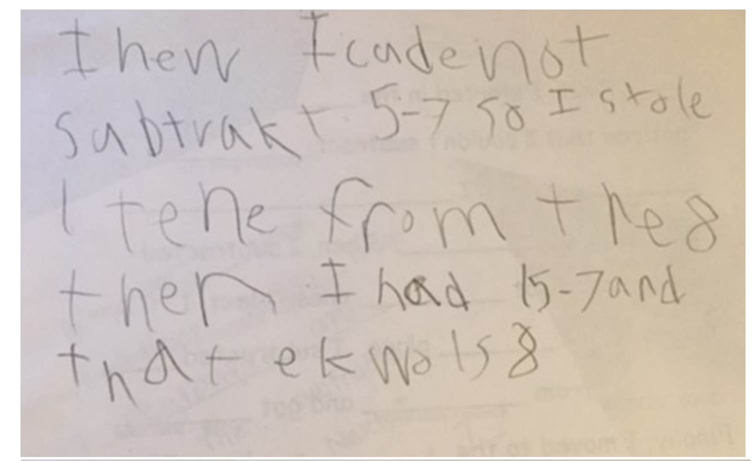
Rigor of Instruction and Intervention

There are some teaching and learning practices that should be avoided if the goal is student learning and understanding. There are many teaching practices that will help students learn at

Figure B: Written Response Frame

First, I started in the ________ place. I noticed that I couldn't subtract __________ (number + place value) from __________. So, I ______________________ ______________________. Then, I subtracted the ones and got __________ ones. Next, I moved to the ________ place. I subtracted ______________ (number + place value) from __________ (number + place value) and got __________ (number + place value). Finally, I moved to the ___________ place. I subtracted __________ (number + place value) from __________ (number + place value) and got __________ (number + place value). The difference equaled ________.

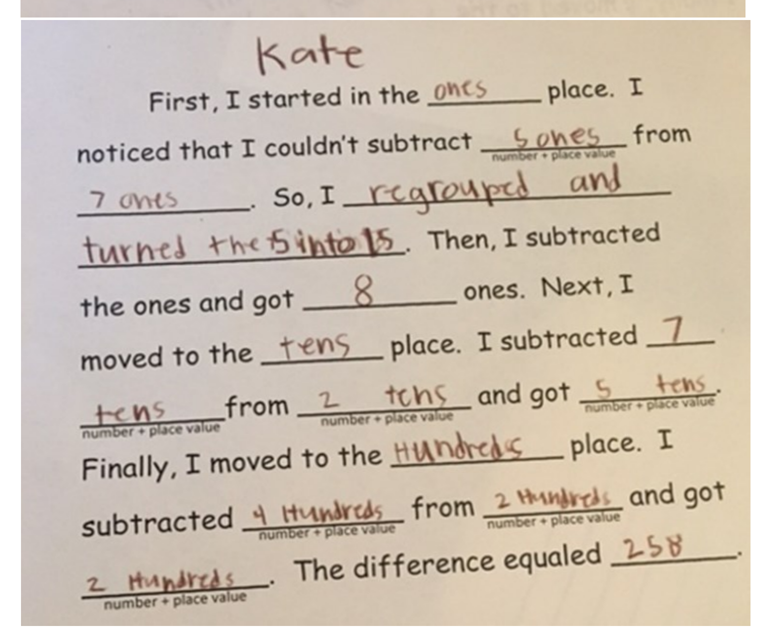
The student responses were collected and used as a formative assessment to guide future instruction. Written responses provide greater insight into students' understanding, especially in comparison to purely numerical responses. Below are some examples of the students' responses.



Student Example 1: **Connor's Open-Ended Written Response**

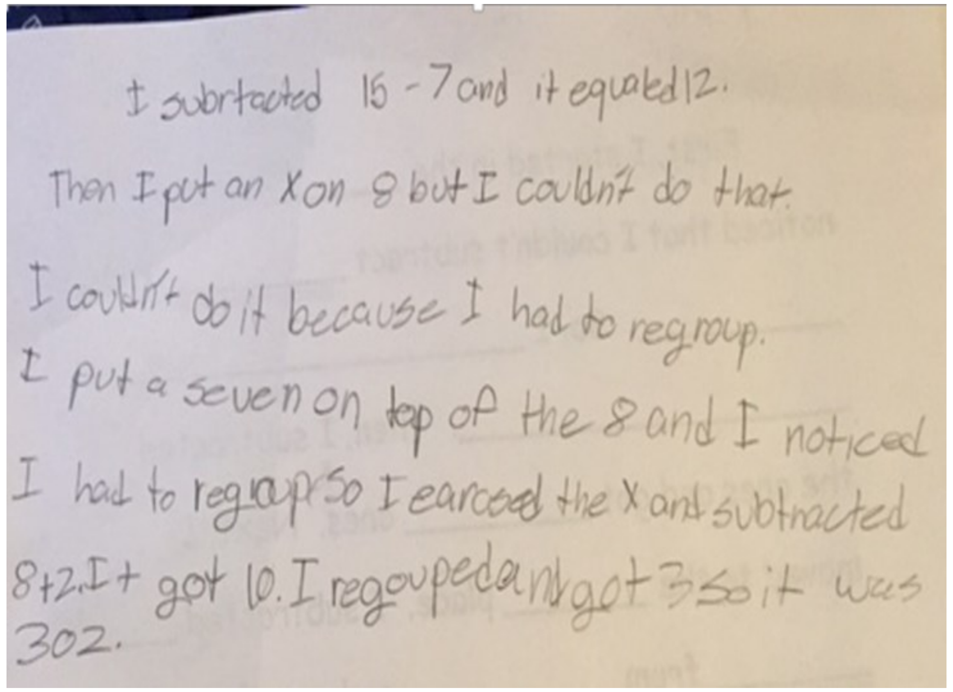
"I knew I could not subtract 5-7 so I stole 1 ten from the 8. Then I had 15-7 and that take away was 8.

His partial explanation demonstrates his understanding of regrouping.



Student Example 2: **Kate's Written Response Frame**

Note: Kate's explanation indicates that she took "7 tens from 2 tens" and "4 hundreds from 2 hundreds". This may be a simple error, but requires additional information to know for certain.



Student Example 3: **Allison**

Allison is a perfect example of how useful asking students to write about their thinking can be. The explanation provided insights into the student's misconceptions and lack of understanding.

Reflecting on students' strategies

Through listening to students' verbal explanations and the collection of their written responses, we were able to gain useful formative assessment information. The student work samples were extremely useful in demonstrating concrete evidence of students' thinking processes and mathematical understanding, which in turn would be used to support group decisions and to adjust instruction in the areas of subtraction, place value, and regrouping. Furthermore, as the students orally explained their reasoning to their partners they were able to clarify their thinking and justify their understanding. We encourage you to use the Mathematically Speaking template and written response frames as avenues for facilitating students' mathematical reasoning. These instructional practices provide opportunities for students to demonstrate their conceptual understanding of mathematics concepts. Verbal and written demonstrations of mathematical understanding are invaluable for ascertaining students' thinking and determining next steps for instruction.

analysis of this data showed the disparity between what our school had intended to do with SBG and what was actually occurring. All math students in the school were divided into three groups depending on grade level. *Table 3* displays a summary of data collected. You will notice that there was a much greater correlation between grades and course proficiency in Group 3. The students in this group received math instruction for the academic year solely from teachers heavily implementing strategies this article highlights—grading for learning and Depth-of-Knowledge-aware instruction and assessment design. We believe an analysis of other schools would likely produce similar results.

	Group 1	Group 2	Group 3	School Total
Students receiving an A	69%	39%	20%	43%
Top 20% Winter 2014 NWEA	12%	16%	23%	17%
Students with C or higher	94%	80%	45%	74%
Top 60% Winter 2014 NWEA	55%	54%	66%	77%
Table 3				

Classroom Grades vs. SAGE Data

We later analyzed our individual classes of 8th grade students for the Spring 2015 SAGE test administered between April 27, 2015 and May 8, 2015. *Graph 4* and *Graph 5* each show a very high positive correlation—with correlation coefficients of 0.872 and 0.882 respectively—between class grade average (horizontal axis) and 2015 SAGE score (vertical axis). The vertical line ($x = 70$) marks the line between C and D grades. The horizontal line ($y = 499$) denotes the cut-off for proficiency on the SAGE test. A few points on the graphs are squares; these represent SAGE scores that decreased from 2014 to 2015. All circular points indicate SAGE scores that increased. Note that with only a few exceptions, students who averaged C, B, or A in math class also demonstrated proficiency on SAGE. This and the high correlation coefficients indicate strongly that we enjoy a high validity of grades. Because of this we also enjoy the benefit of using classroom grades to predict SAGE performance, but much more important, we know what a student has learned and whether we have been able to prepare them for future learning and eventual success in the students' future endeavors. This is because learning broadens horizons whereas lack of learning narrows such.

lower than 25. Both practice and learning outcomes use this rubric. Regular classroom tests will assess between one and five learning outcomes—identified from the corresponding state math core by teachers working collaboratively within the school (or district). Each learning outcome is usually assessed with 7 to 9 queries. When grading these assessments, multiple scores are reported to students disaggregated by each individual learning outcome. This tells the students, teacher, parents, and any other responsible party specific information about the students' learning.

It is important to remind the reader that there is a variety of DOK infused into the assessments. The practice is also designed to include a variety of DOK. This part can be a challenge dependent on what resource materials are available to teachers at their school. Teacher buy-in and the amount of work and cognitive effort teachers are willing to invest are also crucial factors. The rubric defined above, coupled with good assessment design that properly uses DOK, provide for a very valid grading system that communicates to stakeholders specific information about student learning.

The Traditional Grading Scale and Standards Based Grading

The traditional grading scale with A, B, C, D, and F has worked well with our SBG to communicate to stakeholders overall student performance across several learning outcomes. Whereas each individual learning outcome has a proficiency score assigned, the overall grades communicate much. Our school uses a 90-80-70-50 scale. This means that individual scores are 4 – A, 3 – C, 2 – D, and 1 – F. This has meaning when several learning outcomes are averaged. An A means that students have mastered at least 60% of the concepts, with near mastery on the rest. A B means that a student has mastered at least 20% of the concepts with near mastery on the rest. A C does not require any mastery, but near mastery on almost all learning outcomes. A D shows that a student is deficient across several learning outcomes, and an F shows this even more so. Therefore, A means Superior, B is Good, C is Average, D is Below Average or Deficient, and F is Failing to master concepts. Students with As, Bs, and Cs are progressing and those with Ds and Fs are in need of intervention—academic and often motivational.

Classroom Grades vs. NWEA Data

The first step to making a change is to realize that there is a problem. We analyzed data from classroom grades and NWEA—an adaptive test similar to SAGE in a few ways. The

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WARNING! EXPLICIT CONTENT...With Linear and Exponential Functions

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Warning: Explicit Content! What better way to grab the attention of teenage students who are otherwise indifferent to learning mathematics? There's nothing like hooking your students as they walk into the classroom making them curious about what their teacher has in store for them that period. That's the way I introduced a transition activity between two units in my Secondary Math I class last year – those being the units on Sequences and Functions.

Let me start with a little background behind the creation of this activity. I have been teaching the Utah Common Core for the last four years and have witnessed my ninth grade students consistently struggle with a concept that I thought was simplistic by nature. Of course, it could be that my approach to teaching some of these concepts has been teacher-directed and mostly rote memorization of an algorithm… BORING. I used to show different forms of sequence notation when defining arithmetic and geometric sequences and then would have students complete examples mimicking what I had done. I would write the appropriate format on the board and keep it up all unit long so that the different forms of each equation would sink in – if not by having them write it in their notes, then by subliminal messaging by keeping them front and center on my white board for about a month. When it came for an assessment, I allowed the students to bring in a hand-written note card so they wouldn't get flustered and frustrated with trying to remember the details of the format. Most students demonstrated proficiency on the sequence unit exam. Then, I presumed the following unit on functions would flow easily since the process for writing equations for functions was so similar to that of writing equations for sequences. However, proficiency on the function exam was erratic at best. And, I was getting the same results year after year. I couldn't figure out what was wrong. It seemed so obvious and logical to me. I had given them everything they had needed to be successful. I explained the difference between the forms and showed many examples. Still, I was not being effective. Then I decided that maybe they weren't grabbing the material because there was no transfer of understanding from one unit to the next; students were just copying notes and

affected by grades that were reported to them and their parents, but that were not accurate indicators of learning. This increases the deviation of grades from understanding and knowledge to work ethic and participation. Grades should tell what a student comes to know and understand; not behavior, not participation, and not a record of i's dotted and t's crossed.

In order to make the change to where grading is based on proficiency, one must get used to the idea that not every student is an A-student unless every student learns at a high enough level. We need to honestly look at what grades actually mean. A teacher may tell the parents of C-students that either their studnents tested well and didn't do so well on the homework, or that they did well with homework completion, yet they did not master the concepts as well. Standards Based Grading (SBG) provides the framework for increasing the validity of grades but does not guarantee it. Our school adopted SBG for all teachers, yet teachers found ways to not change their grading practices and continued assigning A-grades to students who later went on to show level 1 proficiency on end-of-level tests. This discrepancy cannot be completely attributed to the common scapegoat of "That kid just doesn't test well" when more accurately it should be said, "That teacher just doesn't grade well." A teacher should know, within a small error, how well their students would do on an end-of-level state test before the students ever sit down to take the test.

