Utah Mathematics Teacher
Fall/Winter, 2017-2018
Volume 10

Utah Council of Teachers of Mathematics
j = 1950
UCTM

http:/utahctm.org

UTAH MATHEMATICS TEACHER

Volume 10, Fall/Winter 2017-2018

Table of Contents

UCTM President's Message

Karen Feld, President, UCTM

Many of you know that one thing UCTM has been focusing on is allowing our students to have access to high quality mathematics instruction and to improve equity in the classroom. Many of you have already taken the Access and Equity course that has been provided through the USBE. At the NCTM conference this past year, Matt Larsen, the NCTM president, spoke of access and equity, but he explained that the NCTM board wanted to focus more on just allowing access and equity to increase in our mathematics classrooms, they also wanted to increase empowerment for students. Thus, access and equity has now been re-framed as access, equity, and empowerment.

This 2017 UCTM conference is a tribute to the need for access, equity, and empowerment in our classrooms. As a UCTM board, we believe that all students can learn mathematics. To do this, we need to team up as general educators and special educators to determine the best ways to make this possible. As we work together, we can help each other understand how students learn and what resources need to be available to help every student succeed. The need for access, equity, and empowerment in our classrooms is essential to help our students learn mathematics. This conference will be a great way for you to deepen your understanding of how to help all students learn mathematics, and how to work with other general educators and special educators to let this happen.

One resource that I refer back to time and time again is NCTM's publication of *Principles to Actions.* This book has helped me to improve my understanding of access and equity, and has also helped me as a middle school math teacher to understand things I want to improve on, such as improving my questioning strategies in the classroom. If you haven't yet read this book, I would encourage you to do so. It would be a great book to read as a collaborative book study with your colleagues in your school, district, or others across the state. Along with this book, NCTM is putting out books called *Taking Action with Principles to Actions.* These books will focus on implementing the effective mathematics teaching practices found in *Principles to Actions.* These will be a great resource to help us know what the teaching practices look like in a mathematics classroom and how to become a more effective mathematics teacher.

My hope is that we continue to learn best practices for mathematics instruction together. I hope that we will continue to learn what is best for kids, what will help them to succeed in learning mathematics, and best practices that increase our ability to teach them. I hope that we will learn how to rely on each other to improve our teaching and allow access, equity, and empowerment to be present in our classrooms, schools, districts, and our state. Thank you for all you do for mathematics education. It is a privilege to learn along with you and to learn from you.

Letter from the Editor

Christine Walker, Utah Valley University

The beginning of the school year often triggers an ingrained instinct to buckle down and get to work. As we embark on a new school year, this is a good time to re-focus on our goals. A new academic year is often filled with anticipation, excitement, anxiety and relief; however, it is also a time of renewal and a fresh start. We hope this journal aids you in your fresh start this year.

Our theme for the 2017 UCTM conference and journal is *Maximize Mathematics Learning for All Students*. This year, we had the unique opportunity to collaborate with the Utah State Board of Education SSIP implementation team to discuss how to help all students access and master grade-appropriate mathematics. How do we do that? More importantly, what can we do as teachers, staff, parents, community members, and other stakeholders to lead to improvement of educational outcomes? The position of the National Council of Teachers of Mathematics, in *Access and Equity in Mathematics Education*, is that "[t]o close existing learning gaps, educators at all levels must work to achieve equity with respect to student learning outcomes. A firm commitment to this work requires that all educators operate on the belief that all students can learn."

Our lead article this year comes from Adam King titled "Three R's for #ChangingMathAttitudes." The article asserts that in order to truly change math attitudes, we all need to build three R's (relationships, relevancy, and resiliency) in our family, our students, our community and even ourselves regarding mathematics. It is a must-read for anyone who deals with, and wants to improve, deep-seated negative attitudes towards math.

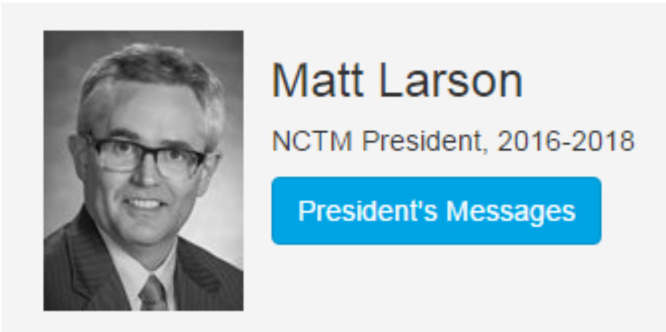
Moreover, in "I Think I Can, Therefore I Can: Developing Positive Cycles of Disposition," the authors conclude that as teachers, we have this unique opportunity to build an environment in our classrooms that promotes a positive mindset, encourages risk-taking, values students' strengths, and develops mathematical knowledge through play. In the brilliant words of Theodore Roosevelt, "Believe you can and you're halfway there." Mathematical knowledge through play is often achieved through engaging tasks and "NCTM (2014) explains that the regular use of high level tasks that promote reasoning and problem solving is a keystone to creating a classroom where students have opportunity to engage in high level thinking." "Pythagorean Triple Threat" provides one such high level engaging task that allows students to explore some basic geometrical properties of Pythagorean triples, while trying to discover the shortest path between two places on a map.

Have you heard of invisible mathematics? Nor had I, but I strongly encourage you to consider that for many students, invisible mathematics can act as an obstacle for progression. As the author of "Invisible Mathematics" suggests, unless we address this,

invisible mathematics will be a barrier for students' sense making of mathematics and perseverance in problem solving. Quoting the author, "If we all own the problem, then we can all be part of the solution."
We close this journal with two key articles that are a must-read. Going Back to School: Lessons Learned by a University Professor in a High School Classroom highlights the insights gained from a university professor who successfully taught mathematically underprepared students. And finally, in Targeted Math Intervention (TMI): An Intervention Program for Struggling Secondary Students introduces a research-based program that can be implemented across the state of Utah that has already shown increased levels of student growth. Considering the timing of this journal's release, these articles are apropos. We hope you enjoy this journal and always, please consider submitting your own articles, or serving as a reviewer for future journal articles.

Finally, a very sincere thanks to Steve Jackson who did the production in preparation for a print and an online publication.

Note: Any mistakes are the sole responsibility of the editor and will be remedied in the online journal. Please send corrections to Christine.walker@uvu.edu.



Letter from NCTM President, Math Education *Is* STEM Education!

Matt Larson, NCTM President

May 17, 2017

What design principles would you include to ensure that an effective STEM (science, technology, engineering, and mathematics) program builds mathematics understanding?

I ask because I was recently asked to be part of a discussion on "Design Principles for Effective STEM Programs that Build Mathematics Understanding." My argument is that there is only one fundamental and critical design principle necessary to make certain that a STEM program builds mathematics understanding. I wonder if we agree.

I address the STEM question with reluctance. Our past three NCTM presidents have written messages, published articles, testified on Capitol Hill, or presented on the topic of STEM education. In addition, our NCTM teacher journals have published numerous articles and have produced focus issues related to STEM education. STEM is frequently a program strand at the NCTM Annual Meeting or Regional Conferences. The "STEM ground" would seem to have been well covered by NCTM.

Despite all these efforts, the questions concerning STEM and the requests to speak and address STEM education just keep coming. It is clear that resolution on how STEM education fits with our goals for mathematics education still lacks clarity in the minds of many.

STEM education is a focus of many policy makers, business and industry leaders, philanthropic foundations, and education leaders because the data indicate there will be accelerated growth in the number of STEM jobs the economy will generate over the next decade, particularly compared to other professions (see, for example, STEM 101: Intro to tomorrow's jobs). Additional data indicate beginning salaries and salary growth for STEM majors will outpace those for other majors and careers.

Let me make one thing abundantly clear: I support STEM education—including science, technology, and engineering. But I support STEM education, as Michael Shaughnessy wrote, from the perspective of "political advocacy." As mathematics educators, it is incumbent on us to be advocates for STEM education because advocacy for STEM education is advocacy for mathematics education.

Among other STEM related recommendations, NCTM's 2017 Legislative Platform, specifically advocates for “adequate investments in the programs authorized by ESSA that serve as the basis of federal support for local education, including specific programs for STEM (science, technology, engineering, and mathematics) education and STEM subjects.”

However, as we look beyond advocacy, one significant challenge associated with STEM education is how it is defined and implemented in districts, schools, and classrooms. There is no universally agreed upon definition of what constitutes STEM education. This complicates matters and allows each entity to define STEM education in its own way to fit its experiences, biases, and agendas—NCTM included. In some cases this leads to math or science classrooms where students build bridges or program robots, but fail to acquire a deep understanding of grade level (or beyond) math or science learning standards.

Could K–12 math classrooms fail to have students engaged and learning the mathematics content and practices necessary to advance in the curriculum, but have integrated some technology, engineering, coding activities, or connections to science and be called a “STEM Program”? If students are not equipped to pursue a post-secondary STEM major and career, is it really an effective K–12 STEM program? My answer is no. No number of fun activities or shiny technology will overcome this fatal shortcoming.

Levi Patrick, chair of NCTM’s Professional Development Services Committee, pointed me in the direction of Rodger Bybee’s recent book, *The Case for STEM Education: Challenges and Opportunities* (NSTA 2013). Bybee is a respected science and STEM educator, and in this book he argues that the “purpose of STEM education is to develop the content and practices that characterize the respective STEM disciplines” (p. 4). Under this definition a highly effective K–12 mathematics program, built upon what we know constitutes the elements of effective mathematics programs, is an effective STEM program.

Of course, the problem with Bybee’s purpose of STEM education is that it isn’t consistent with the definition and vision many others have of STEM programs. Many individuals, particularly those outside of mathematics education, when they think of STEM education, focus specifically on curriculum integration, technology integration, and critical-thinking skills.

NCTM certainly supports curricular connections, appropriate technology integration, and critical thinking, but not at the exclusion of mathematics learning. Appropriate integration of technology in support of mathematics learning goals as well as the need to make curricular connections, both within mathematics and to contexts outside of mathematics, have been guiding principles since *Principles and Standards for School Mathematics* (NCTM 2000) and were reinforced in *Principles to Actions* (NCTM 2014).

The mathematical practices outlined in the standards of many states and *Common Core State Standards for Mathematics* have much in common with the scientific and engineering practices of *Next Generation Science Standards*. Both sets of practices emphasize the importance of understanding problems, developing and using models to solve problems, constructing viable arguments based on evidence, and critiquing the reasoning of others. When we engage students in the standards for mathematical practice, we are making connections to and supporting science education. Implementation of the recommendations in *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME; [SIAM 2016]) provide yet another opportunity for mathematics teachers to make meaningful connections to science (and other disciplines) in support of STEM educational goals while maintaining the integrity of mathematics learning standards.

Maintaining the integrity of the mathematics learning standards is our responsibility as mathematics educators. For example, I frequently hear someone state, "I need a STEM program that teaches algebra." I would argue a high quality algebra course already is a STEM program. The request for a "STEM program that teaches algebra" is driven by the belief that integration is the defining characteristic of a STEM program. Instead, I believe the more appropriate request would be to seek a high quality algebra program that supports STEM through its connections to appropriate applications and integration of technology.

If in the "STEM program" the mathematics isn't on grade level, or if the mathematics isn't addressed conceptually but rather as a procedural tool to solve various disjointed applications, or if the mathematics is not developed within a coherent mathematical learning progression, then the "STEM program" fails the fundamental design principle.

The attention mathematics education gets from STEM is primarily positive. But we need to keep in mind that there are also downsides. The possibility that we might neglect the full development of students' mathematical understanding in order to integrate STEM "activities" into an already overpacked curriculum is real. In addition, STEM education narrowly emphasizes learning mathematics for the workplace and for the scientific and technical communities.

We must always keep in mind that we also teach mathematics for social justice. We teach to empower students in their personal lives. Mathematics is an important part of cultural heritage, including an understanding of the multiple contributions various cultures have made to mathematics. These purposes for teaching and learning mathematics must remain part of our curriculum during an era that emphasizes STEM preparation.

The mathematics design principle of an effective STEM program that builds mathematics understanding is just that: it is a program designed to develop the content and practices that characterize effective mathematics programs while maintaining the integrity of the mathematics. Other design principles, for example, curricular connections and the appropriate integration of technology, are merely vehicles to ensure students learn important mathematics at a deep level and are confident in their ability to use mathematics to be empowered in their own lives.

If we fail to support each and every student in developing a positive mathematics identity, a high sense of agency, and a deep understanding of mathematics, then we will have failed our students, denied them future opportunities, and ultimately failed to build the mathematical foundation necessary for the STEM outcomes that policy makers envision.

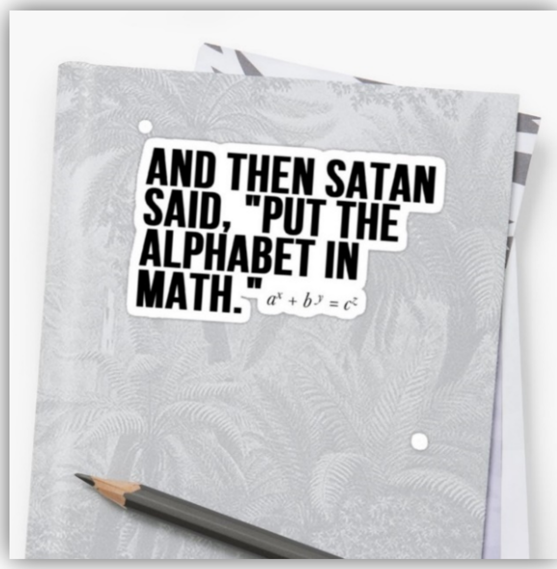
While it is true that advocacy for STEM education is advocacy for mathematics education, it is equally true that advocacy for mathematics education is advocacy for STEM education. As you receive pressure to "STEM-up" your classroom, I urge you to keep this fundamental and critical design principle in mind.

I encourage you to post a response to this message and share your challenges and successes related to STEM initiatives in your district with the mathematics education community.

Three R's for #ChangingMathAttitudes

Adam Dwight King
Davis School District

One day in 6th grade, I was sitting at church in Sunday School of all places, where something happened that shook me to my very core. For some random reason, the small talk turned to math, an older boy wrote a math problem – with the letter "X" – on the board.



A LETTER.

I was shocked… I thought I was very good at math, but this was beyond me. I asked what the letter meant, and was told it could mean anything. The older boys responded by writing more problems with more letters on the board. It was too much for my concrete mind. I didn't get a straight answer, so I quit pushing. But I was scared to death of this new concept called "algebra."

Fast forward to the first day of 7th grade. I got my schedule and went to my first math class: algebra. Wait… I had signed up for pre-algebra! What was going on here? I cowered in the corner away from all the 8th and 9th graders who obviously deserved to be in the class. Why did I, a lowly 7th grader, get placed there? I spoke to the teacher afterwards, and she said it was likely based on my test scores along with a recommendation from my 6th grade teacher.

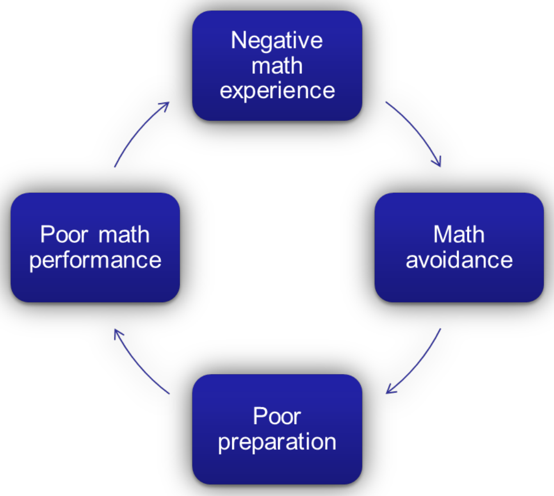
It didn't matter. I was convinced it wasn't the place for me. I went to the counselor's office and changed my schedule back to pre-algebra, where I thought I should have been in the first place. I was much more comfortable the rest of the year. I learned quickly and continued on as a great math student. I even learned about how letters are an important part of math (and no, they were not put there by Satan!). In fact, I was great at math for many more years, clear into high school. I worked hard and learned well, and when there was a challenge, I overcame it. I tutored other students, took honors and Advanced Placement classes, and even ended up with college credit!