Standards Based Grading

Two years ago, our school adopted a school-wide SBG plan. Instrumental in this change were many discussions among faculty about what grades actually mean. These discussions were very educational for teachers. Our staff started questioning practices we'd had for years, decades for some teachers. Our staff now uses a four-point scale where 4 is mastery, 3 is near mastery, 2 is approaching mastery, and 1 is minimal. Students' grades are comprised of two categories: practice and learning outcomes. Each department in the school chose their grading percentages and defined rubrics for what constitutes 4, 3, 2, and 1. True SBG would employ a separate grading rubric specific to each learning outcome. The math teachers in our school have set grading standards that include 75% learning outcomes and 25% practice. A true SBG scheme would be 100% learning outcomes and 0% practice. We retain a little incentive for completing practice. Our rubric is based on the old Criterion Referenced Test proficiency levels used previously by the State of Utah. We have set a 4 at 75%, a 3 at 50%, 2 at 25%, and 1 for any percentage

Because it is often difficult to distinguish between the different DOKs, it is recommended that the reader considers learning more about Webb's depth of knowledge than this article provides.

The Blueprint for Assessments

	DOK 1	DOK 2	DOK 3
Math 7	12% – 24%	48% – 60%	20% – 26%
Math 8	20% – 30%	40% – 50%	20% – 26%
Sec Math I	16% – 24%	44% – 56%	24% – 28%
Sec Math II	16% – 24%	44% – 56%	20% – 26%
Sec Math III	10% – 20%	40% – 50%	30% – 36%

Table 2 (USOE 2015)

We have found that a good assessment blueprint to gauge student understanding has target values of about 25% DOK 1 questions, 50% DOK 2 questions, and 25% DOK 3 questions. Compare this to the Utah State Office of Education SAGE Blueprint in *Table 2* and you should see that it is quite similar. It is difficult to create higher DOK questions for some concepts because they require different depths of knowledge. When assessing a concept we usually use 7 to 9 queries. If the concept is naturally high in cognitive-demand, we can design the assessment with as few as 4 queries, and for lower cognitive-demand concepts we use as many as 12 queries. We will discuss how to grade the assessment in the grading section of the article.

Grading for Learning

Why do we give students grades? It is a communication system between stakeholders with built-in consequences. When students fail, they do not receive credit. When students pass, they have learned the content to varying degrees—A through D. Or at least this is what we want it to be.

Effort Based Grading and It's Shortcomings

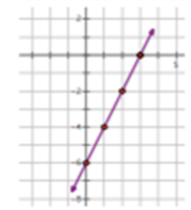
It reflects well on a teacher when the students have good grades. Also, students who receive the good grades are equally happy. Consequently, there are many teachers who genuinely believe that they should give high grades based on effort rather than on learning or understanding. This quickly develops into *perceived* effort and hoop jumping. There is a compounding effect as students move on to the next teacher. Students and teachers find that students did not learn what was required, or at least they did not retain it. This often leads to later teachers giving breaks and lowering acceptable levels of performance to be "nice" to the students

examples and were not relating the parts of the equations to the multiple representations of a sequence or a function; there was no anchoring of the knowledge, nowhere to file that information in their brains; it was boring for the students to just watch and listen – like pointing and shooting while not even aiming or knowing the purpose of the shot. Thus emerged the idea of using a discovery approach so that students could make their own connections instead of me trying to do it for them.

The design of my activity, "Investigating Multiple Representations," had students focus on arithmetic and geometric sequences separately. Each activity was split into three parts. The students first completed a table where they were given a sequence in four forms: table of values, list of values, picture pattern, and a graph. For each they found the common difference, wrote an explicit equation in the format $a_n = a_1 + d(n-1)$, then simplified the explicit equation – all tasks they were readily able to do based on the previous unit on sequences. (See the heading of the table for the arithmetic activity below.)

<table>
<tr><th>Sequence</th><th>Common Difference</th><th>Explicit Equation</th><th>Simplified Version of Explicit Equation</th></tr>
<tr><td>Ex.
<table><tr><th>Term</th><th>Value</th></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>3</td></tr><tr><td>3</td><td>-4</td></tr><tr><td>4</td><td>-11</td></tr></table>-7
-7
-7</td><td>-7</td><td>$a_n = 10 - 7(n-1)$</td><td>$a_n = 10 - 7(n-1)$
$a_n = 10 - 7n + 7$
$a_n = -7n + 17$</td></tr>
</table>

The second part of the activity asked the students to locate and highlight certain features of each pattern, those being the common difference, the first value of the sequence, and then the zeroth value of the sequence. They were also posed questions that asked them to articulate what each part of each equation represented and then to write what they thought an equation "template" could be for the simplified version of the explicit equation. The idea was for students to have the "a-ha" as to where *slope-intercept* form may have come from. It really anchored the idea of the meaning of each part of *slope-intercept* form and gave them a different way to file that information in their brains for later use. The third part of the activity was for the students to practice their understanding of how all of the representations relate to each other by toggling back and forth from one type to another. In this section, students completed a sequence, table, graph, and equation based on being given only one type and having to find the other three. (See the starting example in the image below.)

Sequence	Table	Graph	Equation
Ex. -4, -2, 0, 2, …	x | y 1 | -4 2 | -2 3 | 0 4 | 2		$y = 2x - 6$

The geometric version of the activity had the students go through the same thought processes, but with the formats $a_n = a_1 \cdot r^{n-1}$ and $a_n = a_0 \cdot r^n$. It then encouraged students to relate the notation in the sequence equations to that of the function equations using x and y instead a_n and n.

Upon completion of the activity, I asked the students to share what they had learned or what they had noticed. Most commented on the use of the different colors to highlight certain features of the equations. They said that it made the parts of the equation more obvious and that they finally understood that a_1 and a_0 stood for the first and zeroth values. Perhaps the most satisfying outcome was the profound ways students articulated their responses to the question about how to write a template for the simplified version of the explicit equation. Some used words, some symbols, and still others wrote paragraphs trying to explain what to do. My personal favorite was when one student asked me something like, "So if a_0 stands for the zeroth value and a_1 stands for the first value, then does a_2 stand for the second value, and a_3 for the third, and so on?" I asked him to demonstrate his thinking to the class by using one of his "future term" equations with one of the examples and the students were amazed at how it worked every time. At that point, I knew the activity was a success.

In my opinion, the beauty of the activity was its scaffolded design and the way it addressed different learning styles. It allowed students to use and transfer their prior knowledge and understanding of sequences to making connections between multiple representations of functions. In addition, it emphasized the visual and linguistic modalities that foster success for most students. It is with great hope that this activity will help all students develop a deeper understanding of the relationship between equations and functions and give them confidence in learning to look for patterns and make sense of what those patterns imply.

Depth of Knowledge (DOK) and Mathematics

There are four depths of knowledge levels for mathematical tasks. These are summarized in *Table 1*. Recall (DOK 1) includes questions that require the students to reproduce a memorized fact or complete a simple one-step algorithm or use a formula. A skill or concept (DOK 2) often involves an algorithm as does DOK 1; however, the problem often requires students to decide which algorithm or procedure to use, thus testing further understanding on the students' part by knowing for what the concept is useful. It also includes using more complex algorithms to arrive at a solution or using a solution rather than just finding the solution. Strategic Thinking (DOK 3) requires more of the students' reasoning with the concepts learned. A problem may be approached using various methods and often requires justification of answers or processes or the critiquing of others' reasoning. Strategic thinking may include using concepts for something that students did not learn directly. Extended thinking (DOK 4) may include planning and thinking usually over a longer period of time. DOK 4 assessments are very difficult to test. They are usually included as classroom learning tasks or as extended learning opportunities that can include out-of-class work. These help develop and solidify understanding of the concept. SAGE does not utilize DOK 4 for math.

DOK for Mathematics Tasks	
DOK 1	Recall
DOK 2	Skill/Concept
DOK 3	Strategic Thinking
DOK 4	Extended Thinking
Table 1 (Webb 2002)	

It is important to distinguish between depth of knowledge and difficulty. Depth of knowledge deals with the cognitive demand of the task independent of actual answers given; the portion of students who answer correctly determines difficulty—easy tasks are answered correctly more often and difficult tasks are answered correctly less often. It is possible for a DOK 1 task to be difficult and a DOK 3 task to be fairly easy. For example, a DOK 3 question could ask a student to critique others about their responses on what is special about the number pi, but a DOK 1 question asks for the first ten decimal digits of pi (which is a horrible question to put on a test).

Assessment Design and Grading for Learning in a Mathematics Classroom: What are we communicating to stakeholders?

Edwin Goodrich, Eric Gubler, Roosevelt Junior High, Mathematics Teachers, Duchesne County School District

Introduction

Mathematics assessments should be created using various depths of knowledge questions. Honest and consistent grading practices that focus on students' learning goals provide valuable information for parents, teachers, and students across grade levels. In order for students to test successfully at high cognitive levels, they must be exposed to cognitively demanding tasks and expected to reason through such. Although this article is written with the perspective of a mathematics curriculum, it has applications to other disciplines.

An Analogy for Grades: The Safety Inspection

When you take your vehicle in for a safety inspection each year you should expect that the mechanic would catch anything that could make your car unsafe. Some car owners do, however, want their car to pass regardless of how dangerous it may be to avoid the cost of repair. Regardless of customer preference, mechanics need to make safety their priority and not just pass a vehicle to be driven for another year. A car may pass its safety inspection, but the mechanic to whom the car is taken might mention a thing or two that passed minimally but would be worth fixing soon. This honesty is much appreciated. Another instance occurred in which the ball joints on the front of a Crown Victoria had passed a safety inspection, but less than a month later suffered a mechanical failure. This situation could have been much more dangerous, even fatal, if circumstances had been slightly different. In one situation, the mechanic's honesty made it possible to repair an issue before it became a problem. In the other, the mechanic's neglect allowed an issue to become a problem with serious consequences and the potential of an irreversible tragedy.

Arriving at our Destination Before We Get There

Ron Marrelli, Christie Tolbert, Mike Spencer — Juab High School, Juab School District

As teachers, our ultimate goal should be to help students take responsibility for their own learning. In past years we have tried to accomplish this by assigning grades based on performance on summative assessments as well as the number of completed homework assignments, how well deadlines were met, classroom participation, or whether students show up before the late-bell rang. In some cases our grading practices did not encourage students to take responsibility for their own learning. Getting a good grade became a 'hoop' to jump through, a reward or punishment based on how well students did what they were told, rather than an opportunity to learn. Unfortunately, rather than encourage learning our grading practices became a barrier to this goal.

As teachers we should always set high expectations for our student. Paul Tough, in *How Students Succeed* and *Mastery Learning* indicates that students will meet high standards for learning if: 1) Our expectations are focused on learning. 2) We hold them responsible to meet these standards. 3) They know what a high level of proficiency looks like. 4) They know what is required to meet a high level of proficiency. Our previous grading practices were more directed towards 'doing' rather than learning.

Additionally, evaluation of student performance, both formative and summative, should be carefully designed to not undermine perceptions of competence and future expectations. Standards assessed must have clearly defined criteria for mastery, and must be accompanied by specific, timely feedback. Assessments and assessment items should be varied as well as ongoing in order to provide multiple opportunities for students to demonstrate competence on all standards.

We believe that all students can learn. Some take longer than others to understand some standards at a high level of proficiency. We want our students to know that while we anticipate they will make mistakes we expect them to quickly correct these mistakes by providing multiple opportunities to demonstrate proficiency. As a math team we plan for 'unfinished' learning by making adjustments to our instruction based on feedback from formative and even summative assessments. We want our students to know that learning, real learning, is difficult and that failing an assessment does not equate with being a failure. We believe that our grades should reflect these views.

The four essential questions of professional learning communities define what an empowered student looks like: 1) What do I need to know? 2) What will it look like when I do know it? 3) What will I do if I don't know it? 4) What will I do if I already know it? *(Cultures Built to Last,* Richard DuFour and Michael Fullan). Ultimately parents, guardians, students, and teachers, should all be able to answer these questions from their individual perspective. In our own school, we looked at the system we had in place and realized that what we were doing with our grades was obscuring the ability of our stakeholders to do this. The following is a description of the changes we have made to empower all involved to answer these questions.

What do I need to know?

Our gradebooks have become our method for helping students answer the first two PLC questions. Instead of listing assignments, participation, extra credit, etc., our gradebooks only include assessments in the forms of quizzes and unit assessments. Assessment titles reflect the learning goal being measured. By removing items that do not measure learning, we have provided a platform that communicates student progress to all stakeholders. Gradebook can be referenced at any time to identify content students are responsible to know as well as levels of understanding.

How will I know if I know it?

We have developed levels of proficiency based on a four-point rubric that help guide students in determining what proficiency looks like for each learning goal. As learning progresses, we encourage students to self-assess their own proficiency and diagnose areas of weakness. We also formatively assess and record student progress based on short, frequent quizzes aligned to our learning goals. Grades are fluid and are changed as students demonstrate learning.

What will I do if I don't know it?

Quiz scores are an estimate of what students know at a given moment, and students are encouraged to reflect on their understanding. "Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning" (*Principles to Actions,* NCTM). If grades are truly reflective of understanding, then

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grading practices must be dynamic and representative of learning whenever it takes place. For this reason, retakes are allowed on every quiz. Retakes are carefully designed to not give students a false sense of competence. If a student scores higher, they are given a higher grade. Students are not penalized for multiple attempts to demonstrate learning.

Students are encouraged to proactively correct misconceptions prior to the unit assessment. Students should know their level of understanding throughout the learning process and use that information to advance their learning. As cited in *Principles to Action*, "Thinking of assessment as limited to 'testing' student learning rather than as a process that can advance it has been an obstacle to the effective use of assessment processes for decades."

Teachers should know how students are progressing towards unit goals before the summative assessment. While formative assessment is needed, individual student understanding may be underestimated and class-level understanding can be misdiagnosed without continuous formal feedback. Frequent quizzing provides data to use for formal feedback.

Because grades should reflect learning, homework is not included in the final grade. Homework is used as a self-assessment tool for students to evaluate their progress. As students leave the class, they are told that their homework is not to complete the assignment that night, but to make sense of what was done in class and be prepared to demonstrate that understanding the next day. In this way, homework has been individualized. What a student needs to do will look different for each individual as they assess their **own** understanding of the learning goals.

What will I do if I do know it?

The most powerful conversations are had with students that demonstrate understanding of learning goals. They should be aware that current understanding does not always equate to long-term fluency and that "repeated retrieval not only makes memories more durable but produces knowledge that can be retrieved more readily in more varied settings and applied to a wider variety of problems (*Make it Stick,* Brown, Roediger, McDaniel)." Continued and spaced practice, even though difficult at times, is essential. For these students metacognition plays a dominant role in understanding and retention. Answers to questions, such as 1) Can I create questions about this learning goal that I could not answer? 2) Can I connect these ideas using multiple representations? 3) Did I get lucky? 4) Can I connect these ideas to earlier learning goals? 5) Are there homework questions that I am unsure of? should be continually evaluated.

As teachers we feel that grading practices can, and should, empower students. Earning a grade should never be a ‘point game’ or a ‘hoop to jump through.’ However, traditional grading policies and practices have not permitted us to end this game. Grading practices should be aligned with our beliefs of what a grade should represent, student learning. All stakeholders should know by the student, by the standard, at any given moment, what a student understands and what they don’t understand yet. This is paramount for the implementation of immediate targeted interventions. Changes came about through learning focused on the standards, clear and consistent communication of grades, meaningful homework, and fluid grades. This has allowed students to know where they are throughout the learning process of each standard being taught. Arriving at the destination before we ever got there!

Formal assessments are used to help students continually refine learning on an ongoing basis and guide instruction. By scoring these self-assessments and quizzes on a four point scoring rubric our grades communicate a clear and consistent message to all stakeholders. This has empowered students to be an active participant in their own learning and has helped embed responsibility for learning the content in the process. As our grading practices moved more to reflect what students need to learn rather than what they have done, students have realized that homework and quizzes are not just something to do and turn in to get points, but it is apart of the process to help meet the learning goals. This has been a slow change because it is not only a change in grading practice but also a cultural shift in student and teacher thinking of what a grade really represents. Students are now coming with focused questions, can articulate their own deficiencies, and are actively working to advance their own learning. It shouldn’t matter when the student learns the material being taught, the focus should be that they have learned it.

As you read this article, you may think, “duh, of course, we should be focusing on the learning and not the doing.” We challenge you to look at your own grading policies; are you really about the learning, or is doing the work all that counts? It has taken our department a few years to let go of the doing and truly focus on the learning but the results have been transformative.

The pedagogy of FACT embeds three evidenced-based practices: (a) concrete-representational-abstract (CRA) sequencing of math concepts (Miller & Mercer, 1993; Witzel, Mercer, & Miller, 2003), length-based models and number line development; (b) Self-Regulated Strategy Development framework for instruction (SRSD) (Graham & Harris, 2005); and (c) writing-to-learn content, an educational practice in which teachers assign writing tasks to help students deepen their understanding of subject matter (Klein & Yu, 2013). Students are taught through a series of six lessons how to self-regulate the process of using writing as a learning activity and to connect their procedural understanding with underlying conceptual knowledge of fractions (Bailey, et al, 2015; Hallett et al., 2010) (see Figure 3).

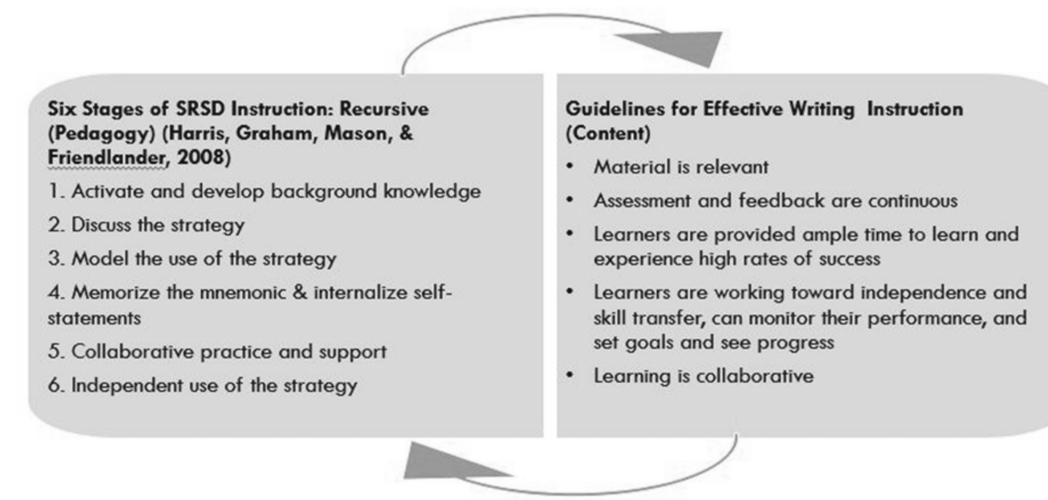


Figure 3. SRSD and Guidelines for Effective Writing Instruction

Takeaways for Embedding Writing into Mathematics Instruction

Students who struggle with mathematics can be more successful when they are provided with explicit strategic support for constructing written arguments to effectively engage in problem solving and critical thinking (Bhatia, 2004; Ferretti & Lewis, 2013; FritjterenDam & Rijlaarsdam, 2006; Hillocks, 2006; Resnick, 1987). Through the construction of written arguments, FACT is an approach that helps struggling learners transfer their content knowledge and skills to new contexts when asked to solve novel fraction problems, and it provides teachers with an instructional framework for going beyond the current practice of basic skills remediation in writing and math.

being taught when having to construct written arguments (Bangert-Drowns, Hurley, & Wilkinson, 2004; Graham & Hebert, 2011; Graham & Perin, 2007; Hand, Villaneuva, & Yoon, 2014; Klein & Yu, 2013).

FACT: An Intervention Designed to Provide Intensive Instruction

Kiuhara, Witzel, Dai, Rouse, & Unker (2015) designed a small group intensive intervention (i.e., Tier 2) called FACT to help students who struggle with mathematics better understand the characteristics of fractions and how fractions differ from whole numbers. FACT scaffolds and embeds the construction of arguments as a learning activity within a framework of mathematics problem solving. FACT is an acronym representing four steps: F = Figure out a plan; A = Act on it; C = Compare reasoning with a peer, and T = Tie it up with an argument. Figure 2 presents the Utah Core Standards in Mathematics, Writing, and Language that are aligned in FACT. Through the problem-solving process, students are reminded to (a) make specific decisions about which information is most important when solving a fractions problem, (b) make explicit connections between ideas, as they commit them to text and organize them into a coherent argument, (c) engage with a peer to reflect, critique, and reexamine ideas, and (d) think about what ideas mean, as they put them into their own written words.

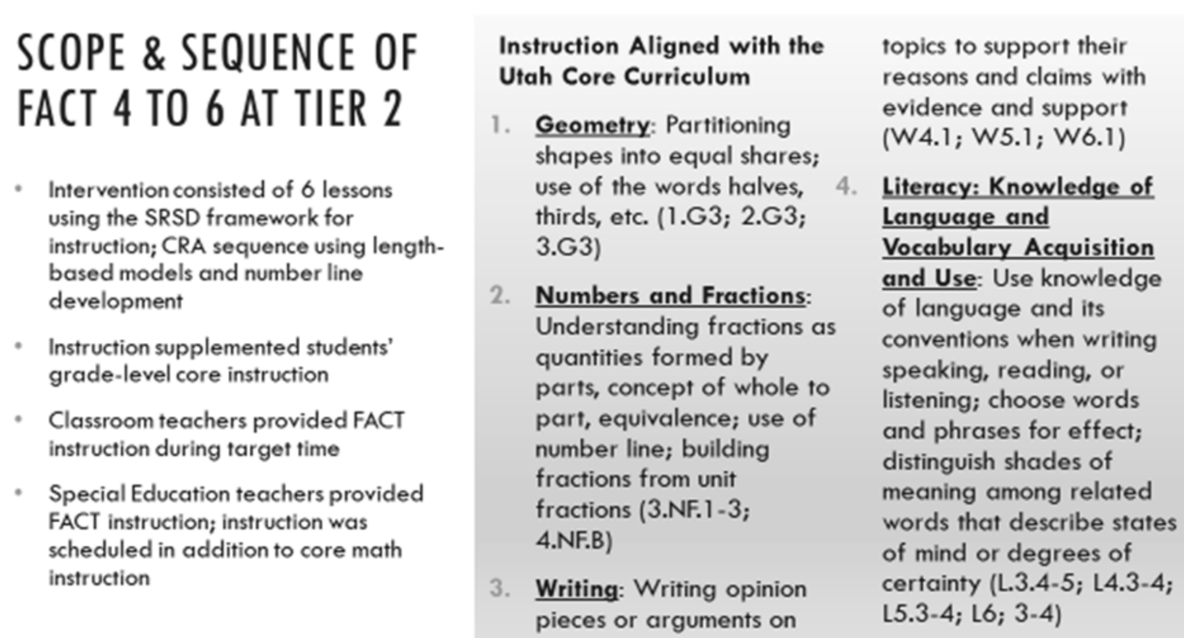


Figure 2. Utah Core Standards Embedded in FACT

Growth Mindset

Sallianne Wakley, Mindy Robison — Canyons School District

How does the use of assessment in my classroom foster a growth mindset? This paper will focus on specific strategies teachers can use to increase student confidence and willingness to make mistakes in order to learn mathematics. Specifically, we will focus on student feedback and the role of self-evaluation in assessment.

According to Carol Dweck, “The growth mindset is based on the belief that your basic qualities are things you can cultivate through your efforts” (p.7). Students and teachers with a growth mindset are concerned with improving, where those with a fixed mindset believe that intellect is “carved in stone” and are concerned about how they will be judged (Dweck, p. 6). Using the work of Carol Dweck, Jo Boaler applies the growth mindset to assessment in mathematics and identifies assessment for learning as a “form of assessment that gives useful information to teachers, parents, and others, but it also empowers students to take charge of their own learning.” (p.94). Boaler identifies critical components of assessment for learning:

- Students need clear communication to know what they are learning and how they will get there through feedback
- Students need to be aware of where the are in the learning process

We will look at each component of assessment for learning and identify simple strategies teachers can utilize to create a classroom focused on using assessment to communicate using feedback and create awareness for student and teacher success.

Assessments to Communicate

In the book *Principles to Actions*, the National Council of Teachers of Mathematics (NCTM) identified productive beliefs for assessment. The first two beliefs, “The primary purpose of assessment is to inform and improve the teaching and learning of mathematics,” and “Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction (p. 91)” support Boaler’s critical components for assessment for learning. Consider the following situation: A teacher gives a quiz to a student, after the student takes the quiz the teacher writes a C on the quiz and gives it back to the student, the student sees the C and throws the quiz away. Has the test provided a means to communicate to the student and the teacher what the student has learned and where he/she is going? Maybe, but is there a more effective way? Boaler, citing research done by Ruth Butler, found that students who received comments instead of a grade increased their performance significantly (p. 99). Students need to receive specific feedback on their assessments for learning that makes them more aware of where they are on their learning path. Students need to be aware of each mistake and provided feedback to learn from their mistakes. Boaler, quoting Dylan Wiliam, states, "Feedback to learners should focus on what they need to do to improve, rather than on how well they have done, and should avoid comparison to others" (p. 100). A student with a growth mindset seeks to get better through feedback, allowing for improvement. When a student and teacher in the same classroom have a growth mindset the potential for growth is exponential. *The simple strategy: give specific feedback on assessments for learning instead of a letter grade*.

Assessments to Create Awareness

Teachers are not the only ones that can provide feedback and assess student understanding. Students can self-assess and evaluate their own performance. In John Hattie's work he identifies the zone of desired effect as effect size above 0.40, through his research he found that self-reported grades (students estimating their own performance) have the impressive effect size of 1.44 (p. 44).

In *Principles to Actions*, another productive belief on assessment is, "Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning" (p. 92). In regard to assessment, Boaler explains, "In studies of self-assessment in action, researchers have found that students are incredibly perceptive about their own learning, and they do not over or underestimate it. They carefully consider goals and decide where they are and what they do and do not understand" (p.96). If students are able to accurately self-assess what tools do teachers need to provide to allow students this type of opportunity? One strategy, identified by Boaler, is "traffic lighting" where students are asked to put a red, yellow, or green cup on their work to assess their level of understanding of new work (p.98). This simple strategy allows students to take a moment and assess their understanding, as well as provides the teacher an opportunity to receive feedback and then adjust instruction accordingly.