However, that one choice in 7th grade came back to haunt me. Because I had backed up that one class, I put myself on a track that didn't allow me to take the higher math class in 12th grade that I really wanted and needed. But I had no way of knowing then. All I cared about was my fear.

How could that one conversation give me so much math anxiety, despite my years of confidence and success? It was so uncomfortable and debilitating. I hadn't experienced it before and didn't know how to handle it then. It created a crossroads in my life and I wish I would have chosen differently.

This is #MyMathStory. Yes, even I have experienced math anxiety.

I quickly fell into the math anxiety cycle and took my turn going around. One negative math experience immediately led to avoidance. Fortunately, I got off pretty quickly. Not everyone does. Some people have been going around in circles for most of their lives.



We could spend days describing all the problems in society surrounding math: test scores, changing standards, parent attitudes, job readiness, pop culture attitudes, and even the proverbial "that's not the way I was taught!" You have seen it all already. There is so much to overcome! It is my belief that ***math skills (and scores!) will not improve until math attitudes do***, and that this applies to both students and adults. So, instead of admiring the problem, let's look at some solutions. Let's discuss some "power" tools we each have to make a difference in our area of control.

In order to truly change math attitudes, we need to build three R's: relationships, relevancy, and resiliency.

<u>Relationships</u>

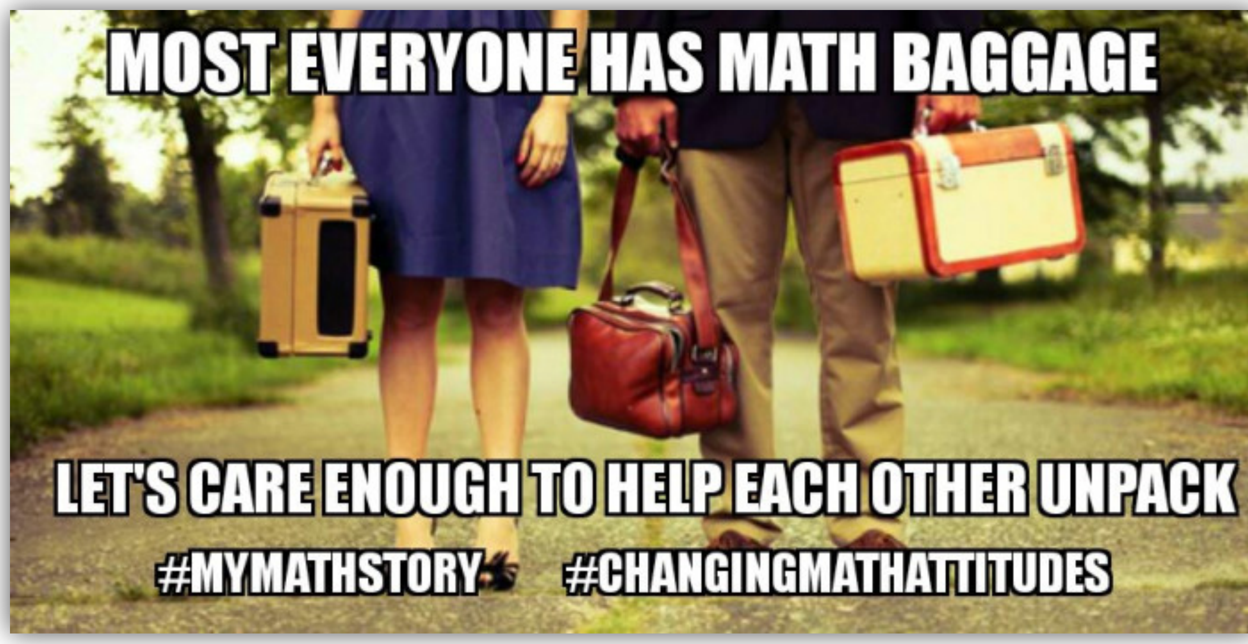
Who can forget the YouTube video sensation of Travis and Chelsea Chambers, the traveling couple from Logan, Utah? They took the world by storm in 2011 with a four-minute viral video[1] where Travis provided some "affectionate teasing" to his wife as she struggled to answer how long it would take them to drive 80 miles while travelling 80 mph. She actually employed some pretty decent background knowledge by comparing her own running pace, the weight of different vehicles, the speed of the tires, and the unit rate of one mile per minute. But she couldn't get the conceptual or procedural answer right. Here's some math for you: almost 12 MILLION people have watched the video and laughed at her as she struggled. That's a similar quantity to popular music videos and new movie trailers! Twelve million people tuned in to see how foolish she looked! Their comments seemed to be split between derision towards Chelsea for her lack of math skills, and amazement that Travis could put his wife through that. Quite a few of them wondered if the couple's relationship was ruined.

Travis and Chelsea Chambers, YouTube

[1] https://www.youtube.com/watch?v=Qhm7-LEBznk&t=1s

Travis and Chelsea's antics seemed to have started a new trend – making fun of people about math. Thank goodness for social media, which makes this process so much more efficient! In 2016, another video popped up, this one with Brad in his British accent filming his girlfriend Jen as she struggled to decide how many pieces of pizza she could eat.[2] He asked if she would want her large Hawaiian pizza cut in to 8 or 12 pieces, and she chose 8 because she didn't think she could eat 12. He tried repeatedly to help her understand that it was the same amount of pizza no matter how many slices you cut it into, laughing the entire time.

So what of these videos? What do they reveal about our culture? Some people have wondered if they are faked. Even if that were the case, it seems that how they are received is more telling than how they were made. Based on my experience over the years with math attitudes, it is likely that a good portion of the people laughing at these struggling YouTubers actually struggle themselves. We have all seen these widespread negative math experiences and how profoundly they can impact a person. I call these negative experiences "math baggage."



I think most everyone has some math baggage. I do! And I think that is why our culture has such a problem… because of that baggage, it becomes easier to downplay our good math abilities or cover/excuse our low math abilities. We hide our math anxieties with uneasy humor and direct it on other people, thankful to take the pressure off ourselves.

If we are going to change math attitudes, we need to build better, safer relationships. We need to be brave enough to unpack our math baggage and help others do the same. When we are open and honest with ourselves and others, we will find more commonalities and less reason to hide behind a fixed mindset.[3] In fact, Brené Brown would

---

[2] https://www.youtube.com/watch?v=Fkqg6HE888A. (Language advisory)
[3] Dweck, C. (2006). *Mindset: The new psychology of success*. New York: Random House.

tell us that such vulnerability can lead to true self-improvement and greater opportunities.[4] Even math teachers have math baggage, and sharing it doesn't lessen our authority, it helps us be more relatable.

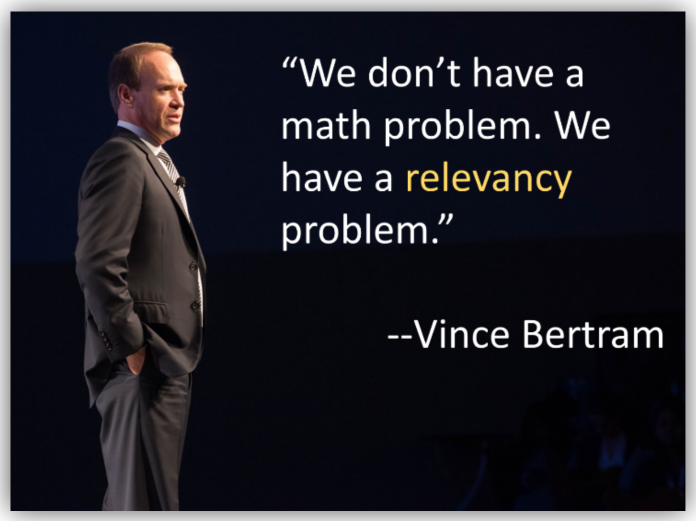
Remember, *supporting productive struggle* is one of the eight Mathematics Teaching Practices[5] espoused by The National Council of Teacher of Mathematics. Making fun of someone while they struggle is not going to help them learn and succeed, and will damage a relationship beyond repair. Furthermore, we can't build relationships that promote encouragement and willingness to struggle if we laugh at videos like these in front of the students (even if it isn't directed specifically at them). They will worry we will do the same to them, and continue to hide their needs from us. Also, we cannot reinforce the dichotomy that someone either gets math or they don't. The elitist math attitude is just as damaging as the defeatist math attitude, and they play off of each other.

We build safe relationships by truly listening and understanding. We build relationships by being consistent and kind. We build relationships by being careful with humor. And we build relationships through praise and appreciation. Doing this one person at a time will help us change the negative math attitudes and cultural problems evident in the YouTube trends above.

There is a happy ending for the Chambers family: In a follow up interview, Chelsea noted that she was initially shocked and then mad about the video, but then realized that she and her husband's relationship was deeper than math. Even with all the jokes about him being in the doghouse for his efforts, they seem to be doing well now, and she is working with her daughter to have improved math skills and attitude. By owning her personal struggles, she can form a safe and caring relationship and be a true math mentor.

<u>Relevancy</u>

Vince Bertram, president of Project Lead the Way, tells a story about his three-year-old son. Like many of us educators have experienced, he worried almost fanatically about skills of his own child. They had been working on subtraction, and the boy had struggled with the type of problem: "If you had nine pencils, and I took away four, how many would you have left?" Vince asked his son a similar question as they sat around the dinner table one night hoping for a breakthrough, and the boy responded, "But Dad, I only need one pencil!" Later that evening, they watched sports together on television and Vince was amazed to see how his son's demeanor changed. He was quoting statistics and comparing player averages



[4] https://www.ted.com/talks/brene_brown_on_vulnerability

[5] (2014). *Principles to Actions: Ensuring mathematical success for all*. Reston, VA: NCTM, National Council of Teachers of Mathematics.

enthusiastically and effortlessly! Vince thought, "He is great at math. He loves math. He just doesn't care about pencils!"

I once had a junior high math teacher describe to me a damaging experience from one of her classes. Like each of you, she has worked hard all year to teach the skills and help the students develop an appreciation of math. To that end, she asked one of the counselors to come in and present to the class about future careers. Unfortunately, he undid all her efforts with one statement about how they wouldn't need much math in their lives (accompanied by the typical disdain and math joke). How sad! This is the battle we fight though; where one detrimental statement can be more reinforcing than dozens of successes and evidences to the contrary.

My brother hated math in school, and it didn't like him either. He was smarter than he gave himself credit for, but he squeaked by with lots of humor, low effort, and little understanding. As many students do, he chose a career that he assumed had little math; in this case, construction. Boy, was he wrong! Fortunately, he had a wise foreman and mentor who showed him how math applied to everyday situations: how much fractions and decimals mattered in measurement, how you could use the Pythagorean Theorem to make sure a wall was at a 90-degree angle, and how to use sine, cosine, and tangent to figure out obscure angles and distances. We have had many a discussion (usually while working on a building or repair project, come to think of it!) about the procedural versus the conceptual aspects of math. He is fully aware that he did not understand math until he needed it; he just wishes that it was taught to him in school with more practical application and relevancy. He didn't do well with strict memorization of steps without the context that he employs every day now.

Last year as we were hanging crown molding in his kitchen, we used a compound miter saw to cut angles in three dimensions. It's not as straight-forward as you would think! Even with an online program to help simplify the calculation part, we still needed to have a visual and conceptual understanding of how it would all fit together. We definitely used the old adage "measure twice, cut once" as our mantra! Just the other day, we were at my house again replacing a rusty old swamp cooler with a more efficient attic fan. I smiled with pride as I watched him quickly use math to measure, cut and fold the new hardware to help me improve my house (and hopefully save some money!). Thank goodness for construction workers who appreciate and understand math!

Math on my roof!

One math attitude that needs to change is the idea that someone will never use math. Another is the idea that someone needs to know it all, including calculus and trigonometry. Both are damaging, and we need to find some middle ground. Not every person will need it all, but we shouldn't allow our students to be limited because they think they won't ever use it. Careers in STEM fields are growing exponentially faster than non-STEM, and many of the jobs our students will have one day haven't even been invented yet! When the proverbial "I'm never going to use this" comes up, tell them you will do a little research and share. Or better yet, involve all the students in a challenge to discover for themselves.

One thing is for sure: doing math "because I said so" is NOT inspiring in the slightest. We can do better than that! Vince Bertram realized that much of our cultural math problem stems from relevancy… or lack of it. He noted "There is nothing we teach that doesn't have application in the real world. We just need to show them how to make that connection."[6] His story above has a happy ending: his son is now studying business and technology at Purdue University, and you can bet that there is plenty of math involved.

Resiliency

Once upon a time, a young girl was great at math. She was in gifted and accelerated classes, loved to learn, and very confident in her abilities. Then something changed. One day in a junior high math class, she didn't understand a concept. This was a very rare thing for her; she didn't know how to be wrong or fail. After some deliberation, she finally got up the courage to ask her teacher for help. His response was that she was just a "stupid girl" and would "never be good at math."

Ouch.

And she was done. Even after years of success and confidence, this one experience ruined her with math. Completely. It absolutely stunted her math growth for the rest of her schooling and beyond. Twenty years later, she still admittedly suffers from math anxiety.

Last year at the UCTM conference, I told that story during my #ChangingMathAttitudes breakout. One participant seemed moved and raised her hand to comment. She noted that she had a similar experience, also in junior high, also with a male teacher. However, when she was told that she could never do math, something swelled inside her and she decided to prove him wrong and be awesome at math. She grew up to be a math teacher, and I am sure her experience made her much more compassionate and understanding of her students and their unique needs.

Both of the male teachers listed above had a number of math attitudes that needed changing. It's very apparent that neither of them applied the first R of "Relationships." Someone in a position of mentorship should never say such debilitating things, even as a joke. But there is something deeper here. How is it that the same experience could damage one person so deeply and empower the other one?

The answer is resiliency.

Resilience is the capacity to recover quickly from difficulties. This is huge in today's world, in all areas of life. It seems that someone is always getting offended or giving up on something because it is too hard, and that everyone is a victim to something nefarious. We need more resiliency to snap back from setbacks. In fact, it is often those setbacks that make us stronger.

This is especially true for math. Lack of math resiliency dooms us to repeated trips around the anxiety cycle, continuously getting worse with each rotation. However, if students are developing

[6] Bertram, V. (2017). Presentation to principals in Davis School District.

resilience, having a negative math experience doesn't automatically start them on the merry-go-round. In fact, it often propels them to greater understanding and a sense of accomplishment. Take that, math avoidance!

For example, on one occasion last autumn I observed an amazing math lesson about dividing fractions. The students each did their own visual model on paper, and the teacher went around and took a few pictures of their work to put on the board digitally. Each student came up to the front when their problem was on the board to explain their answer and reasoning about how they solved it. The first student got the answer right and had modeled it effectively. The class cheered for a job well done, and she sat down. Then the second student came up.

His answer was wrong.

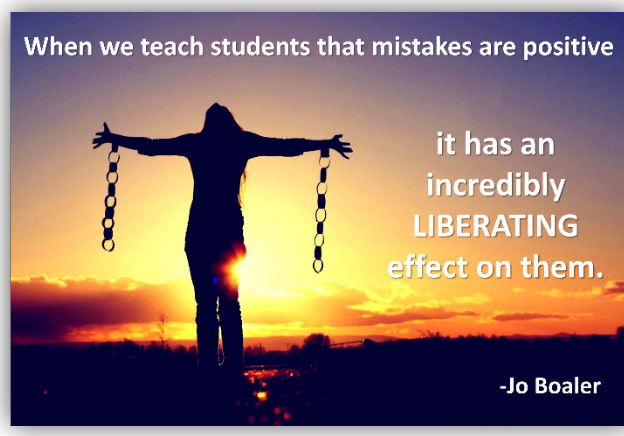
He explained his reasoning, which was fairly sound and mostly correct. In fact, he had just done one part backwards. When the mistake became apparent, the teacher praised the student for his effort and for the things he had done well. Perhaps most importantly, she praised him for sharing his mistake with the class so they could all learn from it and improve. I thought, "what a supportive teacher, helping him to feel better about what he did wrong so it didn't ruin him." I was honestly worried that it would, especially in front of the whole class. But what happened next floored me.

The teacher asked the class to cheer for the student and his mistake.

And they did. Genuinely. Yes, they cheered him on for making a mistake. And it was a positive experience. He displayed resilience.

I am sure this classroom culture didn't happen overnight. I am sure it took repeated practice and over-the-top reinforcement to change those attitudes. I am also sure that the teacher chose this student to come up on purpose because she noticed the mistake. It was all worth the effort. This amazing teacher knew that to develop resilience in these students, she had to build a classroom where struggling and mistakes are just as important as the final answer.

Changing how students view mistakes is one of the best ways to foster resilience. Indeed, the idea that mistakes are bad and should be punished is one of the most detrimental math attitudes. Jo Boaler noted: "When we teach students that mistakes are positive, it has an incredibly liberating effect on them."[7] We can do that through group and individual messages about brain growth, appropriate grading surrounding homework, and letting the students see our own mistakes and thought-process.



What a difference this would have made with the girl in the story above who was ruined through one mistake! Imagine the path she could be

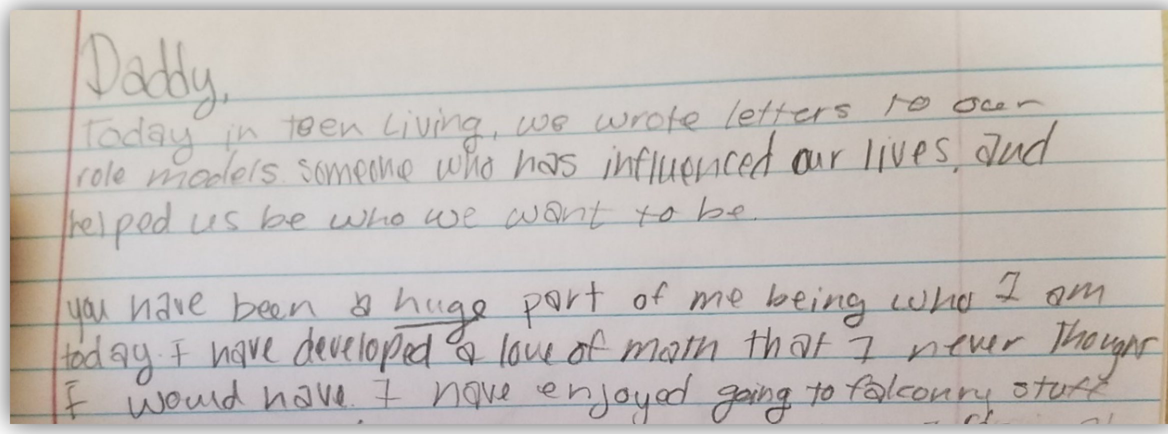
[7] Boaler, J. (2016). Mathematical mindsets: Unleashing student's potential through creative math, inspiring messages and innovative teaching. San Francisco, CA: Jossey-Bass. p. 15.

on now had she received a different response in that one pivotal moment. The good news is that she is aware of that need today, and works hard with her own children to encourage the mindset and support that she did not receive.

Do the three R's actually work?

Relationships, Relevancy, and Resiliency: three powerful tools for #ChangingMathAttitudes. But do they actually work?

You be the judge. Below is a note my 9th grade daughter wrote to me last year.



Daddy,
Today in teen Living, we wrote letters to our role models. someone who has influenced our lives, and helped us be who we want to be.

you have been a huge part of me being who I am today. I have developed a love of math that I never thought I would have. I have enjoyed going to falconry stuff

"I have developed a love of math that I never thought I would have."

Not only does she do math because she has to, she loves it. She sees the power and enjoyment of it. Yes, we did have some struggles over the years, and a few tears shed. However, through relationships, relevancy, and resiliency, we have seen a math attitude truly change. I hope you get an experience like this occasionally. Treasure it forever. This is why we teach.

We can make a difference. We have to make a difference. It might be in our family, it might be in our students, it might be in our community, and it might even be in ourselves.

Thanks for all your efforts!

Adam welcomes stories, ideas, and feedback at aking@dsdmail.net
His blog can be found at https://changingmathattitudes.wordpress.com/
Join the Facebook group "Changing Math Attitudes" at
https://www.facebook.com/groups/1653035008300751/

I Think I Can, Therefore I Can:
Developing Positive Cycles of Disposition

Barbara Child and Arla Westenskow

> *"I hate math! I hate math! Math is so stupid!", yelled Lisa as she entered my room for her tutoring session. This was the same girl who two years earlier, as a second grader, had weekly bounced into my room eager to play math games and to learn. During the previous two years, I watched as this happy second grader lost confidence and became more and more frustrated and defeated. What happened?*

Research suggests that Lisa's attitude towards mathematics was not atypical. In the United States, elementary students tend to enter school with positive attitudes towards mathematics and are eager to learn. However, many students fail to maintain their positive disposition as they progress through the grades (Cotton, 2004; Stipek, 2002). Students' attitudes towards mathematics affects their motivation and their confidence while performing mathematical tasks (Van De Walle, 2004). As a result, some students develop avoidance habits and math anxiety (Clayton, Burton, Wilson& Neil, 1988). Students with math anxiety are hesitant to perform mathematical tasks in front of their peers, often perform poorly in testing situations, and may develop patterns of learned helplessness (Beilock, Gunderson, Ramirez, & Levine, 2010; Brady & Bowd, 2005; Gresham, 2007; Trujillo & Hadfield, 1999; Vinson 2001). These reactions hinder the student's future learning and limit their ability to learn. The students become locked into cycles of failure such as the cycle shown in Figure 1 (Westenskow, Moyer-Packenham & Child, 2017). In contrast, students with a positive disposition see mathematics as useful, worthwhile, and attainable and are motivated to engage in mathematics (Gadanidis, 2004). These students experience cycles of success such as shown in the cycle in *Figure 1*.

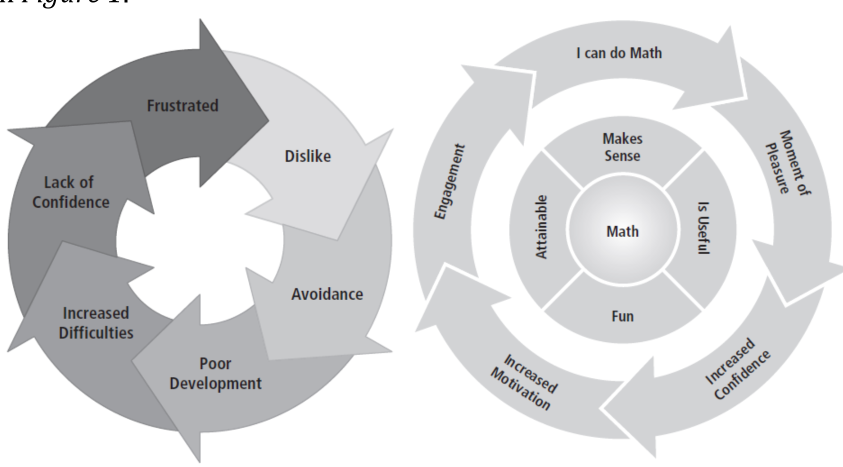


Figure 1: Cycle of Failure and Cycle of Positive Mindset or Disposition (Westenskow, Moyer-Packenham, & Child, 2017, p. 2)

To be effective, mathematics interventions must focus both on improving mathematics understanding and on breaking cycles of failure. In our work with interventions, we have identified four important factors that help break cycles of failure and promote success; 1) develop a positive mindset, 2) encourage risk taking, 3) value students' strengths, and 4) develop mathematical knowledge through play.

Develop a Growth Mindset

> *Lexi was a 5th grade Native American who was behind in mathematics due to multiple absences and was referred to me for tutoring. During our first meeting, she was obviously uncomfortable answering questions about mathematics. She cleared her throat frequently and asked to get a drink several times. Finally, I asked her, "Are you nervous? Do you like math?" Once she realized it was safe to acknowledge her feelings, she began to take a greater role in her own learning. She began to notice how concepts were connected. She became more confident in her ability to do math. As her confidence increased she often tried to trick me while doing activities. One day I called her attention to it and she started to laugh. Her teacher looked up from her desk and noted this was the first time she had heard her laugh in two years.*

When Lexi first began the tutoring sessions, she appeared to have what is termed as a fixed mindset. She believed she didn't have the "math ability". During the tutoring sessions, we saw changes that suggested she was beginning to develop more of a growth mind set. She became more vocal in explaining her thinking and was excited to discover it was correct. Jo Boaler (2016) states that students need to have a "growth mindset", a belief that they can learn at high levels and that the harder they try the smarter they will become. Growth mindsets promote positive attitudes towards mathematics which promotes achievement. Studies have been conducted to show that students with a "growth mindset" earn higher math grades when compared with students who have a fixed mindset, even though they had equivalent math achievement scores (Dweck, 2007). As students develop confidence in their ability to do mathematics, they begin to experience more success which helps children develop a positive attitude. If children exhibit growth mindsets, they are more likely to overcome misunderstandings and stumbling blocks along the way. They will be more inclined to persevere until they master the skills and concepts they are trying to learn.

Encourage Risk Taking

> *Sydney was a 2nd grader who struggled learning math facts and doing basic computation. Her "holes" were so deep that her mother questioned whether she might be dyslexic. A diagnostic assessment revealed problems with counting and the structure of numbers. Initially, due to her anxiety, Sydney would only come to tutoring with a friend. That proved to be counter-productive because the friend answered the questions before Sydney was able to figure them out. She would guess when doing activities because it wasn't worth the risk of looking "dumb" to her peer. Once she was comfortable working alone with me, she was willing to take the risk needed to learn the concepts.*

When Sydney was in a comfortable environment she was able to engage in the risk-taking needed to learn the concepts. When students either don't understand math, or dislike it, they will avoid it. This avoidance is often linked with fear of failure which can cause a hesitancy to engage and take risks. It's important for the teacher to establish a climate where children feel that mistakes are okay. Making mistakes causes the synapses in your brain to spark and grow. Mistakes are opportunities to learn.

Value Students' Strengths

> *Third grader Alicia struggled learning her addition and multiplication facts. Following her IEP (instructional education plan) Alicia began a program in which she worked through a series of leveled multiplication worksheets. Time tests were used to monitor her progress, but very little progress was seen. Then a new teacher, while working one to one with Alicia, observed that even though Alicia struggled with the facts she was able to mentally solve complex story problems. As a result of this discovery, the educational team switched from focusing on Alicia developing facts to capitalizing on her problem solving abilities and to help her develop new computation strategies. Alicia began to flourish and by fourth grade, she was one of the top performing students in her classroom.*

Alicia's story demonstrates the danger of focusing only on what a student cannot do and not identifying the strengths of the student. When a student's disability defines the student, interventions may limit rather than support the student. Alicia's initial program was not only unsuccessful, but also limited her participation in regular classroom mathematics instruction. Because of her disability, she was denied access to the mathematics taught at her grade level. In her second plan, Alicia was identified as a successful learner of mathematics, but needed additional support to make mathematics more accessible to her. We are not suggesting that fact memorization is not important, but rather than using a "deficit" model, intervention should capitalize on what the student can do. When students struggle with mathematics, they should think of themselves as mathematical thinkers who learn differently.

Develop Mathematical Knowledge Through Play

> *Jeremy cried all the way to his first summer tutoring session. On the second to the last day of the ten day program, Jeremy's mother called his teacher to report that Jeremy was again crying. He had taken ill and was crying because he would not be able to go to tutoring and play math games with his teacher.*

What changed? Although many elements influenced Jeremy's change in disposition, an important element was developing mathematical concepts through games. Research indicates that the use of games as a form of mathematics instruction increases motivation and confidence (Ke & Grabowski, 2007; Young-Loveridge, 2004). As one of the classroom teachers involved in a summer tutoring program reported, "Students will do hard math and keep trying if they can have a chance to flip an ant into the pants (a game activity)." (Westenskow, Moyer-Packenham & Child, 2017, p. 7). Winning promotes students' confidence in their mathematical abilities and helps break the negative disposition cycle. However, the games must be structured in a style that not only promotes learning, but also develops confidence. Years after it happened, we have had parents and teachers describe the fear and embarrassment they experienced in elementary school while playing "Around the World", a game in which two students stand together and compete to be the first to answer a multiplication fact. Games should be a positive experience for both the student who wins and the student who loses. The purpose of mathematical games should not be to test students' abilities, but to teach and practice mathematical concepts. Games used in the classroom should always contain an element of luck; luck which makes it just as likely that the struggling student will win as the gifted student. This can be done by structuring the procedure so that all players develop correct answers and the luck of the dice or the cards drawn determine the winner of the game.

It is difficult for children to maintain the positive attitude with which they entered school. This is especially true when students struggle. By building an environment which promotes growth mindset, encourages risk-taking, values students' strengths, and develops mathematical knowledge through play, you can rekindle a child's positive disposition for mathematics.

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, *107*(5), 1860-1863.

Boaler, J. (2016) *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.

Clayton, G. A., Burton, K., Wilson, B., & Neil, M. (1988). *Effective mathematics teaching: Remediation strategies: Grades k-5.* Southeastern Educational Improvement Lab, Research Triangle Park, NC.

Cotton, T. (2004). Inclusion through mathematics education. *Mathematics Teaching 187 (June), 35-40.*

Dweck, C. S. (2007). The secret to raising smart kids. *Scientific American Mind*, *18*(6), 36-43.

Gadanidis, G. (2004). The pleasure of attention and insight. *Mathematics Teaching, 186,* (Mar) 10-13.

Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. *Early Childhood Education Journal, 35*(2), 181-188.

Ke, F., & Grabowski, B. (2007). Gameplaying for maths learning: Cooperative or not? *British Journal of Educational Technology*, *38*(2), 249--259.

Stipek, D. (2002). *Motivation to learn. Integrating theory and practice.* Boston; Allyn and Bacon.

Trujillo K.M, & Hadfield, O.D. (1999) Tracing the roots of mathematics anxiety through in-depth interviews with preservice elementary teachings. *College Student Journal, 33*(2), 219-232.

Van De Walle, J.A. (2004). *Elementary and middle school mathematics: Teaching developmentally.* Boston, MA: Pearson Education.

Vinson, B.M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. Early Childhood Education Journal 29(2), 89-94.

Westenskow, A.W., Moyer-Packenham, P.S., & Child, B. (2017). An Iceberg Model for Improving Mathematical Understanding and Mindset or Disposition: An Individualized Summer Intervention Program. *Journal of Education 197*(1), 1-9.

Young-Loveridge, J. M. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quarterly*, *19*(1), 82-98.