& Fuchs, 2015; Hecht, Close, & Santisi, 2003; NMAP, 2008). Moreover, students who struggle with learning, particularly understanding fractions as numbers, exhibit several challenges when learning mathematics: (a) they have limited background knowledge and language and exhibit challenges with working memory and processing speed (Gersten, Beckmann, et al., 2009); (b) they have difficulty engaging in mathematics reasoning, as well as solving more complex mathematical problems involving multiple steps and skills (Jitendra & Star, 2011); and (c) they have difficulty planning, self-regulating their learning process, and generating and evaluating their own and their peers solutions (Fuchs & Fuchs, 2003; Graham & Harris, 2005; Montague & Jitendra, 2012).

Mathematical Practices and the Role of Writing

The NCTM Principles and Standards for School Mathematics now identify constructing arguments and critiquing the reasoning of others as essential math practices. With the adoption of the Utah Core Standards for Mathematics, teachers are expected to incorporate these practices into classroom instruction as early as Kindergarten (see Figure 1).

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them***
2. Reason abstractly and quantitatively
3. **Construct viable arguments and critique the reasoning of others***
4. Model with mathematics
5. **Use appropriate tools strategically***
6. **Attend to precision***
7. Look for and make use of structure

Look for and express regularity in repeated reasoning

Figure 1. Eight Mathematical Practices with Four Practices Embedded in FACT

Research evidence suggests that using precise mathematical language plays a critical role in developing students' mathematical understanding (Gersten et al., 2009; Star et al., 2015; Woodward et al., 2012) and significantly predicted children's ability to calculate fractions (Namkung & Fuchs, 2015). Additionally, the role of writing when used as a learning activity (a) allows students to examine relationships among ideas, evaluate their thinking processes, and engages students in the meta-cognitive processes of learning; (b) helps students better understand concepts, commit facts to memory, and learn strategies for reasoning; and (c) supports students' critical thinking and provides opportunities for them to acquire a deeper understanding of the concepts

Understanding Fractions Through Writing: A Tier 2 Intervention for 4th to 6th Grade Students

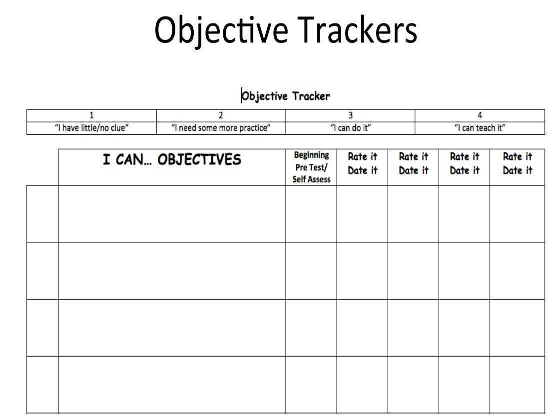
Sharlene A. Kiuhara, Ph.D., Department of Special Education, University of Utah

Across the nation, more than half of 4th and 8th grade students are failing to meet even basic levels of proficiency in mathematics (National Assessment of Educational Progress [NAEP], 2004; 2011). In Utah, 58% of typically achieving students, as well as 87% of students with disabilities in Grades 3 to 8 and 10 are lacking foundational skills to be proficient at their respective grade levels; by the time students complete their 10th grade year, only 4.5% of students with disabilities are performing at grade-level standards (Smith & Gallo, 2015). These data suggest that students are unable to demonstrate conceptual and procedural understanding of foundational arithmetic operations, which include the estimation of whole-numbers, decimals, fractions, and percents (NAEP, 2013). A critical goal for K-8 mathematics education is to ensure students develop proficiency with fractions, a critical foundational skill required to perform successfully in more complex and advanced mathematics, such as understanding ratio, proportion and percent, as well as algebra and other higher-level math (Fuchs & Fuchs, 2015; Fuchs et al., 2015; Namkung & Fuchs, 2015; NCTM, 2007; NMAP, 2008; Siegler et al., 2010). As the difficulty in mathematical skills and concepts continue to increase at each subsequent grade level, so do students' inability to master skills at even basic levels of achievement.

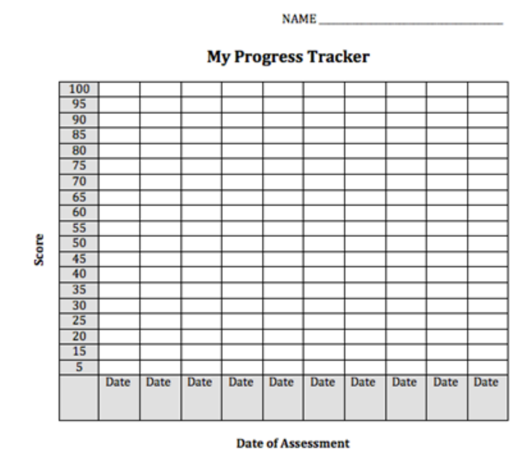
Why Fractions are Difficult for Students with Learning Problems

Students first enter "Fraction Land" (term coined by Ms. Keri Hohnholt, 6th Grade teacher during professional development seminar) in the 3rd grade, a place where their prior knowledge of whole numbers no longer apply to interpreting and measuring the magnitude of fractions (Fuch et al., 2013; Siegler et al., 2012). Although fraction knowledge includes understanding part-whole relationships using area models (e.g., "I ate half the donut"), successful ordering of fractions from smallest to largest depends largely on formal instruction of equivalence and the inversion property of fractions using linear models or number lines, which are introduced at the 3rd Grade of the Utah Core Curriculum for Mathematics and becomes a critical foundational skill for developing fraction competence (Fuchs et al., 2013; Fuchs, Namkung,

Another strategy is to provide an "Objective Tracker" to students (see Figure 1) as used in Canyons School District. A teacher creates a list of objectives in the form of "I can" statements for students in mathematics using Utah State Core Standards. Students self-assess at different intervals during the learning process and rate themselves on a 1 to 4 scale on the "I can" statement.



(Figure 1)



(Figure 2)

Another self-assessment strategy that creates student awareness of their learning is having students graph their progress. For example, if students are taking benchmark tests three or four times throughout the year they will need to monitor their progress in the interim of the benchmarks. Students can use a simple graph to monitor their progress and monitor progress toward mastery (see Figure 2).

A final strategy suggested by Hattie is for students to create a goal called their "Personal Best" (p.165). Students set individual goals that are specific to their achievement. Setting "Personal Best" goals had high positive relationship to educational aspirations, enjoyment of school, participation in class, and persistence on task (Hattie, p. 165). Boaler, citing the work of Wilhelm & Black, explains that students need to move from passive to active learners taking responsibility for their own progress and teachers need to be willing to lose some of the control of what is happening. Boaler cites a teacher that says, "What it has done for me is made me focus less on myself and more on the children. I have had the confidence to empower the students to take it forward (p. 98). *The simple strategies: "traffic lighting," objective tracker, graphing progress, and "Personal Best."*

The growth mindset focuses on learning from mistakes and improving but we can only grow and learn when we know our mistakes. Boaler states that students are often unsuccessful "not because they lacked ability but because they had not really known what they were meant to be focusing on" (p. 97). If assessment is going to be a map to success both students and teachers have to clearly know the outcomes. Focusing on feedback, self-evaluation, and clear outcomes provides students the opportunity for assessment to become an integral part of improvement. Consequently, assessment no longer resembles the unproductive belief identified by NCTM as "something that is done to students" but a process to foster growth and understanding.

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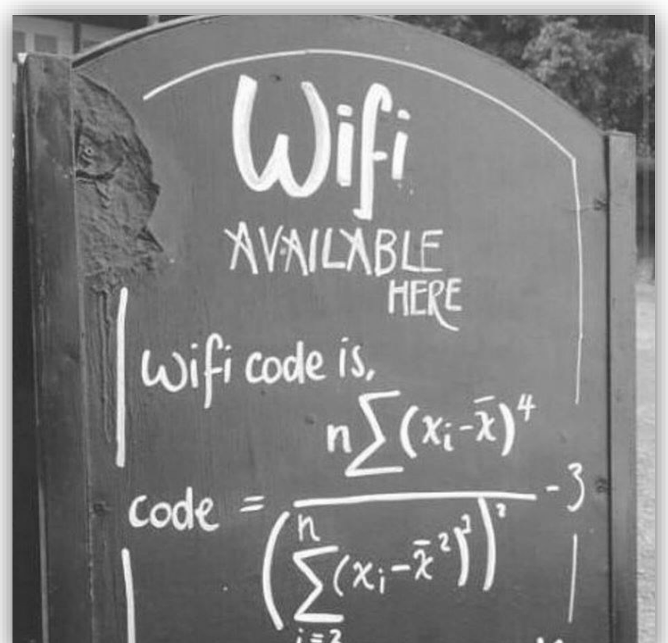

Factor completely: $6x^4 + 40x^3 - 14x^2$ $\quad 40a^3 + 94a^2 + 48a$

Find the product: $(-9b + 4)(-5b^2 + 7b - 3)$

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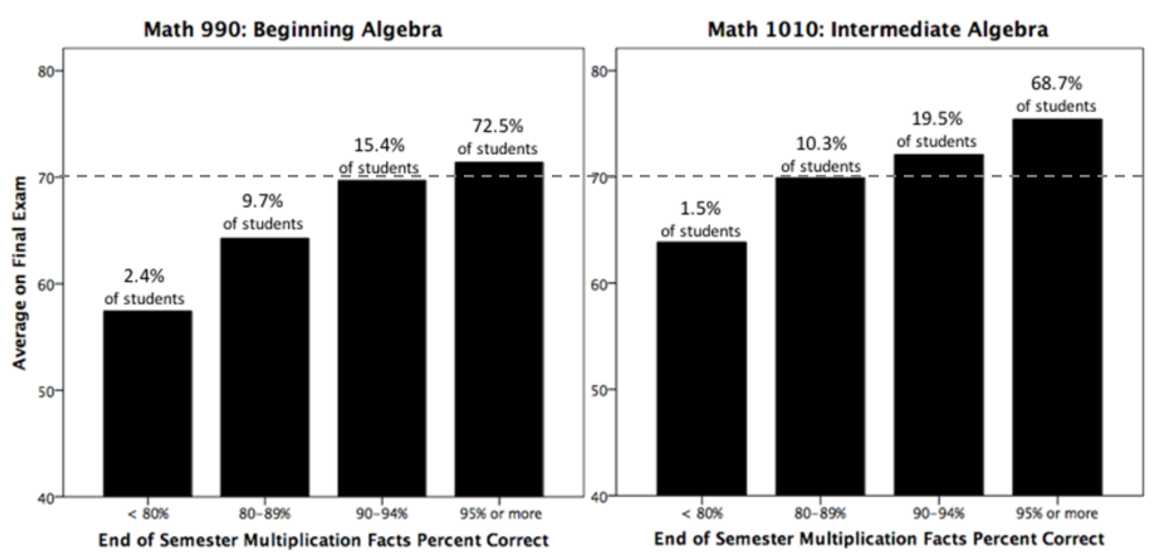


Figure 2: Percent of correct multiplication facts at end of year by average score on final exam

What should be clear from these figures is that multiplication facts accuracy is crucial to successful outcomes in developmental math courses. On average, students with lower accuracy (i.e,. lower than 90%) had a much greater chance of failing the final exam. In fact, accuracy was more important than speed for demonstrating competence in applying math concepts, much like reading accuracy is related to reading comprehension!

While students who struggle with multiplication facts are likely to have other gaps in math knowledge as well, they certainly won't be able to tackle algebra problems with confidence if they can't perform basic math operations fluently and accurately. Though there may be controversy over whether, how, or when students should memorize math facts, it is clear students need to be fluent and, more specifically, accurate in their recall of multiplication facts before they enroll in college math courses. This implies that students need ongoing facts practice without calculators throughout middle and high school to train and retain those skills. After all, a calculator doesn't help with all math. For example, these Intermediate Algebra (Math 1010) exam problems become much more difficult if one struggles with remembering multiplication facts and cannot use a calculator.

Simplify the following expressions:

$$\sqrt[4]{\frac{81}{256}r^8t^{20}} \qquad \frac{4{,}800{,}000{,}000 \times 0.00000054}{0.00006 \times 180} \qquad \frac{64x^3y-72x^2y^2+4xy^3-x^2y}{8x^3y}$$

Creating YOUR Leveled Classroom

Tom Morrell — Bear River Middle School, Box Elder School District

Due to the current structure of our school system, I have many students in my 9th Grade Secondary I class who did not show proficiency in Math 8. Many of you have students in a similar position: they have progressed to the next level without showing proficiency in the previous level. Regardless of their skill level, it is my job to prepare my students for Secondary II. To combat this problem I spent the summer of 2013 creating my leveled classroom and the 2013 – 2014 school year implementing, tweaking, and refining it.

My leveled classroom may look very different from your leveled classroom. Sometimes it is helpful to see what someone else has done in order to see where you can adapt and adopt the ideas. With that in mind, I will describe the process that I went through and why I made the decisions that I made. After that I will provide examples of how the entire classroom has become a leveled experience for my students.

My Problems:

- How do I motivate my students?
- How do I help my struggling students?
- How do I challenge my advanced students without demotivating my strugglers?
- How can I be better at creating consistent assessments that truly show my students' understanding? (Not "easy" or "tough" assessments, but assessments where the grades actually reflect a student's understanding of the material being tested, as opposed to assessments that are easier/tougher depending on my mood the night I wrote them.)
- How can I reflect understanding in a district-set non-negotiable grading scale and grade breakdown? (For example, some tests I was giving were tough! If a student was answering half of those tough questions correctly, I felt like they were doing pretty well. However, numerically, they failed the test.)
- How do I take Depths of Knowledge and make them more math-friendly?