Pythagorean Triple Threat

D. Aidan Gray, Weber State University
Rachel M. Bachman, Weber State University

As teachers, we hope to instill a love and appreciation of mathematics in our students. Let's face it though – we're still likely to encounter some inertia (“This is boring”, “This is too hard”, “I'm not good at math”) from students when it comes to reaching that goal. Providing a steady stream of engaging tasks that allow students to explore, practice, and discover various elements of mathematics helps manage some of this resistance (NCTM, 2014). This article provides one such task to help engage students with intriguing mathematics. The task allows students to practice Pythagorean triples and explore some basic geometrical properties while trying to discover the shortest path between two places on a map. The task acts as a catalyst for discovering (or in the very least reinforcing) that the shortest distance between two points is a line, and the sum of the lengths of two sides of a triangle is longer than the length of the third side.

Creating the Task

I am currently a preservice elementary education major at Weber State University pursuing a Level 2 Mathematics Endorsement. An assignment in one of my mathematics endorsement courses prompted me to design a lesson around the use of a high level task. Initially I was just trying to come up with something that would allow students to practice navigating a rectangular coordinate system to address the sixth grade content standard 6.NS.C.8 (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBPCCSSO], 2010). After running into some dead ends, I remembered a problem/example from a physics textbook that I had encountered a few years back. It involved using displacement vectors to navigate around an island placed in coordinate system. Light bulb!

I envisioned a real-world task involving the transport of goods from one island to another which begged the question “What is the most efficient route?” I experimented with a few different sketches, each including a starting island (marked with a dot), a final destination (marked with an X), and some islands in between. I knew I wanted the students to come up with a variety of ways to solve the problem so that a rich discussion was possible. For this to happen, the final map needed to have more than one obvious solution. Figure 1 shows the image I finally decided upon (note the image is shown in full scale in the Appendix for classroom use).

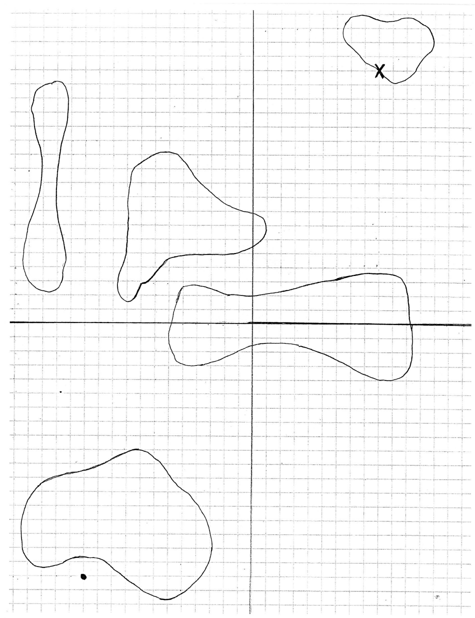


Figure 1, Image used for the task

After playing around with the idea for a while, it became apparent that I needed to establish some specific rules for this activity. I wanted to facilitate a class discussion about the observations the students made while trying to find the shortest path. To do this, the path segments needed to be easy to compare. This is what I came up with:

1. Start at the dot. End at the X.
2. Paths must be made up of straight line segments.
3. Any segment must start and end in the corner of a grid square.
4. If a diagonal segment is used, it must be the result of a Pythagorean triple or one of its multiples.
    - (3, 4, 5), (5, 12, 13), (8. 15, 17), (7, 24, 25)
5. Find the shortest path from the dot to the X.

The purpose of the fourth rule was so to ensure that all the path lengths would be whole numbers and, therefore, easily comparable.
What started off as a sixth grade problem involving rectangular coordinate systems had morphed into an eighth grade problem involving right triangles, Pythagorean triples, and shortest distances. The actual task addressed Grade 8 content standards 8.G.B.7 and 8.G.B.8 (NGACBPCCSSO, 2010). Though the purpose of the activity had changed, I decided to keep a set of axes on the map to give the feel of navigating a coordinate system.

Using the Task

When the time came to finally present this to my class, I was curious how it would unfold. My professor had established a classroom atmosphere which encouraged working together and exploring problems in small groups or as a class. When I handed out the activity, I reminded the class to feel free to work with others. As it turned out, silence filled the room for the majority of the time. Even the students who typically enjoy working together were quietly absorbed in their independent efforts. Perhaps a bit of competition (whether with others or just with oneself) spurred the desire to work alone. As I walked around the room, I noticed some students planning their paths out before committing them to paper. Others took more of a trial and error approach. Only after students had decided on their final paths did they break the silence and begin to share them with their classmates.

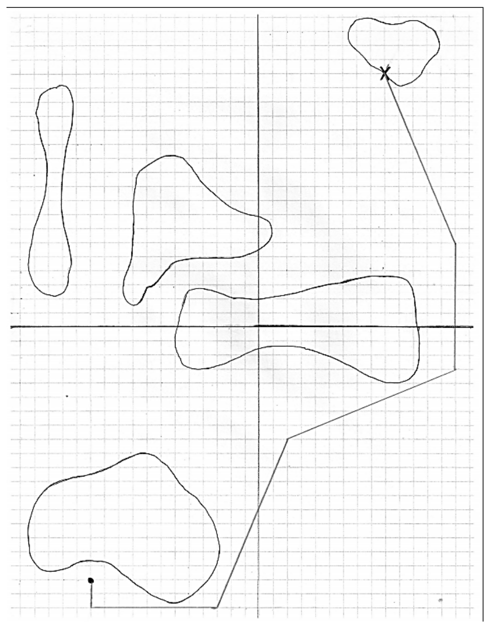


Figure 2. Shortest route found by Student 1

While some of the students started sharing their ideas with their neighbors, I walked around the classroom looking for student work that would foster our class discussion. I used the suggestions from Stein and Smith (2011) to select and sequence the sharing of student work. While I observed a few different types of routes (see Figures 2 and 3), I decided to invite the student with a longer route to share first (see Figure 2). The student showed some of his earlier paths before showing how he revised them into shorter ones. Since his final path seemed to use more diagonal segments, I asked if there was an advantage to using diagonals rather than a combination of horizontal and vertical segments. This opened up a nice little class discussion about the issue.

Student 1:	Yes, it seems like there was.
Me:	Did anyone else notice that, too?
Student 2:	Yes, that's actually how I found my shorter paths.
Me:	Would you say more about how you used this idea to find shorter paths?
Student 2:	Well, for example, I noticed that if I went up 12 units and over 5 units, I could save by just using the Pythagorean triple 5, 12, 13. I would erase my 12 and 5 and replace them with a diagonal of 13.

Student 3:	Yes, that's what I did, too. When I finished a path, I would go back and look at my vertical and horizontal lengths to see if they resulted in some multiple of the Pythagorean triples I know.
Me:	How did you know that the diagonal was always going to be shorter than the vertical and horizontal change?
Student 4:	I could just see it.
Me:	I mean, what properties about triangles would help you know that?
Student 2:	The sum of any two sides lengths of a triangle is longer than the length of the remaining side length.
Student 5:	Or the shortest distance between two points is a straight line.
Me:	Hmmm, interesting.
Student 3:	Wait, is that why the shortest distance between two points is a straight line?
Me:	What do the rest of you think?

The conversation continued further about the connectedness of these two geometric principles. Eventually, I invited another student to share her thinking about the task. She explained how she arrived at her final route by envisioning the dotted path shown in Figure 3. As the class had already discussed, this was technically the shortest distance from the dot to the X. Her strategy was to build a path around the islands most similar to this, and her thinking resulted in the shortest route found in the class (shown in figure 3).

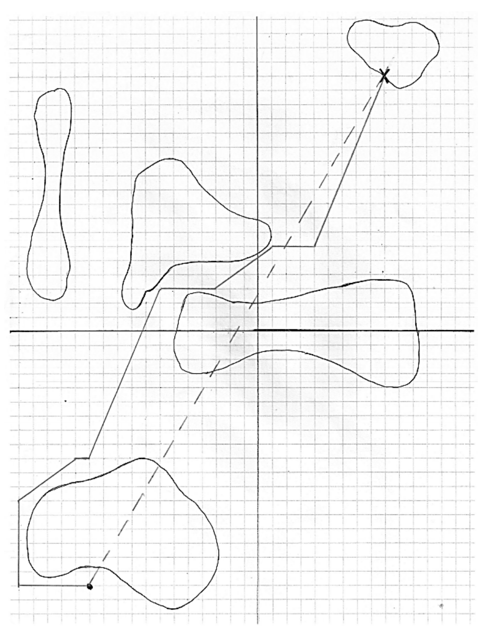


Figure 3. Shortest route found by Student 4

Reflecting on the Task

Perhaps the most important parameter in this task was the rule which imposed using Pythagorean triples on diagonal segments. It allowed students to focus on the task of finding the shortest distance rather than worrying about calculating hypotenuse lengths. By keeping that focus, it gave

students the opportunity to realize, “Hey, if I use more diagonals on my way to the X, my path will be shorter.” This created a great platform for the class discussion that followed the activity.
Since I kept the coordinate axes on the map, I later realized that the task could be extended to have students describe their paths in terms of coordinate points. Extending it in this way may be a good way to review coordinate plane concepts from the sixth grade curriculum.
I also encourage teachers to develop their own versions of this map to add variety and accommodate the needs of their specific classrooms. Furthermore, students could be asked to develop their own maps to challenge fellow classmates.
NCTM (2014) explains that the regular use of high level tasks that promote reasoning and problem solving is a keystone to creating a classroom where students have opportunity to engage in high level thinking. Such tasks have “multiple entry points… and foster the solving of problems through varied solution strategies” (p. 17). The Islands Task provides a low floor entry for students as every student can find at least one way from the starting island to the finishing island, and it sets a high ceiling by encouraging students to search for shorter and shorter ways (McClue, Woodham, & Borthwick, 2011). This search for a shorter way causes students to pay attention to the structure of the paths they are building in order to find shorter routes. The multiple entry points and opportunity to investigate different solutions help set the stage for a meaningful discussion. Through the use of an open-ended, intriguing task that students want to investigate, previously resistant minds are pulled into the beautiful world of mathematics.

References

McClue, L., Woodham, L., Borthwick, A. (2011). Using low threshold high ceiling tasks. NRICH. Retrieved April 28, 2017 from http://nrich.maths.org/7701

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring Mathematical Success for all.* Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

Smith, M. S., & Stein, M. K. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Invisible Mathematics

Andrew R. Glaze, Utah State University

Abstract

In this paper, the topic of invisible mathematics is introduced and explored. The author defines invisible mathematics as numbers, or operations which are hidden from view. Invisible mathematics act as stumbling blocks for students and must be addressed for students to make sense of mathematics and persevere in problem solving. The first topic explored is the number 1 and the underlying meaning of the invisible 1 as a coefficient, a divisor, and an exponent. The second and third topics explored in this paper are the invisible operations of addition and multiplication. Topics introduced span multiple grade bands and address student misconceptions resulting from a misunderstanding of the invisibilities. Finally, the author presents a perspective for addressing misconceptions and a few suggestions from his practice which aid students in understanding invisible mathematics. Because the topics in this paper are not exhaustive, the reader is invited to explore invisibilities in his or her own area of expertise.

Keywords: multiplication, addition, number operations

Invisible Mathematics

Who doesn't like a good magic show? The mysterious ways that a magician pulls a bird out of a hat or causes a person to disappear into thin air are intriguing. It is magic after all and magic is supposed to be mysterious. Mathematics, on the other hand, is not supposed to be mysterious. And unlike a great magician, a great mathematics teacher should be in the business of revealing the mysteries, not creating them.

Unfortunately, the nature of mathematical representations and notations promotes a certain amount of mystery for students. The language of mathematics has its own set of invisibilities. This paper is about invisible mathematics, what it is, where it is found, and how to make it visible.

What is Invisible Mathematics?

We begin with a formal definition. Something invisible is unseen, imperceptible, or hidden ("Invisible," n.d.). In mathematics, invisibilities exist. They are numbers, or operations which are hidden from view. Furthermore, invisible mathematics can be obstacles to students.

Think for a moment about the invisible obstacles your students incur when solving equations, making calculations, graphing, etc. You may find that in many cases they are stumbling over mathematical properties, numbers, or operations that are hidden from view. This paper addresses only a handful of the invisible obstacles of which you might already be aware. The examples in this paper are not intended to be an exhaustive list of invisibilities. Rather, the intent of the few examples which follow are to draw your attention to the practice of looking for invisibilities and helping students recognize and overcome them.

This paper addresses the invisible 1, invisible exponents, and invisible operations. The reader may note that some topics addressed in this paper are procedural in nature. This is because procedural fluency is one very important mathematical strand (National Research Council, 2001). Similarly, the practice standards introduced as part of the common core state standards (National Governor's Association & Council of Chief State Officers, 2010) address the importance of making sense and perseverance in problem solving as well as attending to precision. Procedural fluency is key to perseverance and attending to precision mathematically.

The Invisible 1

The number 1 could possibly be the most omnipresent numeral in mathematics. Predating the number zero by centuries (Joseph, 2000), it forms the basis for a myriad of numbering systems (Dehaene, 2011). So common is its presence that we often take it for granted. Three places where the number 1 often remains invisible are as a coefficient, as a divisor, and as an exponent.

Coefficient

Consider the situations where the number 1 may be a coefficient. In a student's earliest introductions to algebra, the number 1 makes its appearance as a coefficient to the oft used variable x. The coefficient of a variable describes how many or how much of a variable exists. Yet the invisible coefficient also describes how many or how much of that variable exists. For example, x means $1x$.

By itself, the invisible coefficient is not terribly mysterious. When combined with a negative sign, however, it becomes problematic. How often have you observed a student struggle to solve a simple equation of the form $-x = b$? Students who demonstrate competence solving a myriad of other equations stop dead in their tracks at the sight of an equation of the form $-x = b$. As with all one-step equations, solving for the variable requires the effective use of additive or multiplicative inverses. But what is the multiplicative inverse of the negative sign? Is it a positive sign? To solve an equation of the form $-x = b$, a student must understand that it really represents the equation $-1x = b$, in which case the multiplicative inverse is -1.

In later courses, the coefficient of -1 hides invisibly in such places as polynomials and matrices. Recognizing that $f(x) = -x^2$ is $f(x) = -1x^2$, for example, aids in student sense-making of graphical behavior. Instead of memorizing the fact that a negative coefficient performs a vertical reflection about the x-axis, a student can come to the same conclusion by understanding that the argument of a function is being multiplied by -1.

Divisor

Another common misunderstanding of the invisible 1 occurs when the 1 is in the denominator. This may be because students already have weak understanding of rational numbers (Mazzocco & Devlin, 2008). Consider again an equation of the form $ax = b$ where a is a fraction such as $\frac{2}{3}$. In the case of $\frac{2}{3}x = b$, one can write $\frac{2}{3}x = \frac{b}{1}$. The change in representation is relatively minor. Pedagogically, however, the change can aid a student in realizing a solution path. Using the property of multiplicative inverses, a student can multiply both sides of the equation by $\frac{3}{2}$ and understand how to evaluate the right side of the equation $x = \frac{b}{1} \cdot \frac{3}{2}$.

Even when solving the equation $\frac{2}{3}x = b$ by dividing both sides of the equality by $\frac{2}{3}$, a student must reckon with an invisible number 1. Consider the solution $x = \frac{b}{\frac{2}{3}}$. From a procedural standpoint, one will recognize the applicability of the "invert and multiply" rule only if one can recognize that an equivalent equation is $x = \frac{\frac{b}{1}}{\frac{2}{3}}$. From a more conceptual perspective, one might approach the equation $x = \frac{b}{\frac{2}{3}}$ from an iterative perspective by asking "how many times does $\frac{2}{3}$ go into b" or a measurement perspective when asking "how many groups of $\frac{2}{3}$ can I make out of b?" Recognizing the invisible 1 in the divisor is the same misunderstanding which keeps students from performing routine procedures with rational numbers.

Exponents

A third, and equally vexing location for an invisible 1 is in the exponent of a term. Like the coefficient and the divisor, its presence is trivial when not noticed, but is very important nonetheless. In middle grades, recognizing the 1 in the exponent manifests itself as an essential skill when simplifying expressions using properties of exponents such as $\frac{a}{a^n}$. Applying the quotient of powers property, one would recognize the solution as a^{1-n}. In later courses, an understanding of the exponent of 1 plays an essential role in calculus when finding derivatives and integrals. $\frac{dy}{dx}ax = a$ because subtracting 1 from the exponent of x leaves x^0 which is 1.

Invisible Operations

Having discussed a few applications of the invisible 1, the discussion turns to the invisible operation. As with the section on the invisible 1, this is not meant to be an exhaustive accounting of invisible mathematics. The purpose of this section, however, is to bring forth misunderstandings that occur when students are unaware of the underlying meanings in implied operations, specifically those of addition and multiplication.

Invisible Addition

Thanks to the diligent work of elementary school teachers everywhere, students are familiar with the invisible operation of addition revealed when expressing numerals in expanded form. For example, the number 327 written in expanded form is $300 + 20 + 7$. Two more elusive locations of the invisible addition operation are in mixed fractions and long division.

Let's begin with a mixed fraction (also known as a mixed number). Mixed fractions are fractions of the form $a\frac{b}{c}$. The invisible operation is addition. $a\frac{b}{c} = a + \frac{b}{c}$. So ingrained in our mathematics culture is the representation $a\frac{b}{c}$, that we can take for granted student misunderstanding of the implied operation. Contemplate for a moment the prodigious nature of such a misunderstanding. Not only would a student with that type of misunderstanding not be able to maneuver many basic mathematical operations, but he or she would also not have a basic understanding of fractional operations. To a student who mistakenly interprets the invisible addition sign as a multiplication sign $1\frac{3}{4}$ would not mean 1 and $\frac{3}{4}$ of a whole. Yet students who enter this author's classroom often mistake the operation in mixed fractions for multiplication.

An understanding of the invisible addition sign in mixed fractions opens the door to some creative mathematical problem solving strategies. As one example of problem solving strategies, consider fractional multiplication of the form $f \times a\frac{b}{c}$. The standard United States algorithm dictates that we change the mixed fraction to a fraction greater than 1, then proceed to multiply numerators and denominators. Thus $f \times a\frac{b}{c}$ becomes $\frac{f}{1} \times \frac{ca+b}{c}$. It is an efficient algorithm, but it is worth noting that other strategies do exist. $f \times a\frac{b}{c}$ when combined with the distributive property can be expressed as $fa + f \cdot \frac{b}{c}$. One can argue that the traditional algorithm is more efficient, but this author posits that applying the distributive property can be a powerful tool when estimating or performing mental calculations. Try it. $2 \times 5\frac{3}{4} = 10 + \frac{6}{4} = 11\frac{1}{2}$. Additionally, this would be a powerful way to reinforce student use of the distributive property in early grades.

It is completely understandable that the invisible and implied addition sign in mixed fractions could be confused for a multiplication sign. When one considers all the invisible multiplication signs, one can easily understand why a student would see mixed fractions as following the same rule. This misunderstanding is addressed in the next section when we consider the invisible multiplication sign.

Another elusive location of the invisible addition sign is in the quotient when applying the standard algorithm in long division. As an example, let us examine the common long division algorithm as applied to 184 ÷ 8. Typically, a student truncates 184 after the 8 to determine the number of times that 8 divides into 18. After determining the answer to be 2 with a remainder of 2, a 2 is recorded as the first number of the quotient, then the 4 removed in truncation is now placed behind the remainder of 2 to create the number 24. It is then determined that 8 divides 24 three times. A three is then placed behind the 2 in the quotient to reveal an answer of 23.

The whole process can seem somewhat mystical. There is much more sense to be made if, instead of truncating, we allow students to recognize that when we truncate we are dividing 184 by 8 instead of dividing 18 by 8. The remainder is 24. When acknowledging the expanded form of the quotient, students can write 20 + 3 instead of a 2 then a 3. This author finds that students appreciate a vertical addition of the quotient as displayed below.

$$\begin{array}{r} 23 \\ \hline +3 \\ 20 \\ 8\overline{)184} \end{array}$$

It is worth our time as teachers to help students understand this representation since polynomial division is founded upon this principle.

The long division algorithm reveals one final location of invisibilities in division – the remainder. This author has observed numerous students who can successfully perform a long division algorithm, but stop shy of reporting the remainder in a meaningful form. Take for example the division $82 \div 7$. A common representation of the quotient after applying the long division algorithm is 11 remainder 5. A much more meaningful form of the quotient is $11\frac{5}{7}$. However, it will only be meaningful if the problem is given in context. For example, one might ask "how many groups of 7 can you make out of 82 objects?" When we do not represent the remainder in fractional form, we are treating division as a modular operation. While modular operations are important, we must not treat all division as a modular operation.

Multiplication

As mentioned previously, invisible multiplication signs are prominent. Just look at any given polynomial expression of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1$. Every term in the expression contains an invisible operation. Invisible multiplication signs are found in equations, expressions, and multiplication with parenthesis. We might take for granted that every time a student is asked to either simplify an expression or factor a polynomial, he or she is being asked to perform their own magic trick complete with disappearing operations.

A common problem offered in many text books is the simplification of expressions. When, for example, students are instructed to simplify $3x(x+2)(x-2)$ they are expected to write the expression without any parenthesis. Consider that the expression has three hidden multiplication signs.

Proposed Solutions

If this paper has served one of its purposes, the reader has at least paused once to consider how invisible numerals or operations may cause student confusion in mathematics. At the conclusion of this brief exploration of invisible mathematics, the reader is invited to consider one perspective and two solutions.

The perspective proposed is that we are all in this together. It is an easy thing to blame student misunderstandings on teachers of earlier grades, parents, culture, etc…, but the truth is that we are all in this together. When a student enters our classrooms, are we not all under the same mandate to teach and help her or him learn to the best of his or her ability – even if they are lacking fundamental understanding of previous mathematical topics? If we all own the problem, then we can all be part of the solution.

If we want to teach students with understanding, we should not be shy about revisiting topics from previous courses to solidify student understanding. One way we can do this is by strengthening connections between current topics and topics covered in previous grades. It is naïve to assume that the only time to learn mathematics assigned to a grade band is when students are in that grade. Students frequently come to secondary mathematics classrooms lacking stable prerequisite knowledge for the course. Thus we must strengthen their knowledge of elementary mathematics while helping them continue to explore mathematical topics addressed in current courses. Elementary mathematics are elementary because they form the most rudimentary foundations of the subject. They are not necessarily elementary because they are easy.

In his teaching of mathematics, the author has found two ways to help make invisible mathematics more visible. The first is to explicitly revisit mathematical topics taught in previous courses whenever possible. The second is to invite students to explicitly make invisible mathematics visible.

Revisiting Earlier Mathematics

The study of mathematics offers many opportunities to revisit earlier topics. The following are two ways in which the author visited elementary school topics while teaching high school mathematics. Neither lesson took a considerable amount of time away from the current topic and both helped students to deepen an elementary understanding of mathematics.

When teaching polynomial division one can take a moment to review the conceptual underpinnings of the long division algorithm. Consider the rational expression $\frac{m^2+5m+6}{m+2}$. One way to write the expression in simplest form is to perform polynomial division much the way one would perform long division with rational numbers.

$$m+2\overline{\big)m^2+5m+6}\quad\text{quotient: } m+3$$

The m in the quotient represents the number of times that $m+2$ divides m^2+5m+6 with a remainder of $3m+6$. Similarly, the 3 in the quotient represents the number of times that $m+2$ divides the remainder of $3m+6$. Compare this to the example presented earlier of $184 \div 8$ where 20 represents the number of times that 8 divides 184 with a remainder of 24 and the 3 represents the number of times that 8 divides the remainder.

$$8\overline{\big)184}\quad\text{quotient: } 20+3$$

It is this author's experience that for many high school students, revisiting the division of real numbers while investigating polynomial division helps solidify and reinforce properties of real numbers not explored for multiple years.

Similarly, when teaching the addition of rational functions, one can take part of a lesson to help students remember that a common denominator can also be considered a common partitioning. Consider the addition of the two terms in the expression $\frac{m}{m+2}+\frac{3}{m+3}$. Addition of the two terms requires a common divisor of $(m+2)(m+3)$. This is like finding a common partition when adding the fractions $\frac{1}{2}+\frac{1}{3}$. The 2 and the 3 in the denominators can be thought of as the number of partitions of a whole, while the numerators represent the number of partitions present. Thus, a common partitioning of the whole would be a multiple of both 2 and 3.

Representing partitions of $m+2$, $m+3$, and $(m+2)(m+3)$ is often too abstract for high school math students. However, representing partitions of 2, 3, and 6 is quite manageable and offers students an opportunity to strengthen understanding in an area of mathematical weakness (See Figure 1).

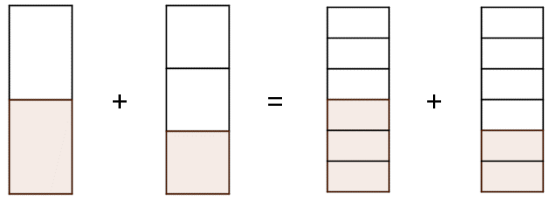


Figure 2. Sample partitions for addition.

The previous two examples are samples of the type of unveiling of invisible mathematics that we as teachers can do in our classrooms. Instructional time in current content area is not significantly depleted and students gain from the added understanding.

Explicitly Making Invisible Mathematics Visible

The second suggestion for making invisible mathematics visible is to make it explicit. The following exercise takes only moments in a lesson, but quickly reveals student misconceptions and understandings. When working with any mathematical representation which has invisible numbers or operations, invite the students to insert the invisibilities. For example, when a student encounters an expression, invite them to insert all invisible numbers and symbols. The following expression displays the type of answers students can generate:

$$-3x^2y = \frac{-1 \cdot 3^1 \cdot x^2 \cdot y^1}{1}$$

Conclusion

Mathematics is a rich and engaging subject. Many of the symbolic representations, however, hide numbers and operations. When we perpetuate the representations of invisible mathematics without assisting students to reveal the invisibilities, we unwittingly become magicians.

As stated previously, the examples presented in this paper are not exhaustive. Mathematics is full of invisibilities. Some topics not explored in this paper, but equally as vexing for students include the index of a square root, logarithmic bases, and the asymptotes of an equation. Look for the invisible mathematics in your own area of expertise. You might be surprised at what you find and begin to look at mathematics in a new way!

References

Dehaene, S. (2011). *The Number Sense: How the Mind Creates Mathematics, Revised and Updated Edition*. Oxford University Press.

Joseph, G. G. (2000). *The Crest of the Peacock; The Non-European Roots of Mathematics* (2nd ed.). Princeton, N.J.: Princeton University Press.

Mazzocco, M. M. M., & Devlin, K. T. (2008). Parts and “holes”: Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science*, *11*(5), 681–691.

National Governor's Association, & Council of Chief State Officers. (2010). Common core state standards initiative. Retrieved from
http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf
National Research Council. (2001). *Adding it up; Helping children learn mathematics*. Washington, DC: Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education.

Going Back to School: Lessons Learned by a University Professor in a High School Classroom

Rachel M. Bachman, Weber State University

In my third year of teaching as an assistant professor of mathematics education at Weber State University, I began a research project at a local high school with the aid of a StepUp Ready grant from Utah System of Higher Education and generous support from the College of Science at Weber State University. For this project, I taught a special section of College Prep Math for high school seniors who successfully completed Secondary Math I, II, and III; expressed a desire to attend college; showed an interest in STEM majors; and had a mathematics ACT score less than 23. The aim of the class was to prepare students to enter college level mathematics with conceptual understanding of important prerequisite mathematics concepts and the use of successful student habits for learning. The year started with 17 high school students made up of almost equal numbers of male and female students as well as equal numbers of Hispanic and Caucasian students. Their average mathematics ACT score was 17.

Never could I have anticipated all that I would learn through this experience. First, I learned that if one is going to try to operate within both an A/B teaching schedule and a university teaching schedule, one should also plan to start coloring her hair. Noise makers, I learned, are a no-no. As one of the students explained, "We will respect anything you bring to help us learn, unless it makes noise. That's just more than we can handle." My students taught me about the extra credit function f with domain being the earned letter grade for the course and range being the expected grade after performing extra such that $f(\text{D}) = \text{A}$. And, of course, because I taught in the 2015-2016 school year, I also learned to "whip and nae nae." Truly, though, the lessons I learned about teaching and learning have reshaped how I structure all my classrooms since, and I am so appreciative for this opportunity. In this article, I describe some of the teaching tools used and developed throughout the year as well as insights gained about working with mathematically underprepared students.

Getting Started

Even in the infancy of this project, I knew I wanted this course to help students approach mathematics through the Standards of Mathematical Practice outlined in the Common Core State Standards for Mathematics (NGACBPCSSO, 2010). These standards reflect the longstanding importance of the following practices in mathematics:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

From the very start of the course, I wanted the students to understand that we were not going to focus on learning rules and procedures to use to find answers, but instead, we were going to be studying mathematics for the purpose of increasing our use of the above practices.
To set an expectation for the use of these practices, the first day of the course I chose an analogy called "Microwavable Math" that I often use in my university classes to help the students understand the shift in mathematics instruction when we focus on the standards of practice. In this analogy, I told students that I was going to teach them to make chicken mole. The next slide showed a picture of a Herdez microwavable chicken mole bowl, and I went through the following absurd description of the instructions:

> Step 1 is to remove the item from the package. The rest of the steps will not work unless you do this. This is a very important step. Are you writing this down? Step 2 is to place the item inside the microwave. This is also equally important. The rest of the steps will not work if you do not do this. Ok, now Step 3 is the tricky one. Are you ready? Here we go. Step 3 says to "Heat on high for 3 minutes*." However, do you see this asterisks? Nod if you see the asterisks. This is what makes this step difficult. We heat on high for 3 minutes only if we have a 950 watt microwave. If you do not have a 950 watt microwave, you will need to do a conversion step first. You need to do this conversion step or your chicken mole will not turn out correctly. Ok, are you ready for the last step? Here we go. This is such an easy step, but this is where everyone always loses points: Let stand for one minute before serving. Who has skipped this step before? Yes, see. I hate taking off points for this, but I will since it is one of the steps. Ok, there are your four simple, well mostly simple, steps for making chicken mole.

Following this ridiculous demonstration, I asked the students that did not know what chicken mole was to name one ingredient in the dish besides the chicken. When they failed, I acted shocked and said, "How do you not know this? I just taught you how to make chicken mole." I also asked them if we did any of the things a chef would do when she makes chicken mole, or if they had a better appreciation for the cultural heritage of the dish. The students explained that I did not actually teach them to make chicken mole but rather how to make microwavable chicken mole. Then I asked, "In your past mathematics classes, were you doing the things mathematicians do or were you making microwavable math?" To help them understand the things that mathematicians actually do, I showed them the list of eight standards of mathematical practice and describe how mathematicians use these practices. I challenged them to work on the following problem as a mathematician would through the use of the standards of mathematical practice:

> The cube root of 68,921 is an integer. Find the integer by eliminating possibilities.
> We had just enough time to recall what integer and cube root meant before the class period ended. The only homework I assigned that night was to work on this problem and come to the next class with a list of numbers the square root could not be.

Realizing the Problem

To start the next class, I had the students talk to the students at their table about their experience solving the cube root problem. What happened next was the first signal of the problem to be addressed. I watched one student turn to the students at her table and say, "I don't know how to solve this" and the other students turn to her to respond, "I don't know either." In fact, this happened at every table in the classroom. I quickly intervened to remind the students that their assignment was simply to figure out a list of things the answer could not be; surely they had figured out some numbers the answer could not be. One brave student offered an insight into her thinking. She shared that she wondered if the problem could be solved by simply dividing 68,921 by three. However, after finding the quotient to be 22973.67 she realized that this number was far too big to be the cube root of 68,921. Several students were nodding their heads and explained that they too tried this approach. I celebrated that we now knew one number it could not be and asked what they tried next. At this point, many of the students looked confused and someone said, "After that, I knew I didn't know how to solve." Again, several heads were nodding, "Yes, our idea didn't work." Someone else added, "I don't remember the formula for this."