Converting your classroom into a leveled classroom may sound daunting, and a lot of my recommendations for the order may appear counter-intuitive, but these steps are ordered in a way that you will be able to make the most important conversions first, then other changes will fall into place naturally as the system takes place.

Phase I – Establish your levels

Step 1: Establish levels at a course-level

Step 2: Establish levels at a unit-level

Step 3: Establish levels at a lesson-level

Phase II – Implementing your levels

Step 1: Level your summative tests

Step 2: Create quizzes

Step 3: Level your homework/practice assignments

Step 4: Have students set academic goals

Step 5: Begin leveling your in-class presentations

Step 6: Leveled station activities

Phase III – The Extra Mile

- Create online answer keys and provide individualized leveled at-home support
- Honors Integration

<u>Phase I – Establish Your Levels</u>

One major issue that I encountered during my first years of teaching is that I felt as though a student's understanding of various concepts did not match up with their grade, often for the worse. The sense of "failure" that my students were encountering was understandably demotivating. The first step in creating a leveled classroom is to define clear expectations for yourself and for each student. You will need to take the time to answer the following three questions:

1) What should a C student be able to do in your classroom?

- This should be the bare minimum that *all* of your students would need to know in order to be successful next year.

2) What should a B student be able to do in my classroom?

- This is what you would like the *majority* of your students to be able to do when they leave your classroom.

3) What should an A student be able to do in my classroom?

- This should describe what your brightest students are able to do that sets them apart from the

assessed prior to college enrollment, and too many students enter college lacking basic math skills (National Center for Education Statistics, 2013; National Science Board, 2006).

During the fall of 2014, we conducted a research study to investigate factors that influenced success in developmental math at USU (Bagley, 2015). We collected data from students enrolled in all sections of Beginning (n = 376), Intermediate (n = 932), and College Algebra (i.e. Math 990, 1010, and 1050) at both the beginning and end of the semester. Here, we discuss findings Beginning and Intermediate Algebra from just one of the measures used in that study: a one-minute timed single-digit (0-9) multiplication facts test. Because calculators are not allowed on exams in developmental math courses, we believed multiplication facts fluency would be an important basic math skill, and as such, a predictor of outcomes in these courses. Our research did not investigate students' strategies or conceptual understanding of basic multiplication.

Figure 1 shows bars indicating the number of correct multiplication facts completed (within one minute out of 100 facts) at the beginning of the year, with bars shaded to indicate the overall percent correct. In general, students with fewer correct also had a greater number of errors, which supported our concern about multiplication fluency. Indeed, beginning of year multiplication fluency (i.e., number of correct or incorrect) was a predictor of course outcomes in all three courses. In fact, for each incorrect item at the beginning of the semester, final grades in Beginning Algebra (Math 990) decreased, on average, 1.5 out of 100 percentage points. Figure 2 shows average final exam scores for percentage of correct multiplication facts (i.e., accuracy) on the end of semester timed multiplication facts assessment.

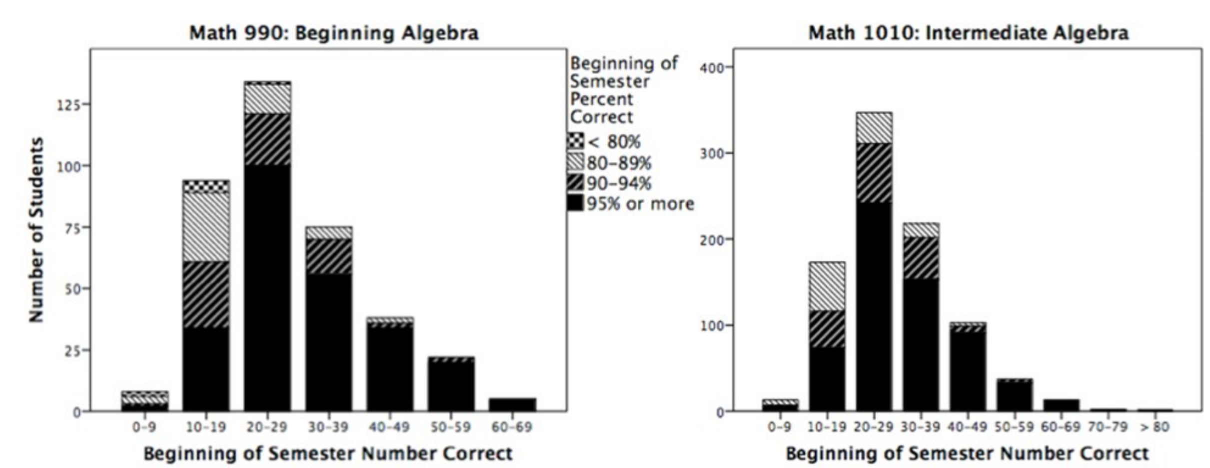


Figure 1: Number and percent of correct multiplication facts at beginning of semester

awareness in early literacy. If so, then math facts may be more like letter naming—a good predictor of later skills but not necessarily one of the crucial building blocks for later mathematical proficiency.

On the other hand, cognitive psychologists have long argued that the development of higher-level skills requires that lower level skills be developed to automaticity. According to Hasselbring, Goin, and Bransford (1988), "The ability to succeed in higher-order skills appears to be directly related to the efficiency at which lower-order processes are executed." Furthermore, Whitehurst (2003) stated, "Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become so routine and over-learned that they become automatic." Ball, Ferinin-Mundy, Kilpatrick, Milgram, Schmid, and Schaar, (2005) state that fluency requires "automatic recall" of basic number facts but refrain from identifying what constitutes automaticity. To investigate the impact of automaticity on mathematical processing, Price, Mazzocco, and Ansari (2013) conducted research using brain imaging and concluded that the region of the brain activated during single digit arithmetic predicted high school math scores on a standardized assessment. For less proficient students, the neuroimaging showed greatly increased activity in the regions of the brain associated with numerical processing, indicating they could be applying procedural strategies to determine answers, while more proficient students activated areas of the brain associated with fact retrieval, suggesting faster response times and less energy required to reach solutions.

This research indicates that without multiplication fact fluency, in which students recall quickly, accurately, and confidently the results of single digit multiplication, students are likely to struggle with new concepts because simple multiplication disrupts problem-solving and uses cognitive processes needed for understanding more complex concepts. Much like needing to know which sound the letter "a" makes while reading the words cat, make, tall, sea, and coat before one can read fluently and accurately, students need to be able to quickly recall multiplication facts to, for example, determine whether fractions are equivalent or identify factors of integers—both building blocks for algebraic operations. Yet, while the Dynamic Indicators of Basic Early Literacy Skills (DIBELS) have helped us define fluency and accuracy cutoffs for early reading skills, it is more difficult to find guidance for what constitutes fluency and accuracy with basic math operations. Unfortunately, automaticity with basic math facts is rarely

rest of their classmates.

Step 1: Establish Levels on a Course-Level - You will need to answer the questions listed above at a course-level so that you can again answer those questions for each unit within your course, and again for each lesson within each unit. Answers will be dependent on subject, grade level, and your personal view of teaching. Having said that, being able to answer these questions is fundamental to being successful as you level your classroom. Here are *my* answers for my Secondary I classroom:

- What should a C student be able to do in my classroom?

  Level 1: A C student must know the basic vocabulary associated with my learning objectives for the year.

  Level 2: A C student in my classroom can perform basic, computations. "Basic" meaning they are uncomplicated by extra steps or barriers that can be created by certain types of numbers (i.e. non-integer) or operations within a given problem.

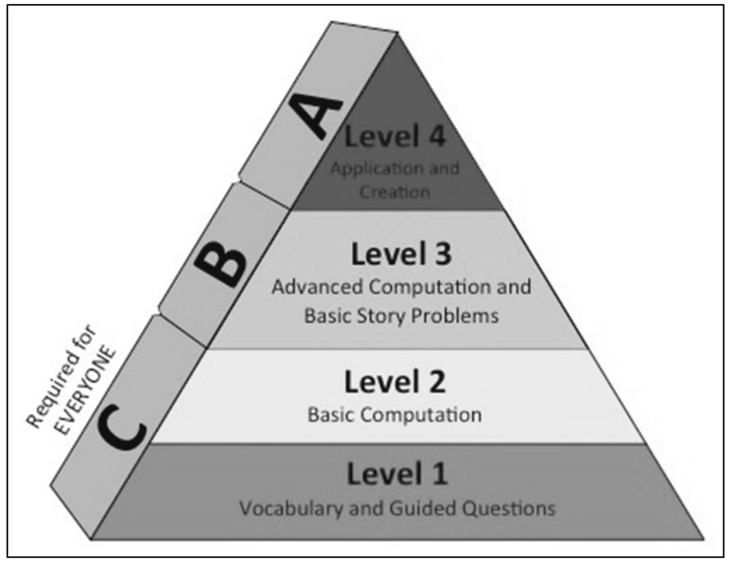
- What should a B student be able to do in my classroom?

  Level 3: In addition to the requirements for a C student, a B student in my class room can also perform *any* computation within the given unit. They can navigate extra steps and non-integer numbers found in the questions, encountered while solving, or in their final answers. They will also encounter non-numeric answers to seemingly numeric questions. (ie "No solution" or "Infinite solutions.")

- What should an A student be able to do in my classroom?

  Level 4: An A student in my classroom can also perform any computation given to them, and be able to answer application and creation-type problems that demonstrates a deeper understanding of the content. This is in addition to the C and B student require ments.

The diagram on the following page visually summarizes how I see my levels playing out in my classroom. I have this diagram displayed in the front of my room for students to see and refer to regularly.



Step 2: Establish Levels on a Unit-Level - Once these questions have been answered at the general-classroom level, you will need to answer these questions for each unit. I will use my "Systems of Equations" unit to continue with my example.

- What should a C student be able to do during the Systems of Equations Unit?

  Level 1: A C student needs to know that a system of equations is two or more equations grouped together in such a way that your goal is to find a solution that works for ALL of the equations within that system.

  Level 2: A C student will be able to solve any system of equations that is clearly set up to be solved by graphing, substitution, or elimination. These systems will have integer-based ordered pair answers. These students will also need to know the vocabulary associated with this unit.

- What should a B student be able to do during the Systems of Equations Unit?

  Level 3: A B student will also be able to solve a single system in multiple ways. They can manipulate one or both equations within a system as necessary. They may encounter and understand systems of equations that have infinite or no solutions.

- What should an A student be able to do during the Systems of Equations Unit?

  Level 4: An A student in my classroom can also solve any system of equations given to them. They can also create a system of equations that would yield a specific answer and can also set up and solve a system of equations from a story problem.

Why Should Students Know Basic Math Facts? Because Multiplication Facts Skills Predict Grades in College Math Courses

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Jason Bagley, M.S. — Utah State University
Christina M. Watts, M.S. - Mathematics Education doctoral student, Utah State University

We can't tell you how many times we've heard, "What do you mean I can't use a calculator in this class?" while teaching developmental (previously called remedial) math classes at Utah State University (USU). Unfortunately for students, most of our higher education campuses in Utah do not allow calculators in developmental math courses—those courses below College Algebra (Math 1050). Yet, in Utah and nationwide, about 70% of students enrolling in college are required to start with developmental math courses, and pass rates in these courses are abysmal--estimated to be about 50-60% (Cutler, 2009; Twigg, 2007). Sadly, only 1 in 4 students who take developmental math courses graduates from college (Bailey, 2009). While many factors lead to this high failure rate, ensuring success with basic math facts may be one important first-step to success.

The Utah Core Standards for Mathematics state that by the end of third grade, students should be able to "fluently multiply and divide within 100" and "know from memory all products of two one-digit numbers" (Utah Core Standards for Mathematics, 3rd Grade, Operations and Algebraic Thinking, 3.OA.7). However, the Standards do not define "fluently" nor explain what it means to "know from memory." Kling and Bay-Williams (2014, 2015) suggest that four "tenets of fluency," specifically flexibility, appropriate strategy use, efficiency and accuracy, are important to consider in determining math fact fluency. Baroody (2006) describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills" and suggests that basic math facts skills are "not merely a collection of isolated or discrete facts but rather a web of richly interconnected ideas." Gersten and Chard (2006) add to this by claiming basic math facts need to be built on a foundation of number sense to ensure young students are competent in mathematics. They also claim that number sense may be a correlate to phonemic

If we do this, our students will find meaning in learning mathematics. As educators, we can help our students to develop these good habits through the Teaching Practices (establish goals, implement tasks that promote reasoning and problem solving, use mathematical representations, facilitate discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle, and elicit and use evidence of student thinking). Finally, we must all advocate for knocking down the obstacles (myths) that prevent students from being successful.

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Part 3: Establish Levels on a Lesson-Level - Finally, these questions need to be re-answered for each lesson or concept within your unit. I found this is best done with a general answer as well as specific sample problems. This is important because, for example, a Level 2 solving systems of equations by *substitution* will look very different than a Level 2 solving systems of equations by *elimination*. The samples below are how I would answer this question for my "Solving Systems of Equations with Substitution" sub-unit.

- What systems of equations should a C student be able to solve using substitution?

  Level 1: A student should understand that "Substitution" is a method by which a studentcan solve a system of equations. Substitution is best used when one of the varia bles is "known". This level will include a guided system in which one variable is already solved for.