Again, I reminded the students that they simply had to rule out numbers the answer could not be and asked if there were any other numbers they were sure it was not. Someone jokingly said, "1," and I asked how he knew. This prompted someone else to say "100" and explain that that 100 times 100 times 100 was bigger than 68,921. At this point, I had the students go back to working in their groups to find more numbers the answer could not be. As I circulated around the room I continued to see some interesting things. One student had a list on her paper checking 99 x 99 x 99, then 98 x 98 x 98, and 97 x 97 x 97. When I asked her about the work, she explained that now she knows the answer has to be less than 100 so her plan was to try every number less than 100 until she found the right one. I asked her if she thought the answer would be 90 something. She said she did not think so, but she did not want to risk missing the number. Another student discovered that the answer had to be odd. When I asked her why she responded, "An even and an even is an even." As a class we worked on how to say this observation using more precise mathematical language.

The students did eventually solve the problem correctly, but not before painting a startling picture of their deficits with the standards of mathematical practice. They showed a clear lack of persistent problem solving with a problem they had not been taught a procedure for solving. They struggled to verbalize their thinking, and lacked the mathematical language needed to speak about their ideas with precision. Some students also missed opportunities to take cues from the structure of the numbers in the problem and notice patterns in the numbers. More than anything, I was surprised by how resistant the students were to experimenting with what the solution could and could not be. It became evident that if I wanted the students to use the eight mathematical practices in this class, I was going to have to teach them how to use the practices. I also quickly recognized that my standard tools for engaging students in the practices were not going to be sufficient; I needed some new tools.

Tools for Fostering the 8 Standards of Mathematical Practice

Timers

One of the simplest tools I discovered to help students persist with problem solving was to use embedded timers in the PowerPoint presentations I used when I wanted students to work on a nonstandard problem without a defined procedure. I had used PowerPoint timers in the past to keep students on task and the lesson moving along (Foord, n.d.). However, the timers had an extra benefit in this high school class. Without a time limit in mind, students often felt overwhelmed by how long it might take to solve a problem. When I said, "Let's think about this problem for two minutes" the students were more likely to actually put in good thinking knowing there was an expected end. Often at the end of the allotted time, students were engaged in solving the problem and would ask for some additional time. I always acted like I was doing them a great favor to extend the time, all the while knowing I actually wanted them to spend more time solving the problem anyway.

Leave Off the Question

Two of my colleagues at Weber State University developed a strategy in one of their classes called "Share What You Know" (Chan & Stern, 2016). They developed this strategy to help students put aside the anxiety of not knowing how to solve a problem and put focus on what they did know about the situation. For example, Stern and Chan posed a problem such as "A rectangular prism has dimensions 2 x 3 x 4. Each side length of the prism is tripled in length. By how much does the volume increase?" Instead of having students work to solve the problem posed, they actually asked them not to solve but instead share what they know about the problem and ideas for solving. They went as far to say the students would not receive full credit if they solved. The students first brainstormed ideas independently or with a partner before discussing ideas with the whole class.
I had watched Stern and Chan use this strategy very successfully in a university class and decided to use the strategy with this high school class to try and increase their persistence with problem solving. However, the strategy did not work quite the same with this group of students as it had in the university class I observed. Even though I instructed the students to not solve the problem, the students were still mostly fixated on solving the problem stated and feeling like they did not know the steps to solve. I needed to find another way to shift their focus from what they did not know to what they did know.

I decided to make one slight alternation to "Share What You Know." Since I did not want them to actually solve the stated problem anyway, I decided to leave off the question and asked "What do you know? What could you figure out?" The first time I tried this strategy I used the context of a rectangular room that was 16 feet long, 12 feet wide, and 8 feet high. Normally I would go ahead and state the question of "What is the total wall area?" along with the dimensions of the room. This time, I just stated the dimensions and had the students brainstorm the things they knew and what they could figure out for two minutes. Then I split them into two teams to brainstorm more information together. At the end of the brainstorm time I asked various questions about the room. The first team to provide the correct answer received a point, and the team with the most points all received Zot candies (a candy that became our class mascot for reasons I am not sure I can explain).

When I walked around the room during the brainstorming time, I noticed a remarkable thing. Almost every student in the class had correctly calculated the total wall area of the room. When I asked this wall area question in the past, only one or two students out of a class of 30 could answer the question correctly. Common wrong answers included the use of the volume formula, finding the total surface area of the room, and providing the area of one wall or the ceiling. Once I even saw a student use the area formula of a trapezoid to find the wall area because "that had to be the formula for wall area because it has three variables." As I walked around the room this time, however, I did not see anyone apply an incorrect formula. I observed the students drawing pictures of the room and knowing what they were calculating. Removing the question seemed to help them shift their focus from what they did not know to what they did know. Even I was amazed by how much knowledge they were able to demonstrate when their anxiety was relieved and they were able to think actively about a problem.

I continued to leave off the question in many more problems in the course. Sometimes I would use the strategy to review a unit. For example, at the close of our unit on quadratic functions, I split the class in half and had them brainstorm everything they knew about $ax^2 + bx + c$. After they had time to prep, the groups took turns offering something they knew. Each time the job of answering rotated to a new person on the team. By the time we finished, the board was full of such facts as the general form of a quadratic, characteristics of the graph of quadratics, several ways to find the vertex and *x*-intercepts, and examples of real world applications of quadratics. I also used the strategy of leaving off the question to begin our thinking about a new topic. For example, to assess how much the students remembered about angle properties, I posed the image seen in Figure 1 and asked them to share what they know or could figure out about the image. They determined the angle and side length measures of the triangle, found the height of the triangle, and calculated the area and perimeter of the triangle. Not only were the students successful in calculating several values, they also constructed viable arguments to justify the values they found. I could not help but contrasting this success with what likely would have happened had I initially asked, "What is the area of this triangle?"

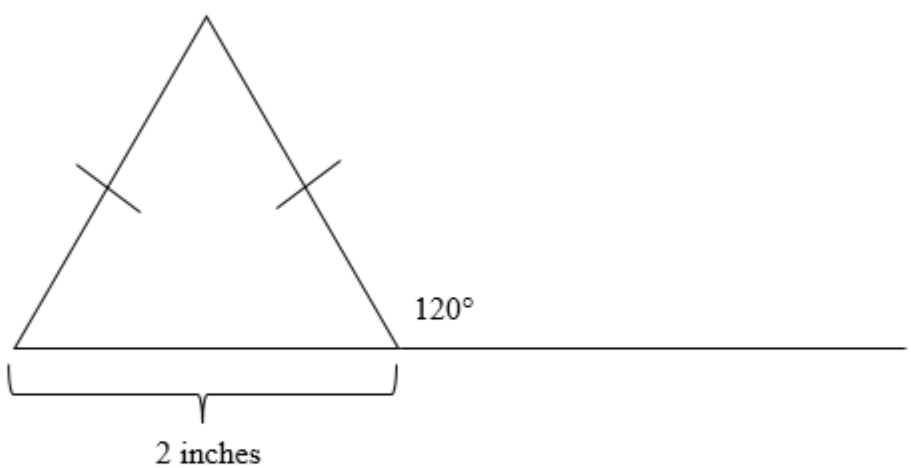
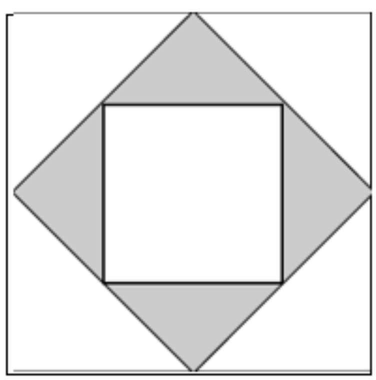


Figure 1. Leaving off the question to find out what students know about angles.

This very simple strategy invited students to contemplate what they knew or could figure out about a context. The students were persistent with problems I have watched students in the past toss away because they did not know how to begin. The students also learned to construct viable arguments and critique the reasoning of others. Because the brainstorm sessions often preceded a team competition, the members of the team had a reason to defend their insights and vet all possible responses. The students also learned to take a deeper look at the contexts posed and gather clues from the structure of the problem. Furthermore, this strategy created a "low floor high ceiling" task as everyone in the class could make some observations about the setting and the top students in the class were challenged to find more and more things they knew.

TAPPS. Midway through the first semester of this project, I learned about a collaborative problem solving tool called Thinking Aloud in Pair Problem Solving (TAPPS) at the 2015 AAC&U Transforming STEM Higher Education conference. The strategy was first introduced by Lochhead and Whimbey (1987) as a way of encouraging students to verbalize their problem solving process. In this strategy, the students work in pairs to solve a problem like the one posed in Figure 2, and each student has a clearly defined role. One of the students assumes the role of the problem solver and is charged with verbalizing all their thoughts while solving a problem. The other student plays the role of the listener and records everything the problem solver does to solve the problem. The listener is not allowed to help solve the problem but can prompt the problem solver to continue talking by using questions such as "What are you thinking about?" and "Why did you do that?"



The figure to the right is composed of three squares.
How much of the large square is shaded?

Figure 2. Image used during TAPPS.

The most noticeable area of growth with the standards of mathematical practice from the use of TAPPS was improvement constructing viable arguments. Without TAPPS, students struggled to start a conversation with their neighbors about their thoughts on how to go about solving a problem. Many students felt they had nothing to say if they did not have a clearly paved path to the solution. I was more interested in them sharing their initial ponderings about the problem, potential paths to a solution, false starts, road blocks, key insights, pictorial depictions, and ideas of what sort of answers might be reasonable. By asking the students to make these thoughts audible *during* the problem solving process, the process began to be visible to others. The students realized that answers did not automatically appear to the "good students;" everyone had to put in work to get to the answer. The students also realized that right answers often occurred after a few wrong ideas were explored. TAPPS really highlighted the important components of problem solving, illustrated the need for persistence, and taught the students how to have conversations with their neighbors about these elements of problem solving.

Class Extra Credit

At the end of our unit on polynomial and rational operations, I decided to pose a challenging problem to the entire class (see Figure 3). As the students worked, I saw an opportunity to support further growth with the mathematical practices. I offered, "This one seems like a challenging problem. Would you like to work on this as a class for extra credit?" They agreed they would. I explained that the extra credit would only replace the grade for a low homework assignment. I also said I would only accept one submission for the entire class; they would have to come to a consensus on the submission.

$$\frac{(x+1)(x-1)^2}{\left((x+1)(x-1)\right)^2} \times \frac{2x(x+1)}{(x-1)x^{-1}} \div \left(\frac{(x-1)(x^2-1)}{x+1}\right)^2$$

Figure 3. Problem used for class extra credit.

After the students worked on the problem awhile independently and pairs, the pairs starting gathering in larger groups and checking their work. Those groups eventually all gathered around one table where the students took turns explaining their steps and defending their answers. At one point I heard a student who rarely volunteered to speak in front of the whole class say, "I'm still confused about this one spot. Explain it to me again. If you can convince me, it is probably correct." I could think of no better example of persevering with problem solving, constructing viable arguments, and critiquing the reasoning of others.

First, Look at Reasonableness.

I have always encouraged the students in my classes to think about the reasonableness of their answers. However, this teaching experience suggested a slightly different take on this strategy. Recall that one of the purposes for this course was to prepare the students to retake the ACT and gain the entrance scores needed for college level mathematics classes. To reach this goal, I studied the mathematics section of the ACT to better understand how to prepare the students. The mathematics section of the ACT features 60 problems to be completed in 60 minutes. Even for a mathematically literate person, it is difficult to work all of the arithmetic for all of these questions. However, upon further investigation, there were problems that did not need a lot of mathematical calculation in order to determine a correct answer. For example, the students might encounter a question such as "There are 600 school children in the Lakeville district. If 54 of them are high school seniors, what is the percentage of high school seniors in the Lakeville district?" and be offered the possible answers of (a) 0.9%, (b) 2.32%, (c) 9%, (d) 11%, and (e) 90%. By first thinking about an estimate such as 60 being 10% of 600, the students realize that the only answer that make sense is 9%. The student is then able to avoid doing unnecessary calculations by first considering what answers would be reasonable.

I eventually introduced a problem solving routine for this class that always began with the students considering what answers would be reasonable. This subtle switch from thinking about reasonableness first, instead of after solving, seemed to make all the difference. Not only did it prompt students to begin a problem from mindful thinking, it helped them remain in this mindful state throughout their solution. Also, when they were prompted to consider this question first, they were less apt to forget to contemplate the reasonableness of their solution after arriving at an answer.

Sometimes I posed problems that focused on developing tools for evaluating reasonableness of answers. Here are some problems of that nature:

1. For each of the following, decide, without solving, if the answer would be less than, equal to, or greater than one.
   a. $\frac{2}{3}+\frac{3}{5}$
   b. $3\frac{3}{10}-2\frac{1}{3}$
   c. $\frac{4}{7}\times 2\frac{1}{2}$
   d. $\frac{4}{3}\div\frac{7}{6}$

2. Decide which of the following values is closest to $\frac{17}{48}$. *Select one.*
   a. 0.27
   b. 0.34
   c. 0.55
   d. 0.625

3. Which is closest to 5% of 1230?
   a. 6
   b. 12
   c. 60
   d. 120
   e. 600

Other times, I directly asked students to show how they would estimate the answer prior to solving.

Building From What You Know

Early on in this course, it became evident that many students felt excluded from mathematics because they could not remember what they were supposed to remember (multiplication facts, fraction to decimal conversions, the equation of a line, the formula for slope, the quadratic formula, and the definition of a function). I also noticed that things the students thought they remembered often were incorrect. One very informative example happened the second day of class when I asked students to complete an assessment where they were asked decimal conversions of unit fractions. One of the students explained that he already knew these because his teacher last year made him memorize these. I asked which he remembered. Mixed in with several correct conversions were statements suggesting 1/8 was equal to 0.0625 and 1/6 was equivalent to 0.6666 (repeating). When I inquired if those values made sense to him, he said, "Sure, I memorized them."

This was extremely enlightening to me. I have always found unit fraction to decimal conversions to be extremely useful for estimating the value of fractions. In my mathematics for elementary classes, I have always asked the students to memorize the unit fraction to decimal conversions for 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/9, 1/10, 1/11, and 1/20. While I did not intend them to memorize the conversions without regard to the size of the fractions and relationships between the fractions, this is what often happened. I learned that I needed to model the learning of these facts through the use of their structures and by building from the facts they knew. Now, several times throughout the semester, I have students practice explaining how they can construct the decimal conversions for fractions by working from the knowledge that 1/2 = 0.5, 1/3 = $0.\overline{3}$, and 1/10 = 0.1. The following captures some of their ideas.

- 1/4 is half of 1/2. It is also half of 50 cents. It is also the value of a quarter. Therefore ¼ is 0.25.
- 1/5 is twice as big as 1/10 so it is 0.20.
- 1/6 is half of 1/3. Because half of $0.\overline{3}$ is somewhat hard to think about, one student suggested thinking of a little more than half of 0.32. This brings us to $0.1\overline{6}$.
- 1/8 is half of 1/4. Instead of thinking about half of 0.25, someone suggested thinking about half of 0.250. Therefore, 1/8 is 0.125.
- 1/9 is one-third of 1/3 so 1/9 is $0.\overline{1}$.
- 1/11 is a little less than 1/10. Also, 11 of them need to equal 1.00. We must multiply 11 by slightly more than 9 to get 100. So 1/11 is $0.\overline{09}$.
- 1/20 is half as big as 1/10 so 1/20 is 0.05.
-

This idea of building what we do not know from what we know reemerged throughout the course. When trying to determine 6 x 8, students learned that they could build from 5 x 8 by adding another set of 8. Calculating 70% of 40 became simple when the students began with the recognition that 10% of 40 is 4 and 70% would be just 7 times greater. When trying to recall the formula for the vertex of a quadratic, students recognized that the vertex was half way between the *x*-intercepts; the quadratic formula held the formula for the vertex.