  Level 2: These systems of equations will have an isolated variable in *at least* one of the equations. Performing the substitution should take place in the first step. Answers will be integer-based ordered pairs.

  Examples and Solutions:

  $$\begin{cases} y = x - 1 \\ x + y = 3 \end{cases} = (2,1) \qquad \begin{cases} x = y - 7 \\ x = 2 - 8y \end{cases} = (-6,1)$$

- What systems of equations should a B student be able to solve using substitution?

  Level 3: These systems of equations will not have an isolated variable in either of the equations, or will have non-integer solutions including fractions-based ordered pairs, "no solution" and "infinite solutions" answers.

  Examples:

  $$\begin{cases} y = 2x - 2 \\ y = \frac{-1}{3}x + 5 \end{cases} = (3,4) \qquad \begin{cases} y = x - 1 \\ -x + y = 4 \end{cases} = \text{No Solution}$$

  $$\begin{cases} x + 2y = 13 \\ 3x - 5y = 6 \end{cases} = (7,3) \qquad \begin{cases} y = 3x - 7 \\ y = 5\,x + 2 \end{cases} = \left(-\frac{9}{2}, -\frac{41}{2}\right)$$

- What systems of equations should an A student be able to solve using substitution?

  Level 4: These systems of equations will come from a story problem, or students may be asked to create their own system of equations.

  Examples:

  You have 12 coins made up of quarters and dimes. The value of your coins total

$1.95. How many of each coin do you have?

Create a system of equations for which (-3, 4) is the only solution. Instead of solving your system of equations to check your answer, explain how else might I check to make sure that your system of equations has that solution.

Phase II – Integrate the Levels Into Your Classroom

It took me one intense year (planning and preparation during the summer, then tweaking and detail work throughout the school year) to fully convert my classroom into a leveled classroom. The refining process continues, but the toughest part seems to be behind me. The full conversion can be a difficult and time-consuming task that many people would say they don't have time for. This is understandable, and I will attempt to help you prioritize the steps so that the conversion can happen in parts that will work for you in your situation. I will also describe the many different ways my classroom has been impacted by levels. Some of these may not work for you and your situation or philosophy, but many of them can be adapted to fit your style and classroom.

Step 1: Level Your Summative Tests

You will want to begin by creating your unit summative tests. I have found that these are the things that most impact students and are the most out of sync with the ideas of leveled learning. Tests should be written very carefully with the levels in mind. Every test needs to be compiled so that 80% of the questions are Levels 1 and 2 type questions. 10% of the questions are Level 3 and 10% are Level 4 questions. I write my tests to be 20 questions each. Applying the percentage breakdown, this means that my tests have 16 level 1 and 2 questions, 2 level 3 questions, and 2 level 4 questions. This method makes it so that a C student who truly tries to master level 1 and 2 questions have the very realistic option to earn an 80% (the lowest B-) on that test. One or two mistakes will keep them in the C range. Similarly B students have a real shot at their B, and A students who took the time to master level 4 type questions should be capable of achieving their A without too many obstacles, but they will certainly be required to stretch and work.

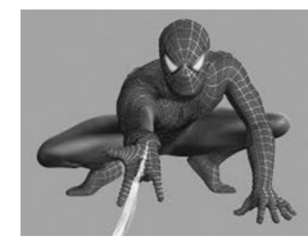
Leveling your tests first will be the least-time consuming and most impactful step you could take. As I compared my old tests with how I had defined my levels I was shocked at how difficult my tests were. I estimate that most of my tests were composed of an average of 30%

•The teacher needs to engage students in tasks that promote reasoning, and should facilitate discourse that supports shared understanding (Myth: Teachers need to tell students the definitions, formulas, and rules)

•The student needs to use varied strategies and representations, justify solutions, make connections to prior knowledge, and consider the reasoning of others (Myth: The student needs to memorize information and use it to solve problems)

•The teacher provides an environment with appropriate challenge, encourages perseverance, and supports productive struggle in learning mathematics (Myth: The teacher makes mathematics easy for the students by guiding them step by step to ensure they are not frustrated or confused)

Using the Webshooter to Collect Ideas on How to Help Students Develop the Process Standards



Resources for Teachers. The internet is replete with ideas that can help teachers develop good math practice, and we are encouraged to spend time looking for resources that will be beneficial. Here are just two resources for teachers:

Math Practice Standards

http://www.insidemathematics.org/common-core-resources/mathematical-practice-standards/standard-2-reason-abstractly-quantitatively

Real World Application Sites

http://gettingsmart.com/2013/12/4-tools-connect-students-real-world-math/

Conclusion

We are engaged in a noble profession. How we approach math will have long-lasting effects on our students. If we can really listen to our students, and discover what is involved in math proficiency we can meet our students' needs, and become their superheroes. We need to concentrate on the Process Standards, and teach our students to develop these good habits (make sense and persevere, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning).

Teaching Practice	Description
Establish mathematics goals to focus learning	Establishes clear goals, situates goals in learning progressions, uses goals to guide instructional decisions
Implement tasks that promote reasoning and problem solving	Engages students in solving and discussing tasks and allows multiple entry points and varied solution strategies
Use and connect mathematical representations	Engages students in making connections to deepen understanding of concepts, procedures, and tools
Facilitate meaningful mathematical discourse	Facilitates discourse by analyzing and comparing student approaches and arguments
Pose purposeful questions	Uses purposeful questions to assess and advance students' reasoning and sense making
Build procedural fluency from conceptual understanding	Builds fluency with procedures on a foundation of conceptual understanding so that over time they become skillful in using procedures flexibly
Support productive struggle in learning mathematics	Consistently provides students with opportunities and supports to engage in productive struggle
Elicit and use evidence of student thinking	Uses evidence of student thinking to progress toward understanding and adjusts instruction continually in ways that support and extend

Using the Batarang to Knock Down the Myths about Teaching and Learning Mathematics

Myths about Teaching and Learning Mathematics. Many parents and educators believe that teaching should be the same as when they were taught – and this should be done through memorizing facts, formulas, procedures, and through drill and kill practice. This can be unproductive and may hinder the implementation of effective practice. Further, it can limit student access to content and practices. It is important that we have productive beliefs about teaching and learning mathematics. Below is a list of mathematical productive beliefs - and the myths they break (Leinwand, S., Brahier, D., Huinker, D., et. al., 2014):

• Mathematics learning should develop understanding of concepts and procedures through problem solving, reasoning, and discourse (Myth: Learning should focus on practicing procedures and memorizing facts)

• Students need to have a range of strategies and approaches to choose from, including general methods and algorithms (Myth: Students need to use the same algorithm and methods)

level 1 and 2 questions, 40% level 3 questions, and 30% level 4 questions. It was no surprise that students were struggling so much to pass my exams! I wanted to challenge my students, but I found I was mostly creating a sense of failure and frustration more so than a healthy challenge or basis for judging understanding.

In my class, tests may be retaken after students complete an additional leveled practice assignment (a study guide of sorts). Students must complete a certain amount of questions that vary depending on their goal. So, students who are trying to pass the test (with a C) do the Level 1 and 2 portions of the practice paper, students trying to get a B also do Level 3, and those students really trying to get an A must complete the entire practice assignment. When I feel they have finished their practice test well enough, and I have checked their work, then they may retake the test.

Step 2: Create Quizzes

Quizzes are a way for me to quickly figure out who needs the most help to understand the very basics. For this reason my quizzes are generally 5 or 6 questions, all of which are levels 1 and 2, and these scores are put in the "homework" section of their grade as opposed to the "test" section. When you write your quizzes remember to keep them simple! Again, you are just trying to figure out who has a very basic understanding and who does not, so that you can begin remediation for those who have fallen behind.

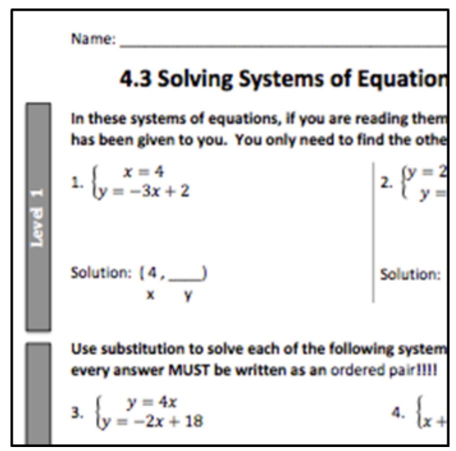
Students who fail the quizzes are generally the same students who haven't done the homework associated with that quiz, or did them very poorly. For that reason I will only allow a student to retake a quiz after they have completed the associated homework assignments and turned them in, OR if they had already completed the assignment they may retake once they have fixed any level 1 and 2 questions that were previously marked wrong. Generally the work that goes into qualifying for a retake helps their grade and understanding much more than the actual quiz retake will. (Shhhh, don't tell them that!) Of course that system only works when the homework has been leveled as well (as described in step 4) however, a similar principle would apply for non-leveled assignments.

Step 3: Level Your Homework/Practice Assignments

This part is the most exciting part for students. They see this as you giving them an option to only do "half of the homework." I have found my students are usually more willing to do work if they think they only have to do half of it. And for me, half of the work is better than none! However, be careful, students will get increasingly lazy. For this reason I require that EVERY level 1 and 2 question be complete. If that is not the case I simply mark the homework as "Incomplete" and hand it back to them.

Each homework assignment I give has clearly defined and designated levels. I have simple text boxes along the left hand side of each assignment so that students can easily see which questions they are responsible for mastering. I also grade these 10 point assignments in a way that will allow their score to align with the district-set grading scale.

- 2 points for "completion" – meaning every level 1 and 2 question is complete. Any assignment turned in where this is not the case is simply returned to the student and marked "incomplete."
- 2 points for completing the assignment on time.
- 4 points for 4 randomly selected levels 1 and 2 questions (mostly level 2) at one point each.
- 1 point for a randomly selected level 3 question.
- 1 point for a randomly selected level 4 question.


Name: ______

4.3 Solving Systems of Equation

Level 1

In these systems of equations, if you are reading them
has been given to you. You only need to find the othe

1. $\begin{cases} x = 4 \\ y = -3x + 2 \end{cases}$

Solution: (4, ___)
x y

2.

Solution:

Use substitution to solve each of the following system
every answer MUST be written as an ordered pair!!!!

3. $\begin{cases} y = 4x \\ y = -2x + 18 \end{cases}$

4.

Grading in this way allows a student who completes the first 2 levels (with a goal of a C) to earn a score up to 80%. 3 levels completed would earn up to 90%, and only students who complete all 4 levels have the possibility of earning 100% on any given assignment. This sets a bar for your high achievers, while also saying to the struggling students "It's okay if you can't answer these really tough questions, let's focus on mastering these level 1 and 2 questions." I will also add that by grading in this way I am able to grade a full class of assignments during the 5 minutes I give them for their self starter. Add a teacher's aide to record those scores and grading has become quick and easy.

Step 4: Have Students Set Academic Goals

At the beginning of each term I have students fill out a basic form in which they circle their grade-goal for the term. (Either A, B, or C). I encourage them to be realistic when setting their goal by having them consider how they did in the year/term before. I suggest taking their last grade and setting a goal of ONE step higher. Last year's A students should consider being

Process Standard	What Does it Involve?	What does it look like?
Making Sense of Problems and Persevere in Solving Them	**Conjecture** = conclude even though some information is missing	Have students try to solve a problem before the solution method is known
Reason Abstractly and Quantitatively	**Decontextualize** = remove from context **Manipulate** = try different symbols **Contextualize** = put back in context	Have students draw/diagram first, manipulate as needed, and then apply it abstractly
Construct Viable Arguments and Critique Reasoning of Others	**Justify** = defend or uphold **Refute** = prove to be false **Explain** = to make plain or clear	Have students either defend their answer, prove an answer is false, or explain their answer
Model with Mathematics	**Represent** = to stand for a term, symbol **Formulate** = to devise or develop	Have students use objects or pictures to display the problem
Use Appropriate Tools Strategically	Use language, materials, and symbols to **record** and **communicate** their methods	Have younger students use concrete manipulatives - as they get older the students should be able to move away from these
Attend to Precision	Understand and be able to communicate with precise language using definitions, **formulating** explanations, **making** and **refining** conjectures, and **constructing** and **critiquing** mathematical arguments	Have students explore the definitions of mathematical terms - and then describe them precisely
Look for and Make Use of Structure	Using mathematical structure to **recognize** pattern recognition and **doing** pattern-generalizing	Have students find patterns through doing several like problems and then apply that new knowledge to other problems to test the theory
Look for and Express Regularity in Repeated Reasoning	**Extending** reasoning and making broad claims from a particular instance	Have students recognize a pattern and make and test a claim based on the recognition

reasoning – students ask themselves, “Does my answer make sense?” In order for students to develop a Productive Disposition, the teacher needs to believe and model that math is worthwhile and useful, that steady effort pays off, and that they are effective at learning and doing math.