Conclusion

The first few months of this teaching experiment were some of the most exhausting days of teaching I have ever experienced. The students in this course did not come preprogrammed with the eight standards of mathematical practice. They did not understand the value of perseverance in problem solving, lacked the conceptual understanding needed to see application of mathematics outside the classroom, were stuck in rule based thinking that inhibited active thinking, struggled to converse with their peers about their insights into a problem, and did not know how to use the structures and patterns inherent in mathematics problems to find solutions. However, even though the students did not begin the school year using the standards of mathematical practice, they grew tremendously in their ability to engage with real mathematics. By the end of the semester, they were debating the value of a product when all its factors had been removed, deriving the quadratic formula, and describing their estimation techniques to determine reasonable answers. I do truly believe, however, that I am the one that learned the most over the course of the year. To all that helped me learn these lessons, thank you!

References

Chan, J., & Stern, E. (2016). Putting your best foot forward: Movement & mathematics in college. In E. Torrence, B. Torrenence, C. S'equin, D. McKenna, K. Fenyvesi, R. Sarhangi (Eds.). *Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture.* pp. 641-648. Phoenix, Arizona.

Foord, D. (n.d.). Countdown timers for PowerPoint. Creative Commons Attribution-NonCommerical 3.0 Unported License. Retrieved from www.a6training.co.uk/resources/ImprovedPowerPointTimers.pptx on March 24, 2017.

Lochhead, J., and Whimbey, A. (1987). Teaching analytical reasoning through thinking aloud pair problem solving. *New Directions for Teaching and Learning, 30,* 73-92.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010).*Common Core State Standards for Mathematics.* Washington, DC: Authors.

Math Intervention (TMI): An Intervention Program for Struggling Secondary Students

Cody Reutzel
VP at Stansbury High School

During the summer of 2015, I was analyzing our Secondary Math I SAGE results. With the student data sorted from lowest to highest, two things struck me. First, even among our lowest performing students, there was usually one reporting category that was significantly lower than the others. Second, rich in data, I wondered what we were doing as a school to help these students target and build the specific skills they were lacking. As professional educators, it is our responsibility to build response systems to target student skill deficiencies, specifically students at the Tier 3 level. Tier 3, meaning students who have experienced Tier 1 and 2 instruction throughout a prior course and still exhibit significant deficits. Passing students onto the next grade level albeit with some amount of in class support and then hoping they gain these much needed skills later is too frequent a practice in many schools. Tier 3 solutions need to be developed to meet the needs of all students. At Uintah High School, that is what we set out to do. In this article, I will first outline our general goals for the program. I will then share the results that have generated so much excitement and a greater resolve to continue our commitment to the refinement of this Tier 3 intervention program. A summary of the concepts and research that guided the development will then follow. Finally, I will describe the specific details and components of the program. I believe this program and the subsequent results can be replicated by other secondary schools who desire to meet the mathematical learning needs of their own students.

Program Goals

This program, which we have title *Targeted Math Intervention* (TMI), was primarily developed by me and Niccole Franc, a devoted and talented math educator at Uintah High School. It also required the support and collaboration of many others including our high school math teachers on the Math I team and Julie Wilde, principal of Uintah High School. Our intent was to design a program to intensively target specific math skills using a highly individualized approach. As this was a new program, we really didn't have a clear expectation for student growth. Our goal in the first year was to, utilizing research-based strategies, implement the best program we could in order to actively learn and refine the program through data analysis, discussion, collaboration, and student feedback with the intent of improved future iterations. After the first implementation, we believed we would have baseline data to drive future adjustment and improvement.

Results of the TMI Mathematics Tier 3 Intervention Program

Our primary tool to measure the student impact of TMI was the SAGE summative assessment. We used SAGE data from the year prior to and the year following implementation of

TMI to measure student growth. Raw SAGE Summative scores from 2016 (pre-TMI, following Math I) were compared to 2017 (post-TMI, following Math II) for each student in the respective reporting category that was targeted and studied. We had students targeting the reporting categories of Algebra, Functions, or Geometry. The scores of the students targeting each reporting category were averaged and compared to the average score of their peers at Uintah High School who did not participate in the TMI program. The difference between the average score of the TMI students and their peers on the same test and in the same reporting category is labeled "School Gap" (Tables 1-3). Using the same procedure, the TMI students were compared to their peers across the state of Utah, labeled "State Gap." The purpose of this approach was to establish the growth of the TMI students relative to their peers. One potential concern in only measuring growth of the TMI students was if the SAGE assessment became easier from 2016 to 2017, the growth could have simply been a function of all students achieving significant gains. By calculating the average score for all students in 2016 and 2017, this concern is dispelled. Even when school and state average scores rose rapidly from 2016 to 2017 (as was the case with the Algebra reporting category), TMI student scores rose sharply enough to not only keep growth pace with their peers, but to accelerate growth as compared with their peers. The School Gap and State Gap are shown for 2016 and 2017, illustrating a reduction in the gap between the TMI students and their peers of 78 points (School Gap) and 73 points (State Gap) in Algebra (Table 1), 170 points (School Gap) and 150 points (State Gap) in Functions (Table 2), and 128 points (School Gap) and 113 (State Gap) in Geometry (Table 3). Additionally, the difference between the TMI students and their peers across the state (State Gap) was reduced by 45% in Algebra, 84% in Functions, and 69.7% in Geometry from 2016 to 2017. Table 4 shows the range of proficiency cut scores for Math II. Notice that the gap reduction for both school and state comparison for each reporting category is similar to, if not larger than, the range of one proficiency level. One obvious limitation of this data is the small number of students in the Functions (*n*=6) and Geometry (*n*=8) groups.

Table 1 – Algebra Averages

Reporting Category: Algebra	**Math I - 2016**	**Math II - 2017**	**Change**
TMI Students (*n=29*)	327	447	+120
School	484	526	+42
School Gap (School - TMI)	156	78	-78
State	488	535	+47
State Gap (State - TMI)	160	87	-73

Table 2 – Functions Averages

Reporting Category: Functions	**Math I - 2016**	**Math II - 2017**	**Change**
TMI Students (*n=6*)	317	495	+178
School	498	506	+8

School Gap (School - TMI)	180	10	-170
State	495	523	+28
State Gap (State - TMI)	177	27	-150

Table 3 – Geometry Averages

Reporting Category: Geometry	**Math I - 2016**	**Math II - 2017**	**Change**
TMI Students (*n=8*)	334	481	+147
School	525	544	+19
School Gap (School - TMI)	191	63	-128
State	496	530	+34
State Gap (State - TMI)	162	49	-113

Table 4 – Proficiency Cut Scores

Math II - 2017	**Proficiency Level**
<507	1
507-583	2
584-647	3
>647	4

Foundational Concepts of the Targeted Math Intervention (TMI) Program

The following concepts and research-based practices guided our design of the TMI program. Each practice was integral to the implementation and future successful replication of this program.

1. Teacher Efficacy

It is no secret that teacher beliefs and expectations have an immense impact on student motivation and achievement. John A. Ross (1994) defines teacher efficacy as, "the extent to which teachers believe their efforts will have a positive effect on student achievement" (p. 3). Eells (2011), Goddard, Hoy, and Hoy (2000) and Bandura (1993) have conducted extensive research that indicates that teacher efficacy is systematically connected to student achievement. My foremost task in implementing TMI was to enlist a teacher who had exhibited high teacher efficacy as well a willingness to try innovative ideas. In fact, finding a teacher willing to implement this new program may indicate that they possess a higher level of teacher efficacy, as research reveals an association between trying innovative teaching ideas and high teacher efficacy (Ross, 1994, p.2). Knowing this, I sought out an effective teacher with high teacher efficacy and who exhibited excitement about innovation in math education. In discussing with colleagues, we reasoned that the right teacher with the right attitude and expectations would undoubtedly yield invaluable gains, specifically in consideration of the student demographic served by this program.

2. Growth Mindset

Teacher belief, closely related to teacher efficacy, is a confidence and expectation that all students can learn. Inherent in the implementation of a Tier 3 intervention program is the unwavering belief that student skills and abilities are not fixed. Psychologist Carol Dweck (Great Schools Partnership, 2013), a leader in the concept of growth mindset, describes the presence of growth mindset as when, "people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point." I was adamant that the teacher as well as the students must have a growth mindset for this program to be successful. The teacher must be guided by this mindset in all student interactions and also work to cultivate this belief among her students. At the core of a growth mindset is the type of praise and feedback given to students by the teacher. Students who receive comfort feedback, essentially helping students to feel comfortable about not being "good" at something, demonstrate considerably less motivation and lower expectations than students who receive strategy feedback, which provides students with specific information about strategies to use and how the teacher will support the student as they progress (Rattan, Good, and Dweck, 2011).

3. Student Self-Monitoring and Feedback

Students need to be active participants in monitoring their own progress in relation to learning targets. Students who do so, experience increased academic achievement. I wanted to ensure that this would happen in a systematic way. The teacher has a responsibility then, to clearly communicate the learning target, what success looks like, and provide feedback to the student about their progress and also how to accomplish the task or target. Time and opportunity must be provided for students to track, document, and reflect on their progress. Frequent informal and formal formative assessment plays a key role in ensuring that both the student and teacher are aware of current progress. When skills are mastered, as informed by these checks for understanding, students must have the opportunity to move on to new units of study. This concept combines two of Professor John Hattie's highest rated educational practices together, teacher clarity and feedback, demonstrating average effect sizes of .75 and .73 respectively. Teacher clarity is defined by Hattie (2009) as, "communicating the intentions of the lessons and the notions of what success means for these intentions" (p. 126). Hattie's notion of feedback is widely known by the idea of "feed up, feed back, and feed forward," meaning that teachers need to continually answer the following questions for their students: "Where am I going? How am I going? Where to next?" (Waack, n.d.).

4. Blended Instruction

Horn, Staker, and Christensen (2015) have written extensively on the topic of blended learning and offer the following definition, "any formal education program in which a student learns at least in part through online learning, with some element of student control over time, place, path, and/or pace" (p. 34). Students develop gaps in their knowledge at various locations along their learning path. Identifying those gaps, providing content specific to those needs, accelerating through concepts already mastered, and extending time on concepts not yet understood can be a daunting, specifically at scale. Horn et al. suggest, "allowing all students to progress in their learning as they master material may be possible in a school with a small student-to-teacher ratio and flexible groupings, but it is taxing on an individual teacher who has to provide new learning experiences for students who move beyond the scope of a course" (p. 10). Advances in online instruction offer a promising solution to these needs when properly utilized and as a support to live instruction. Technological solutions can extend the capability of a single instructor to provide individualized content in ways that catalyze student learning. As Horn and Staker explain, "at its most basic level, it lets students fast-forward if they have already mastered a concept, pause if they need to digest something, or rewind and slow something down if they need to review (p. 10)." Not with the purpose of displacing live instruction, but with the intention of working in conjunction, online learning can serve a valuable function. Quality online instruction can occur in scenarios where, "teachers serve as professional learning coaches and content architects to help individual students progress—and they can be a guide on the side, not a sage on the stage" (Christensen, Horn, & Johnson, 2011, p. 39).

5. "Know your impact" and Adjust

The first of John Hattie's "8 Mind Frames" (John Hattie's Eight Mind Frames for Teachers, 2014) is, "My fundamental task is to evaluate the effect of my teaching on students' learning and achievement." Without consistent, grounded methods to evaluate the effect of a program, we as educational professionals are left to make decisions based on instinct and other anecdotal evidence. While certainly important to the everyday classroom, the task of knowing the impact of instruction could not be more relevant than for implementing a Tier 3 instructional program. Annual summative assessments and pre/post-test cycles that measure growth as opposed to simple achievement are a key element to understanding student impact. In addition, student surveys about teacher effectiveness, online instruction effectiveness, classroom climate, and availability of helpful and timely assistance offer insightful information. Indeed, adjusting to and improving in response to information gathered is the fundamental objective.

Program Description

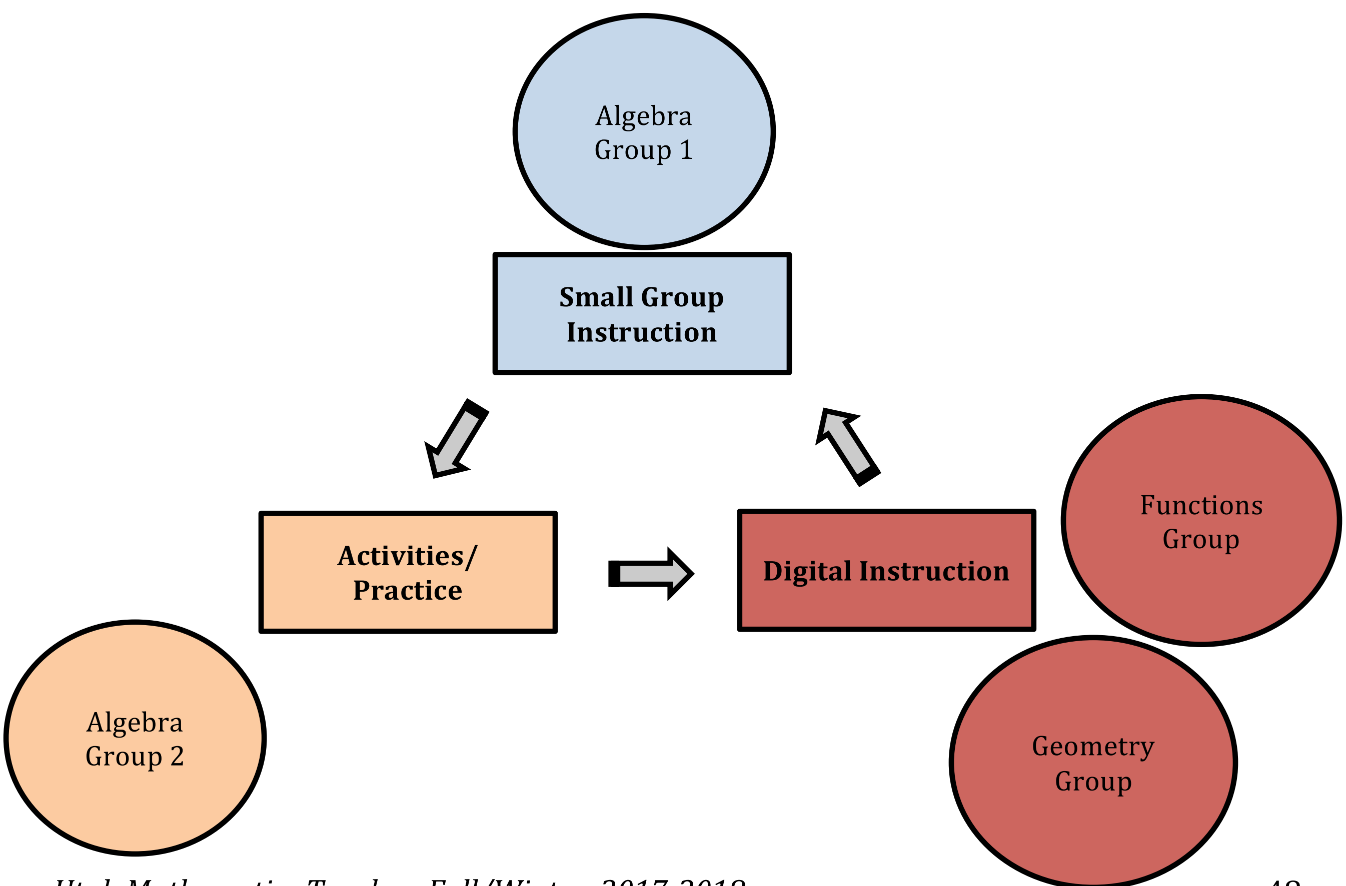
The TMI program we designed is a blended rotation model of instruction. A maximum of 40 students are grouped with other students who had deficiencies in the same reporting category on the Secondary Math I SAGE Summative assessment. No more than 10 in a group. Each group rotates between small group instruction, non-computer based activities/practice, and digital instruction and practice. Small group instruction is a station where students participate in a lesson specific to their (group) needs with a live instructor. The instructor must use formative assessment from all activities in the classroom to identify the highest leverage topics to be taught to each respective group. The non-computer based activities and practice are an extension of the small group instruction. Students apply the information taught in small group instruction to activities either individually or with peers to reinforce the content and skills being learned. The digital instruction and practice station involves students using an online individualized math program to master content and skills specific to their deficiencies. Students who experience difficulty with the digital content are occasionally pulled out for one-on-one instruction.

There are many programs that can be used for the digital instruction station, including Compass Learning, ALEKS, Mathspace, etc. We used Compass Learning for this implementation, but based on student feedback, we will be transitioning to Mathspace in the next iteration. While the choice of the specific digital program does have an impact, any program that is interactive, has the ability to create custom learning paths, offers tools to monitor student progress, provides continual formative assessment, includes solutions for students who don't get it on the first try, and is understood by the teacher, can be effective. I recommend collaborating with a school level team to select a digital program that best meets the needs of your students and teachers.

The following is a class schedule based on a 60-minute class period. On any given day, each group will cycle through three different stations. As "Digital Instruction" is assigned the value of two stations (15 minutes x2), some groups will spend 2 rotations at that station on certain days. Figure 1 is a visual representation of the rotation between stations during station rotation, the largest portion of the class period. There are two "Algebra" groups shown because we had enough students targeting that reporting category that it became necessary to split them into two groups.

8:00-8:09	**"Bell Ringer" activity related to a high leverage math topic**
8:10-8:55	**Station rotation, three 15 minute rotations per day (Figure 1)**
8:56-9:00	**Student reflect/review/document progress**

Figure 1 – Station Rotation



This program can be implemented using two teachers or one teacher and one aide. One teacher is the primary manager of the classroom. The primary teacher is responsible for what we have termed the "front end." The front end is comprised of the small group instruction and non-computer based activities/practice stations.

Front End Responsibilities

- Identify high leverage topics for small group instruction (based on continuous formative assessment)
- Develop and teach small group lessons
- Develop non-computer based activities/practice
- Supervise all students not in Digital Instruction station

As the primary manager, this teacher is also responsible for the general atmosphere of the classroom (including integration of "growth mindset"), developing and teaching high leverage bell ringer activities that will help all students, and ensuring the daily and rotation schedule is implemented with fidelity.

The second teacher/aide is responsible for the "back end," the area where students participate in digital instruction and practice.

Back End Responsibilities

- Supervise students in the Digital Instruction station
- Circulate and assist to ensure students are on task, not "stuck", and engaged
- Occasionally pull-out students who are struggling severely with a concept or task for individual help
- Maintain a system to track the Digital Instruction progress (daily and weekly) of individual students
- Conduct weekly interviews with each student to lead them in self-monitoring/tracking/reporting progress and reflecting on their effort and performance
- Communicate student progress, effort, and performance information to parents on a weekly basis

- Re-assign students who have shown mastery of a reporting category to the next category of need

Our program ran for 12 weeks as a one-trimester class. We utilized a committee made up of all administrators and counselors at UHS to select students for inclusion in the program. Students were selected by sorting all Secondary Math I SAGE Summative scores from 2016 in order from lowest to highest and identifying students at the top of the list. The criteria for inclusion were that the student did not have severe behavior or attendance problems, and was not currently receiving special education services. Each parent was contacted and provided with an explanation of the program, why their student was a candidate, how the program would help their student, and invited to enroll their student in the class. An emphasis was placed on the fact that the intent is learning recovery, not credit recovery, so elective credit would be earned, not math credit. With the permission of each parent, students were then enrolled in the class. It should be noted that almost every parent contacted was excited and grateful to know that our school knew and cared enough about their student to offer this program to them. The lowest reporting category score was identified for each student and students were then grouped accordingly so as to study only the specific content and skills they were in need of. In our case, many students had "Algebra" as their lowest reporting category, so multiple "Algebra" groups were created.

To assess student progress and program effectiveness, a pre-test and post-test were administered. In-class formative assessments were utilized in the digital instruction program and small group activities. At the conclusion of the class, students were able to provide feedback on both teachers' instruction and support, the format of the course, the digital instruction program, and how their confidence and skills had evolved as a result of the class. Needless to say, this information is extremely valuable in driving improvement of the program.

Conclusion

The need to help students by targeting carefully designed high intensity instruction to math skill deficiencies is well established. There are innumerable methods to attempt to help students remediate these deficiencies. The use of research-based concepts and instructional strategies is the most direct, effective and efficient approach. We are extremely motivated by the student growth we discovered as a result of our implementation of the TMI program. With this year of experience, student feedback, and knowledge of significant student impact, we have optimism and increased resolve that we can experience even higher levels of student growth with our next iteration as we work to refine the program and learn from our teachers and students who participated in the TMI program.

Bibliography

B. (2014, September 09). *John Hattie's Eight Mind Frames For Teachers.* Retrieved June 7, 2017, from https://visible-learning.org/2014/08/john-hattie-mind-frames-teachers/

B. (n.d.). *Glossary of Hattie's influences on student achievement.* Retrieved June 6, 2017, from https://visible-learning.org/glossary/

Bandura, A. (1993). Perceived Self-Efficacy in Cognitive Development and Functioning. *Educational Psychologist,28(2), 117-148.* doi:10.1207/s15326985ep2802_3

Christensen, C. M., Horn, M. B., & Johnson, C. W. (2011). *Disrupting Class: How Disruptive Innovation Will Change the Way the World Learns.* McGraw-Hill.

Eells, R. J., *Meta-Analysis of the Relationship Between Collective Teacher Efficacy and Student Achievement (2011). Dissertations. Paper 133.* http://ecommons.luc.edu/luc_diss/133

Great Schools Partnership (2013). *Growth mindset.* Retrieved from http://edglossary.org/growth-mindset/

Goddard, R. D., Hoy, W. K., & Hoy, A. W. (2000). Collective Teacher Efficacy: Its Meaning, Measure, and Impact on Student Achievement. *American Educational Research Journal, 37(2), 479*. doi:10.2307/1163531

Hattie, J. (2009). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement.* London: Routledge.

Horn, M. B., Staker, H., & Christensen, C. M. (2015). *Blended: using disruptive innovation to improve schools.* San Francisco: Jossey-Bass.

Rattan, A., Good, C., Dweck, C.S. (2011). "It's ok — Not everyone can be good at math": Instructors with an entity theory comfort (and demotivate) students. *Journal of Experimental Social Psychology.* http://scottbarrykaufman.com/wp-content/uploads/2012/02/Rattan-et-al.-2012.pdf

Ross, J. A., *Beliefs that make a difference: The origins and impacts of teacher efficacy (1994).* http://files.eric.ed.gov/fulltext/ED379216.pdf

UCTM Awards, 2017

Karl Jones Award - Elementary

Erika Knight

Wasatch County School District/Midway Elementary School

Erika is an exceptional math teacher, one of the very best in the district. Her math scores reflect her ability. Even more importantly, kids love her. Several times we have put kids who struggle with anxiety in her class who previously struggled to come to school. They do not have those struggles with Mrs. Knight. She is warm, fun, and creative. Kids feel safe in her room. It is no surprise that they learn there. In her first few years of teaching, the district was engaged in paradigmatic shifts in math education and Erika fully embraced learning. She consistently sought feedback and coaching to improve her instruction. Her classroom practices quickly became centered on student thinking and rich mathematical discourse. Students in her classroom love to learn math and consider themselves mathematicians. She has set a strong learning and thinking culture in her classroom, but her influence goes beyond. Erika willingly and masterfully mentored pre-service teachers in their university practicum experience. The pre-service teachers left Erika's classroom with deeper knowledge and skills in the why, what and how of math teaching. Erika is the team lead and masterfully guides her team in unpacking standards, selecting learning targets, creating common formative assessments that assess conceptual understanding in addition to procedural fluency and intervening to help all kids learn. Finally, Erika has been willing to support the ongoing district-wide initiative of math professional development in the Comprehensive Mathematics Instruction (CMI) framework. Erika is a model teacher of mathematics and shares her knowledge with others because she is genuinely committed to making math meaningful for students.

George Shell - Secondary

Mike Spencer

Juab School District

Mike is an exemplary teacher in every way. His students are successful. He is totally dedicated to them, often staying late into the evening or arriving early to give his students extra tutoring time. He single-handedly brought AP Stats to Juab. Actually, his AP statistics class is the first statistics class ever offered at Juab. He has worked hard to keep the bar for performance high while keeping the class accessible for all students. His pass rate for students taking the AP exam is very high.

Mike is the department chair at Juab High School. In that capacity he quietly leads by supporting his teachers as a team member. They work together on improving instruction and learning. Mike and his team have changed the grading perception at Juab. A student's math grade at Juab now represents only what a student knows. Both students and parents talk about what the student needs to know to get a grade rather than how many more assignments need to be turned in. He has shared the success of the new grading program by speaking about it at the UCTM conference each year.

Although Mike is the department head and the secondary math specialist for the district, he also teaches a full schedule, including 5 different preps. One of his classes is special ed, self-contained math. In that class he teaches the Secondary 1 math core and his students are succeeding at their grade level. He does task-based instruction in this class and is wonderfully kind and patient.

Mike is a leader in math education in the state. His was a finalist for the PAEMST award in 2015. He serves on the state math leadership committee, and he teaches professional development classes to other secondary math teachers nationally.

Randy Schelble Award – Special Education and Mathematics

The Randy Schelble Award is given in recognition of outstanding achievement in special education and mathematics. Teachers eligible for this award have exhibited outstanding work in special education and mathematics, an ability to ensure that all students learn at high levels, and a willingness to work closely with mathematics education teachers in the state of Utah.

Brenda Bates

Salt Lake City School District

Brenda Bates has been an influential leader in special education and mathematics education in the Salt Lake City School District for many years. After having taught in a variety of special education settings Brenda pioneered as an instructional coach for special education teachers under the leadership of Randy Schelble. During her years as an instructional coach Brenda has been an agent of change in supporting special education teachers in academic units with implementing grade level standards while having high expectations for each and every student. Brenda facilitates Professional Learning Communities for resource teachers and works hard to increase collaboration among special education and general education teachers. She works alongside mathematics coaches participating in and facilitating district-wide professional development and developing instructional resources for teacher and student use. Brenda is passionate about engaging students in productive struggle and encouraging students to use and connect multiple representations along with mathematical discourse. She is well-deserving of this award as she has exhibited outstanding work in special education and mathematics, an ability to ensure that all students learn at high levels, and a willingness to work closely with mathematics education teachers in the state of Utah.

Muffet Reeves- Teacher of Teachers

Travis Lemon

Alpine School District

Travis Lemon has literally done everything a teacher can do to provide success for his students, contribute to the professional development of the secondary math teachers in the state and in the nation, and in so doing bring recognition and honor to himself. Travis was the 2007 PAEMST award winner, that could have been enough but it wasn't. Travis then served on the UCTM board and then as the UCTM president. He is currently on the editorial panel for the NCTM journal, Teaching in the Middle School. This has all been accomplished while he teaches full time at American Fork Junior High School.
In addition to being a full time teacher, he also has been invited (annually) to speak to prospective teachers at Utah State University, Utah Valley University, the University of Utah, and Brigham Young University. When he is not encouraging college students to become math teachers, he is teaching professional development classes on how to implement task-based learning or promote meaningful mathematical discourse.

Every year for the last several years Travis has been invited to speak at the NCSM national conference, the NCTM national conference, and the UCTM state conference. This year he is speaking to the nation's teachers about the coaching cycle. In addition, he has worked with Achieve.org, a national group dedicated to improving education for all students. As part of his work with Achieve, he has been involved with EQuIP (Educators Evaluating the Quality of Instructional Products), an initiative designed to identify high-quality materials aligned to the Common Core State Standards (CCSS).

Travis is also one of the authors and partners of the Mathematics Vision Project (MVP). This project has provided a free secondary math curriculum for many states across the nation. In his work with MVP he has supported and coached hundreds of secondary math teachers.

Don Clark- Lifetime Achievement

Gary Turner

Wasatch County School District - Wasatch High School

There is not an educator more gifted in helping students learn than Gary Turner. He is an icon at Wasatch High School and in the entire community. Over the last thirty-eight years Mr. Turner has taught and tutored countless high school and college students as well as adults seeking to sharpen their skills.
Mr. Turner has numerous strategies that he expertly employs to help students understand even the most difficult math concepts. Students not enrolled in his math classes are graciously helped before and after school along with his own math students. For several years, Mr. Turner has offered night tutoring twice a week in our school library. Students flock to the library to receive his expert instruction. It is well known that Gary Turner can work his magic and find a way to help even the most struggling student grasp a difficult concept.

Between classes one can find Mr. Turner waiting outside his door to greet students with a smile and a conversation. During class Mr. Turner is patient, kind, and funny. He has high expectations for students and treats all with kindness. Mr. Turner is interested in the lives of his students and colleagues. He attends plays, concerts, soccer and football games and countless other extracurricular events to support his students. He encourages his students to take advantage of every learning opportunity they can. For many years Mr. Turner has been our Academic Decathlon adviser and has continuously coached a successful team. Students thoroughly enjoy the extra time they get to spend with their beloved teacher.

The faculty of Wasatch High School looks to Mr. Turner as a leader. They seek for his advice on everything from teaching strategies to having difficult conversations with parents. He approaches even the most difficult problems with a calm demeanor and wisdom. He is constantly searching for new and better ways to help his students. Mr. Turner videos and posts each of his lessons for each of his courses on the web so that students can review any particular concept at any time. This is just one example of the many extra ways Mr. Turner goes above and beyond.

It is impossible to think of math in Heber Valley without Gary Turner coming to mind; he has established a legacy built upon his love for math and students. Mr. Turner is an outstanding math educator who is very deserving of becoming the recipient of the Don Clark Lifetime Achievement Award.

Print

Introducing NCTM's INNOV8 Conference!

Las Vegas, NV | Wednesday, November 15–Friday, November 17, 2017
Breaking Barriers: Actionable Approaches to Reach Each and Every Learner in Mathematics

Innov8 provides opportunities for attendees to:

- Receive hands-on experience implementing research-based mathematics education practices
- Connect with like-minded teachers facing similar problems of practice and collaborate to determine effective solutions to advance student learning
- Return to the classroom, school or district with an action plan and commitment to implement refreshed tools and techniques

For more information please visit the Innov8 page.

Print

See You in Orlando and Chicago!

NCTM's Regional Conferences & Expositions are a great way to interact with thousands of classroom teachers and decision makers in the mathematics education field. Attendance in the past have averaged over 2,500 attendees making exhibiting an easy and effective way to get great exposure for your company's name and products. Maximize your exposure and have your company listing included in the printed conference program books! Deadlines are 5:00 p.m. EST on Thursday, August 3rd (Orlando) and 5:00 p.m. EST on Friday, August 25th (Chicago).