Using X-ray Vision to See Math through the Process Standards and Teaching Practices

<u>Process Standards.</u> The National Council of Teachers of Mathematics have addressed the Common Core State Standards and Mathematic Practices in the book titled, “Connecting the NCTM Process Standards and the CCSSM Practices”. These process standards describe what it means to do mathematics, and should be integrated into classroom instruction. This is important since mathematics content is achieved through the mathematical practice and process. (C. Koeslter, M. Felton, K. Bieda, S. Otten, 2013) These process standards are: Make Sense of Problems and Persevere in Solving Them, Reason Abstractly and Quantitatively, Construct Viable Arguments and Critique the Reasoning of Others, Model with Mathematics, Use Appropriate Tools Strategically, Attend to Precision, Look for and Make Use of Structure, and Look for and Express Regularity in Repeated Reasoning. As educators, we need to plan our lessons with these process standards in mind. Do your lessons incorporate these verbs? (See Page 52)

<u>Teaching Practices.</u> There are eight Mathematics Teaching Practices that are used as a framework for strengthening teaching of mathematics, which when employed in the classroom will strengthen mathematics learning. They are as follows: Establish mathematics goals to focus learning, Implement tasks that promote reasoning and problem solving, Use and connect mathematical representations, Facilitate meaningful mathematical discourse, Pose purposeful questions, Build procedural fluency from conceptual understanding, Support productive struggle in learning mathematics, and Elicit and use evidence of student thinking. This framework goes well with the process standards, and provides teachers with a framework of how they should think about teaching mathematics in their classrooms. (Leinwand, S., Brahier, D., Huinker, D., et. al., 2014)

on the honors track.

Students then list one *specific* thing they will do differently this term to help them reach their goal. “Try harder” is not an acceptable answer to me. I make sure they set a realistic and measurable goal that I can check up on in the future. I collect these papers and record them in my gradebook so that parents can be made aware of the goals set and this can be a focal point for conversations with students and their parents. Once students have set their goals, and they are collected, I show my students the triangle diagram and let them know that their job is to master the levels designated by their goal. So, a student with a goal of a B must become master at levels 1, 2, and 3 questions. They are always welcome to attempt level 4, but their job is to master the first 3.

Step 5: Begin Leveling Your In-Class Presentations

Every question I pose to my students can be categorized into one of my 4 levels, and those levels are color coded. I color code the slides of my presentation to match the level of the question. (The colors also match those in the pyramid, and the pyramid is displayed in the front of my classroom to serve as a reminder of what the colors represent.) In a way this creates a gamification of each lesson. Students will often beg me to “go to the next level!” because they want a bigger challenge. Occasionally I oblige them, other times I can sense that some students need more time on a specific level and will stay on that level until I feel the majority of students are ready to move on. Either way, the excitement to try something more difficult is definitely a fun sight for me.

This year I have begun to show multiple questions of different levels on the same slide so that students can choose which question they would like to attempt. In this very simple way struggling students aren’t overwhelmed, and the advanced students can still be challenged. I go over both kinds of questions while everyone is listening so that the basics (level 2) are still being covered for struggling level 3 students, and level 2 students who don’t feel confident enough to try level 3 at that moment, may feel more confident trying them on their homework.

Step 6: Leveled Station Activities

I have spent the last couple of years trying to find the secrets of creating effective station activities. (That’s a different article for a different day!) One of the keys to station-success in my classroom has been the ability to have stations color-coded by level. Some days I have stations where each station contains multiple levels, and groups have students of varying abilities to assist one another. Level 4 students can still answer a Level 4 question, and they are available to assist a student who might be struggling with a level 2 question.

Other days I have activities where each *station* is leveled and students are allowed to choose which level they go to according to their level of understanding. This allows each groupof students to struggle and help each other at their own level and also advance to a different level once they are feeling more confident, or move down a level if they are feeling overwhelmed.

Phase III – The Extra Mile

Answer Keys and Video Support

Without a textbook for parents to help the way that they used to, I found that I needed to provide support for my students while they were at home. On my classroom website I do a (nearly) daily blog for each assignment I give. In that blog I provide a partial answer key as well as a self-created video where I walk through how I got every answer provided on the key.

The "partial" answer key is more specific than it sounds. In it I provide leveled-support. Essentially I give *every* answer to the level 1 questions, *half* of the answers to level 2 questions, one or two answers to level 3 questions, and I do not give any answers to level 4 questions. In this way struggling students can receive a lot of support and level 4 students are forced into developing deeper understandings as they struggle with their questions.

For those of you readers concerned about students taking advantage and cheating, be forewarned: THEY WILL! However, I never grade questions that I have provided answers for, and it is often easy to spot students abusing the system (especially when one or two clever "mistakes" make their way onto the key…)

Due to low-traffic on my webpage (compared to what there should be for students who are struggling) this year I have included a QR code on the bottom of each assignment that, when scanned, pulls up my walk-through videos on YouTube.

Honors Integration

This year our school is attempting to integrate honors students into regular classes as opposed to having their own separate class. I will hold onto the details of this integration until after we find out if it was successful or not, but essentially honors students have a packet of "Level 5" questions as well as questions that cover the core-specified honors content for each unit done in class. Implementation of this has certainly brought its struggles, but has also

Know what leads to mathematical proficiency. Do your math lessons include activities that lead to Mathematical proficiency? Learning mathematics involves the inclusion of five interrelated strands that make up mathematical proficiency, which are Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. (National Research Council, 2001) Research supports the characterization that learning math is an active process. (Donovan and Bransford 2005) In the table below are the following verbs: connecting, using, formulating, representing, solving, thinking, justifying, perceiving, believing, and seeing. Do your math lessons include activities that use these verbs?

Standards of Proficiency	**What does it involve?**	**What does it look like?**
Conceptual Understanding	**Connecting** Concepts, Operations and Relations	Drawing, Diagraming, Using Manipulatives
Procedural Fluency	**Using** Procedures to Solve Problems	Procedural Checklists
Strategic Competence	**Formulating, Representing, Solving** Mathematical Problems	Know, Know, Operation, Solve it, and Check It
Adaptive Reasoning	**Thinking** Logically and **Justifying** Thinking	Does it make sense?
Productive Disposition	**Perceiving** Mathematics as Useful and Worthwhile, **Believing** that Steady Effort Pays Off, **Seeing** One-self as an Effective Learning and Doer of Mathematics	Teacher Models this Behavior

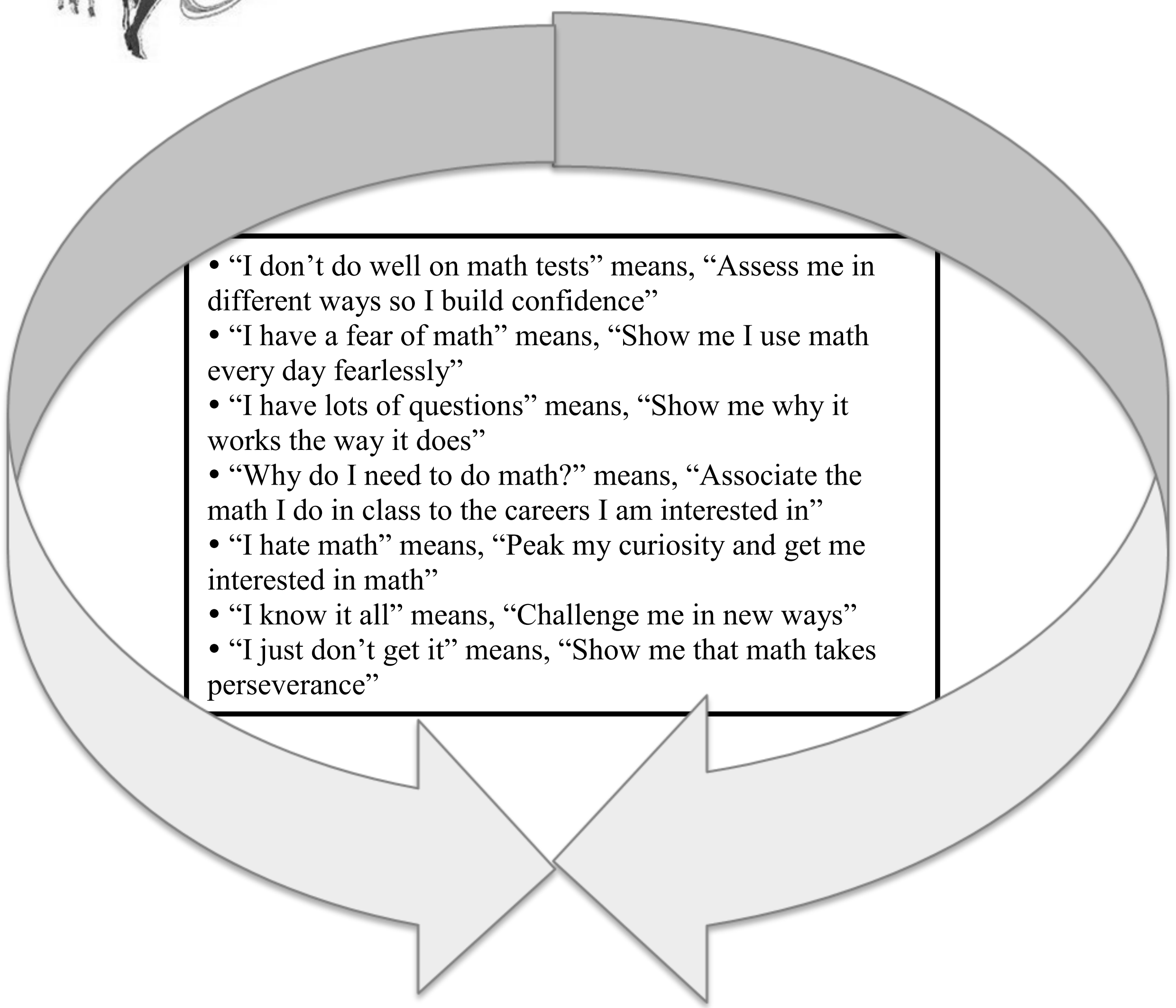
Strategies that lead to proficiency. When we are trying to develop conceptual understanding we provide students with the opportunities to draw, diagram, and use manipulatives. Procedural fluency has recently come under attack, however, we are finding that students need both the understanding of concepts as well as procedures – a simple strategy is to have your students help develop and use procedural checklists, which lay out the procedures step by step. Strategic Competence can be achieved through a simple confidence building strategy called "Know, Operation, Solve It, and Check It". Students first wrap their minds about what is known from a problem – this can include factual information as well as what is being asked of them. From there, students will determine what operation is needed to solve the problem. After solving the problem they need to go back and check the answer, which is a form of adaptive

50% Math Teacher, 50% Superhero—Becoming a Superhero to Your Math Students

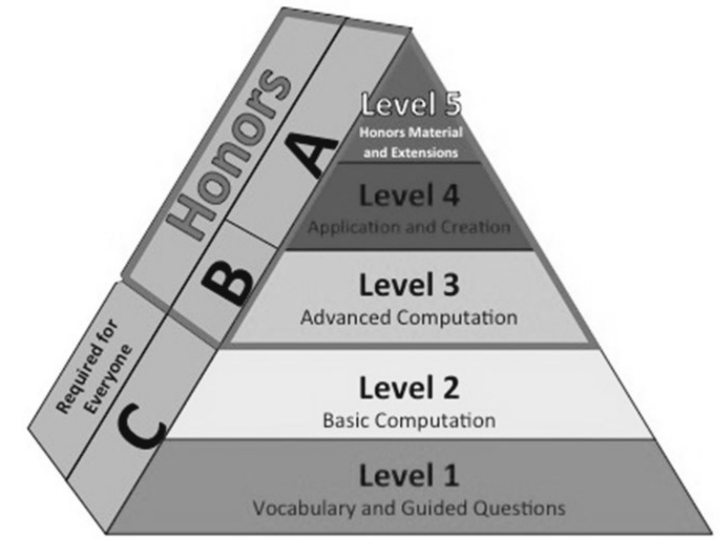
Leilani Nautu — Ph.D., Director K12 Programs, Southern Utah University

Using the Lasso of Truth to Uncover What Helps Math Students be Successful

Know your students. How well do you know your students and how they feel about math? The Lasso of Truth can



helped me to be even more careful about creating an environment where everybody can succeed and learn and grow at their own level of understanding.



The Results and Conclusion

It is difficult to gauge just how successful creating a leveled classroom has been. I don't believe that simply judging success based on SAGE results is the best method to determine success, but we did see a fair increase there. The success is feeling more individual than that. For the most part I am seeing struggling students really trying. My students have set goals and they work hard to accomplish those goals. They know exactly what they need to do, and what kinds of questions they are in charge of mastering, and for the most part they hold themselves accountable for those questions.

My students have a focus that creates mastery of certain subjects. In the past I would catch myself forcing deeper questions on students who were not prepared for those kinds of questions. On many occasions I would witness students as they worked to understand more difficult questions and that new information forced upon them would get jumbled in their mind to the point that they couldn't perform basic computations any more. This is no longer an issue. I can ask students what their goal is and work with them on mastering those kinds of questions. More often than not I am seeing students come to me with a C or D average from the year before, and leave my classroom with a solid B grade, and a solid B understanding of the material. Similarly, traditionally B students are pushing themselves to reach the A they have never had before. And, for myself, that is always a treat to witness.

More than anything, I can tell you that leveling my classroom has been the healthiest exercise for myself as a teacher. It has made me more careful about what I put on assessments (formative and summative) so that my assessments are more consistent. Similarly, homework has become a very good and predictable gauge of how students perform on tests. I am careful to make sure that my lessons have a nice flow, evoke participation and interaction from everyone. And, finally, because of how everything is structured and works together, I truly believe that my students grades are a much more accurate indication of their understanding than they were before, allowing me to know how to better allocate my time and modify my lessons for the following year. My hope is that you have found something in this article that you can implement that will also better your practice as a diverse teacher building a differentiated classroom.

as successful on this concept as others, because it can be confusing. When I saw this particular students performance on the test, I was very curious as to why she hadn't performed better. The results went deeper than I initially believed. What I though the issue would be was that she got confused between when to divide and multiply. Based on our conversation, her confusion was deeper and more complicated. This was an interesting exercise in taking the time to truly understand where a student's misconceptions lie."

One of the most telling examples came from a teacher who shared a story about a student who was able to answer division questions when they were represented as $25 \div 5$, but missed the exact same problems when represented as $5\overline{)25}$. Prior to this assignment, she hadn't focused on this student since it appeared that he had a firm grasp of the concept, but she was concerned that his scores weren't indicative of his conceptual understanding.

When she met with the student and began asking him about the problems, the student began to cry. He explained that he was trying as hard as he could and that every night he did his homework with his father. They would skip these $5\overline{)25}$ problems because his father said they weren't possible, "You can't divide 5 by 25." He became increasingly sad and frustrated that he kept being asked to do something that his father said wasn't possible. After investigating further, the teacher found out that where his father came from, division problems were represented slightly different from the format shown in American schools and textbooks. After clarifying the presentation and the meaning, the student went on to relate the two formats and understand that they both represented 25 divided by 5.

So, what do these examples tell us about how we proceed with interventions? While, initially, taking the time to meet with students who are struggling may seem like an impossibility, the time may actually be very well spent. So often we meet with students and continue to force feed them more problems, manipulatives, and algorithms, when we might be better served by having them do the talking. We have seen the benefits of incorporating activities such as number talks into our instruction, shouldn't we use the same thinking for intervention? Students need multiple opportunities to communicate their mathematical thinking and develop their ability to use precise mathematical language. It is through these conversations that we can truly see and hear what they do and do not understand. It is then and only then that we can provide the appropriate intervention that will allow the student to make progress.

Me:"So the computer told you?"
Sam:"yeah"
Me:" If you had 32 kids in your class, and you needed to put the desks in 4 rows , how many desks would be in each row?"
Sam:"32?
Me:" How do you know that? Do you want to draw that? "
I reexplained the problem, and drew it.
Sam:" 8? Because 8x4=32. 8 people and 4 and then equals, ….I think it is correct…..
Like I just counted 4, no like, (pause he drew 4x4) I counted 1,2,3,4, 1,2,3,4, Now there are 15. It is too less I think. I got it wrong. Cause 4+4=8?"

They continued to dialogue about what the program was asking and the meaning of division and multiplication. In her summary, she concluded that the student had no idea about the meaning of the facts for which he was being rewarded "fruits and coins".

In another example, the teacher began the process with a student based on a recent topic test. Her initial dialogue with the student was focused on surfacing the meaning of the question and how to use conversions to solve problems. The following is the teacher's reflections on the dialogue process.

"And so the conversation continued with trying to understand how she manipulated the answer choices to figure out the conversion. Based on the rest of the conversation, I came to understand that the student just did not have a strong understanding of these things:

1. What the relationship between inches and feet is;
2. When to divide;
3. How to divide and what the result represents.

What surprised me most in this instance was her inability to divide correctly. She was not putting the quotient in the correct place and as a result was confused about what to do in the problem.

These questions I came away with:

1. How could we have come this far in the math curriculum without students having mastered division?
2. How can I improve my teaching of this concept, which is confusing. How can I make this more accessible to students?.
3. How can I help students who can master division facts, carry this skill over to longer division?

This was eye opening to me. There are students in the class that I thought would not have been

Bridging Research and Practice: Growing Greenwood Elementary Teachers' Mathematics Pedagogical Content Knowledge Through Action Research in the Classroom: The Beginning

Emma P. Bullock — Utah State University
Jessie Kidd, Tess O'Driscoll, Alyson Reid—Greenwood Charter School

Introduction

Often mathematics is a source of anxiety for both elementary students and their teachers (Hembree, 1990; Ma, 1999). Frequently, this is due to the lack of pedagogical content knowledge (PCK) (Shulman, 1986) of elementary teachers, who trained as generalists, have not delved deeply into the wonder and power of the patterns found in arithmetic, algebraic reasoning, and geometry (Evans, 2013; Isiksal & Cakiroglu, 2011; Lamon, 2012; Mizala, Martínez, & Martínez, 2015; Newton, Leonard, Evans, & Eastburn, 2012). Without this knowledge, teachers often fall back on the procedural knowledge they remember as students instead of exploring the rich conceptual relationships that lead to the love of mathematics and the mathematical sciences (Evans, 2013; Isiksal & Cakiroglu, 2011; Lamon, 2012). In *Principles to Actions* (2014), NCTM proposed a series of research-based actions for all teachers. However, without effective, situated, longitudinal professional development, this call to action may largely fail in implementation (Desimone, 2009; Doerr, Goldsmith, & Lewis, 2010; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hochberg & Desimone, 2010; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). Out of this desire to implement reform practices in meaningful ways that would positively impact student mathematics learning, the Growing Greenwood Elementary Teachers' Mathematics Pedagogical Content Knowledge Through Action Research in the Classroom Project (hereafter, Growing Greenwood mathematics project) was born. The purpose of this paper is to share the beginnings of this project in the hope that others may see what is possible in practice as educators strive to improve student mathematics learning.

The Seed

The seed for the Growing Greenwood mathematics project began in December 2014 when Jessie Kidd, the principal/executive director at Greenwood Charter School contacted

Emma Bullock at Utah State University. Greenwood charter school was slated to open in August 2015. Jessie was previously the principal of a different school and, like many school leaders before her, wondered if there was a better way to approach mathematics education for both students and teachers. Emma had previously been a mathematics teacher, specialist, mentor of new teachers, principal, and was currently working on her Ph.D in Curriculum and Instruction with a concentration in Mathematics Education and Leadership.

In March 2015, the two met over lunch to discuss Jessie's ideas of partnering with higher education to provide support, as called for in Principles to Actions (NCTM, 2014), for her incoming elementary teachers. In particular, Jessie envisioned a way of bridging the gap between research and practice in a way that did not overwhelm her mostly novice teacher core but still allowed all students to have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources to maximize their learning potential. The two decided to move forward with plans to create such a support with a team of researchers and graduate students at Utah State tailored to the unique needs of Greenwood's situation with the further vision of recruiting other schools/teachers that could also benefit from such an approach. The focus would be on increasing elementary teachers' mathematics PCK through action research in the classroom. The approach would be rigorously studied as to its effectiveness and would include a close collaboration between researchers and classroom teachers/school leaders.

The plan evolved into a three-year longitudinal professional development project that would combine online coursework with on-site professional development/support in lesson study and action research. In addition, continuous support would be available to grade level teams throughout the school year via email, Skype, and phone.

Preparing the Soil

To prepare the soil, Jessie worked with her school's stakeholders to choose a set of curriculum resources that would meet both student and teacher needs. After the decision was made to purchase a set of curriculum materials, Emma and Jessie worked to secure a minimum of funding so that teachers would be able to take the online coursework envisioned and members of the research team could provide the continuous support desired.

Using Student Interviews to Provide Specific, Targeted Math Interventions

Patricia French — Sunrise Elementary, Canyons School District

With the recent focus on assessment, diagnosis, and intervention, teachers are inundated with data. In addition, time restrictions and demands for growth and improved scores can create undue levels of stress for both teachers and students. So how do we determine exactly what is preventing students from understanding concepts and being able to articulate their understanding. Sometimes it may have nothing at all to do with the math!

Let's begin by looking at an example of a case study completed during one of the math endorsement classes. The participant met with the classroom teacher and together they identified a student who was not making progress on the district computational screener. Math facts were not a part of core instruction so the intervention consisted of a designated time for the student to practice the facts using the computer software program, Reflex Math. Based on reviews of the program, students who spent an average of 20 minutes/5 times per week, would progress towards fact mastery within a few months. The teacher expressed frustration that this student wasn't making progress on the progress monitoring prompts. So the process began.

The participant observed the student as he worked through the program. She met with the student afterwards and recorded the following dialogue. (Note: this is just a portion of the conversation.)

Me:" I have never seen anyone work with that program before. Can you tell me about it?"
Sam:" There is a rabbit, and it was going up to get fruit and coins. Like, if I go high, like I might get a coin or something, and then I get an island pass. "
Me: " Okay, so you said that you get fruit, coins, and a pass, but you don't really have anything, so why do you do Reflex Math?"
Sam:"So I can be smart? I do multiplication and division because it helps you a lot. If you don't go to school you will be dumb. My brother got in trouble"
Me:" Does math make sense to you? It's hard for me "
Sam:" Its really hard for me too. It's kinda like you have to subtract and stuff. "
Me:" One of the problems you worked on today was 32/4=8. What does that mean? Can you describe that to me?"
Sam:" Um. (Pause) I don't know.
Me:" What if you had 4x8=32? What does that mean? "
Sam:" It's the answer. 32.
Me:" How do you know that?
Sam:" Because I timesed. I got the answer.

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Emma worked with the research team and Jessie to refine their research questions and the essential details of the professional development plan. In this way, the professional development plan could be rigorously assessed as the plan was implemented and further participants could be recruited. The research questions and sub-questions guiding this study currently are:

1. How does on-site and on-line mathematics professional development support the development of teachers' mathematics PCK during a three-year longitudinal action research project?
   A. How does the project support teacher knowledge of the mathematics education research about children's mathematical thinking and learning?
   B. What is the evidence that teachers are applying the knowledge in the literature to their classroom practices?
   C. What projects do teachers conduct to better understand and instruct students in response to gaps in the research literature?
   D. What is the evidence of the dissemination of the results that the teachers gained from their action research projects?
2. What is the relationship between increases in a teacher's PCK and student mathematics learning?

In total, the envisioned project would take place over three school years with a preliminary pilot period. The first year would focus on the development of teachers' knowledge in the area of Numbers & Operations. The second year would focus on the development of teachers' knowledge in the area of Rational Numbers & Proportional Reasoning. The third year would focus on the development of teachers' knowledge in the area of Geometry & Measurement.

In addition, the equivalent of eleven days spread throughout each of the three years would be devoted to professional development that focuses on collaboration, planning, lesson study, and observations of each other's classroom lessons. The online and on-site team study of these mathematical strands would include action research projects in which the teams would work to answer emergent pedagogical questions (especially with respect to underserved populations such as English Language Learners, low socioeconomic, and Special Education). The results would be shared as broadly as possible through publications and conference presentations.

Planting the Seed

The Greenwood seed was planted in August of 2015. Emma met with the teachers of Greenwood Charter school to provide the first three of the 11 days of on-site professional development. Day 1 focused on the overview of the Growing Greenwood mathematics project and the alignment of their curriculum resources to the Utah State core mathematics standards. At the beginning of the day, teachers were given composition books where they could write any questions they might have over the course of the three days. Then working standard by standard, teachers spent most of the day in grade level teams understanding how the standards were supported through their curriculum resources and how they may need to supplement these resources in appropriate ways.

The morning of Day 2 focused on the Comprehensive Mathematics Instruction (CMI) Framework (Hendrickson, Hilton, & Bahr, 2008) so that teachers would have a common language to use in lesson planning. Furthermore, teachers were introduced to the concepts of mathematical discourse, the differences between conceptual and procedural knowledge, and the ideas of concrete, pictorial, and symbolic mathematical representations. Again, the purpose was not to delve deeply at this time, but to develop a common vocabulary so that teachers would be able to delve more deeply in the future as the project continued. The afternoon of Day 2 gave teachers time to develop 180-day plans based on the Utah State Standards while they continued to delve into the resources that would be available to help them with their classroom instruction. Teachers were also able to watch demonstration lessons, which illustrated the CMI framework, the use of mathematical discourse, the building of conceptual knowledge, and appropriate mathematical representations.

Day 3 was designed to allow teachers to continue their work in grade level teams. This process included work on the first two weeks of lesson plans while discussing practical topics such as typical daily schedules, classroom management, assessment, and grading. At the end of each day, teachers were asked to fill out exit cards describing what they felt they had learned that day and what questions they still had. In addition, teachers were asked what questions had they written in their journals that they were wanting to research more.

Nourishing the Seed

With the seed planted, the research team worked with Jessie to nourish the seed. Emma met with two of the Greenwood teachers, co-authors of this paper, to begin writing about this process. In this way, teachers would begin to see success in sharing their journey through publication and conference presentation opportunities. In addition, teachers emailed Emma with questions and members of the research team would provide summaries of the research literature addressing these questions. Some of the early questions focused around young children's learning trajectories, teaching mathematics in classrooms with emerging readers, and classroom management in the mathematics classroom.

Moreover, Jessie planned further professional development days for observation, support, and follow-up by the USU research team. Days in September, October, and November were scheduled to reinforce the appropriate use of curriculum materials among grade level teams to teach the Utah State standards and to develop the foundations of Lesson Study and Action Research with the faculty. Rather than scheduling full days, the members of the USU research team would come for shorter times over several days in order to work more organically with the teachers, observe classroom instruction, and provide support during team meetings or individually, as needed.

Moving forward, the faculty and staff of Greenwood, along with researchers at USU, are seeking to include other schools and teachers throughout the state of Utah as part of a collaboration towards increasing PCK in mathematics. The team continues to pursue various funding options so teachers can take online coursework towards earning their elementary mathematics endorsement over the next three years. In addition, teachers are beginning to engage in lesson study as a way to prepare for action research projects over the coming years.